

# COMSOL Implementation for Two-Phase Immiscible Flows in Layered Reservoir

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# Outline

- Introduction
- Implementation
- Results
- Conclusion

# Introduction

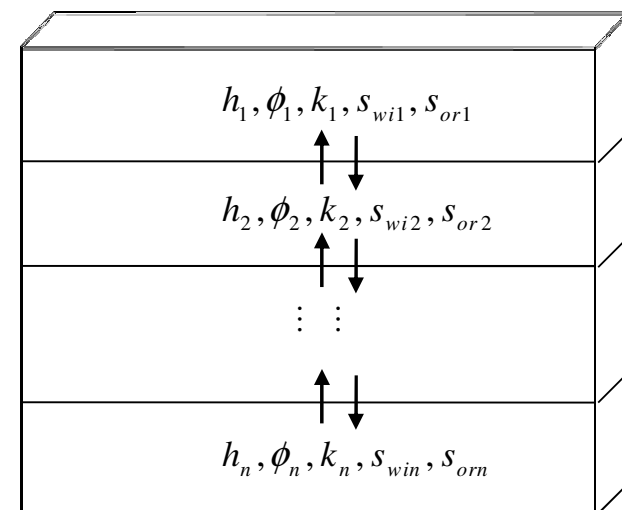
- Waterflooding
- Layered model of reservoir
- Non-/Communicating layers
- Anisotropy parameter

$$E = k_y x_0^2 / k_x y_0^2$$

$x_0$  :Length

$y_0$  :Height

$k$  :Permeability



Scheme of layered reservoir

# Introduction

- Mass conservation

$$\phi \frac{\partial s}{\partial T} + \frac{\partial U_X F(s)}{\partial X} + \frac{\partial U_Y F(s)}{\partial Y} = 0$$

U: total flow velocity

F: fractional flow of water

$\Phi$ : porosity

Sw: water saturation

- Incompressible flow

$$\frac{\partial U_X}{\partial X} + \frac{\partial U_Y}{\partial Y} = 0$$

- Darcy's law

$$U_X = -\Lambda_X \cdot \frac{\partial P}{\partial X} \quad U_Y = -E\Lambda_Y \cdot \frac{\partial P}{\partial Y}$$

$\Lambda$ : mobility

$$E = k_y x_0^2 / k_x y_0^2 \quad - \text{anisotropy ratio}$$

# Implementation in COMSOL

$$\frac{\partial U_X}{\partial X} + \frac{\partial U_Y}{\partial Y} = 0$$

$$\phi \frac{\partial s}{\partial T} + \frac{\partial U_X F(s)}{\partial X} + \frac{\partial U_Y F(s)}{\partial Y} = 0$$

$$U_X = -\Lambda_X \cdot \frac{\partial P}{\partial X}$$

$$U_Y = -E\Lambda_Y \cdot \frac{\partial P}{\partial Y}$$

The screenshot shows the COMSOL model tree on the right side of the interface. The tree is expanded to show the following structure:

- 2D con...
  - Constants
  - Global Expressions
  - Functions
  - Global Equations
  - Geom1
    - Scalar Variables
    - PDE, General Form (pres\_velo)**
      - Subdomain Settings
      - Boundary Settings
      - Point Settings
    - PDE, Coefficient Form (saturation)**
      - Subdomain Settings
      - Boundary Settings
      - Point Settings
    - Expressions
      - Scalar Expressions**
      - Subdomain Expressions
      - Boundary Expressions
      - Point Expressions
      - Interior Mesh Boundary Expressions
    - Equation System
    - Coupling Variables

Below the tree, the 'Scalar Expressions' window is visible, containing the following definitions:

```

krw: krwor1*(1-swi1-sor1)^(-2)*(ss-swi1)^2*(y<=ARFA1)+krwor2*(1-swi2-sor
kro: krow1*(1-swi1-sor1)^(-2)*(1-ss-sor1)^2*(y<=ARFA1)+krow2*(1-swi2-sr
F: krw/meu_water/(krw/meu_water+kro/meu_oil)
lamda: krw/meu_water+kro/meu_oil
U: -k1_x*lamda*px*(y<=ARFA1)+(-k2_x*lamda*px)*(y>ARFA1)
Tpvi: Q*t/PHI
X: x/L
kesi: X/Tpvi
V: -E*k1_y*lamda*py*(y<=ARFA1)+(-E*k2_y*lamda*py)*(y>ARFA1)
    
```

Red arrows in the original image point from the mathematical equations to the corresponding settings in the software interface:


- The first equation  $\frac{\partial U_X}{\partial X} + \frac{\partial U_Y}{\partial Y} = 0$  points to the 'PDE, General Form (pres\_velo)' node.
- The second equation  $\phi \frac{\partial s}{\partial T} + \frac{\partial U_X F(s)}{\partial X} + \frac{\partial U_Y F(s)}{\partial Y} = 0$  points to the 'PDE, Coefficient Form (saturation)' node.
- The third equation  $U_X = -\Lambda_X \cdot \frac{\partial P}{\partial X}$  points to the 'U' expression in the 'Scalar Expressions' window.
- The fourth equation  $U_Y = -E\Lambda_Y \cdot \frac{\partial P}{\partial Y}$  points to the 'V' expression in the 'Scalar Expressions' window.

