A Numerical Solution to the Paraxial Wave Equation in COMSOL

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INTRODUCTION: The Paraxial Wave Equation (PWE) governs how laser light travels through space. In this study we solved an IBVP using COMSOL and the Galerkin methods and compared the solutions to the analytical free space solution for the Gaussian beam. In COMSOL, we used the coefficient form PDE module with a BDF solver and for the Galerkin method we used a Bessel eigenfunction expansion. The variables displayed in all the graphs are non-dimensional. In addition, the propagation distance z plays the role of time in this simulation.

$$\begin{cases} \Delta \bar{V}_1 - 4M\dfrac{\partial \bar{V}_2}{\partial \bar{z}} = 0 \\ \Delta \bar{V}_2 + 4M\dfrac{\partial \bar{V}_1}{\partial \bar{z}} = 0 \end{cases} \quad \bar{V} = \bar{V}_1 + \bar{V}_2 i, \quad \bar{V}_1|_{\partial\Omega} = \bar{V}_2|_{\partial\Omega} = 0,$$

$$\begin{cases} \bar{V}_1(\bar{x},\bar{y},0) = \exp(-\bar{r}^2)\cos(M\bar{r}^2)g(\bar{r}), \\ \bar{V}_2(\bar{x},\bar{y},0) = -\exp(-\bar{r}^2)\sin(M\bar{r}^2)g(\bar{r}). \end{cases}$$

$$g(\bar{r}) = \frac{1}{2}(1+\cos[\pi(\frac{\bar{r}}{a})^6]) \qquad M = \pi\frac{W_0^2}{\lambda F_0}$$

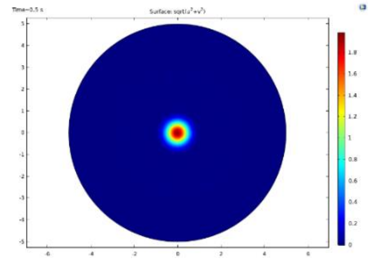

Figure 1. Gaussian Beam at z=0.5 (250m) in COMSOL

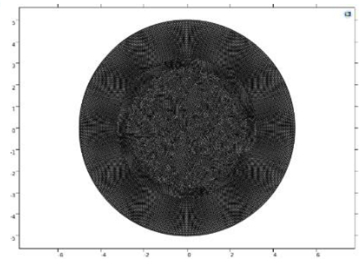

Figure 2. COMSOL 2D disk mesh with 25970 triangular elements and 104512 degrees of freedom

COMSOL Results: F0=500m, W0=3cm, lambda=633nm at beam center (0,0)

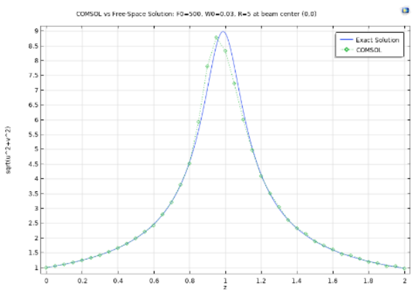

Figure 3. Intensity $(V1^2+V2^2)^{1/2}$ at beam center along propagation axis z

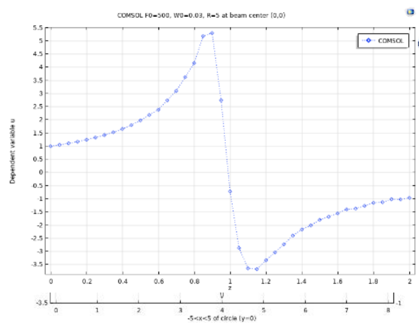

Figure 4. Real part of the solution, V1

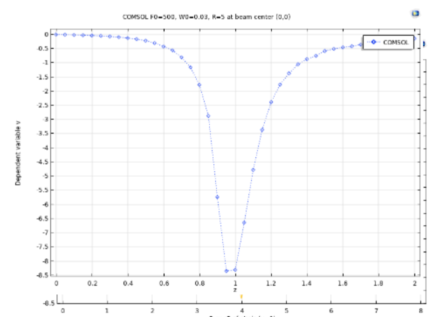

Figure 5. Imaginary part of the solution, V2

Galerkin Method Results:

Bessel eigenfunction expansion: $\bar{V}_1(\bar{x},\bar{y},\bar{z}) = \sum^{\infty}(b_n \cos\frac{\lambda_n^2}{4M}\bar{z} + c_n \sin\frac{\lambda_n^2}{4M}\bar{z})\, J_0(\lambda_n \bar{r})$

$$b_n = \frac{1}{C_n}\int_0^a \bar{V}_1(r,0)J_0(\lambda_n r)rdr, \quad c_n = \frac{1}{\lambda_n^2 C_n}\int_0^a \Delta(\bar{V}_2(r,0))J_0(\lambda_n r)rdr, \quad C_n = \int_0^a J_0(\lambda_n r)^2 rdr, \quad \lambda_n = \frac{j_{0,n}}{a}, \quad n = 1,2,\ldots$$

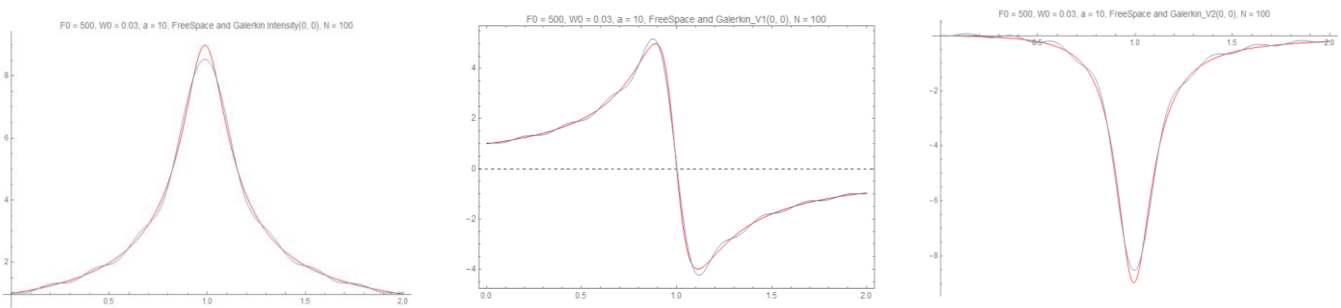

Figure 6: Galerkin Method solution (blue) at compared to analytical solution (red) for the intensity, real, and imaginary solutions.

CONCLUSIONS: At the beam center, we observed an offset between the analytical free space solution and the COMSOL numerical solution, especially around the beam's peak intensity. Also, as the beam moves away from its axis of propagation, we observed larger deviation from the free-space solution and the solution of the boundary value problem. The Galerkin Method solution has a similar deviation from the free-space analytical solution.

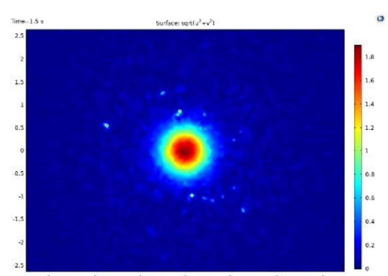

Figure 7: Beam profile at z=1.5 (750m)

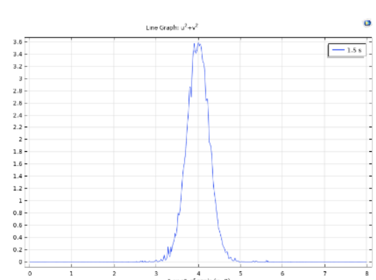

Figure 8: Beam intensity cross section at z=1.5 (750m) from -5<x<5 and y=0

REFERENCES:

1. Andrews & Phillips, "Laser Beam Propagation through Random Media", 2005, SPIE, San Francisco.