# Implementation in COMSOL

- Initial conditions

$$s(t_0) = s_{wi} \quad P(t_0) = P_0$$

- Boundary conditions



A light blue rectangular domain is shown with boundary conditions specified on each side:

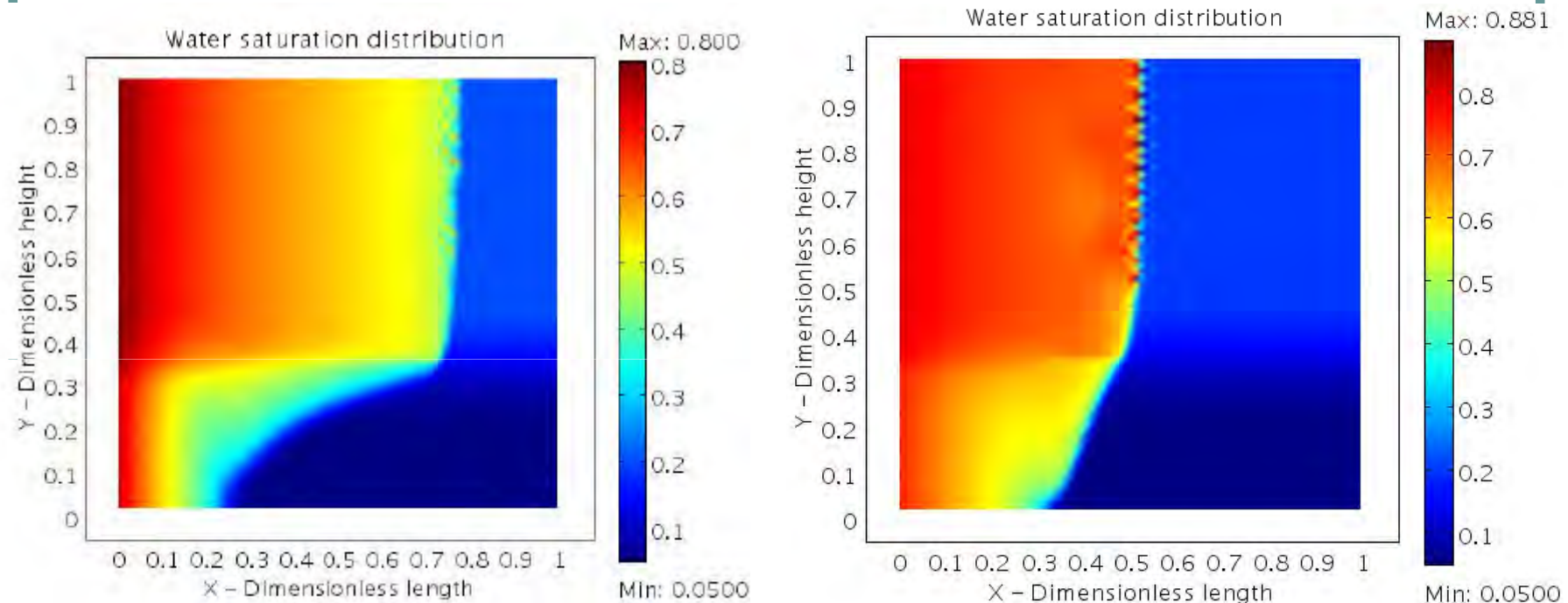
- Top boundary:  $-n \cdot \Gamma = 0$  (red text)
- Bottom boundary:  $-n \cdot \Gamma = 0$  (red text)
- Left boundary:  $s_{in} = 1 - s_{or}$  (red text) and  $-n \cdot \Gamma = Q$  (black text)
- Right boundary:  $s_{out} = s_{wi}$  (red text) and  $P_{out} = P_t$  (black text)

# Results-2 layers

Dimensionless parameters	Layer 1	Layer 2
Fraction of thickness $H$	0.33	0.67
Irreducible water saturation $S_{wi}$	0.05	0.2
Residual oil saturation $S_{or}$	0.25	0.2
Relative water permeability at residual oil saturation $k_{wor}^*$	0.8 (0.4)	0.8 (0.4)
Relative oil permeability at irreducible water saturation $k_{owi}^*$	0.8	0.8
Dimensionless permeability in X-direction $K_X$	0.33	0.67
Dimensionless permeability in Y-direction $K_Y$	0.33	0.67
Dimensionless porosity $\Phi$	1	
Viscosity ratio of water to oil $\mu_w / \mu_o$	1:3 (1:1.5)	
Anisotropy ratio $E$	1000	
Dimensionless injection rate $Q$	1	

$$M = \frac{k_{owi}^* \mu_w}{k_{wor}^* \mu_o}$$

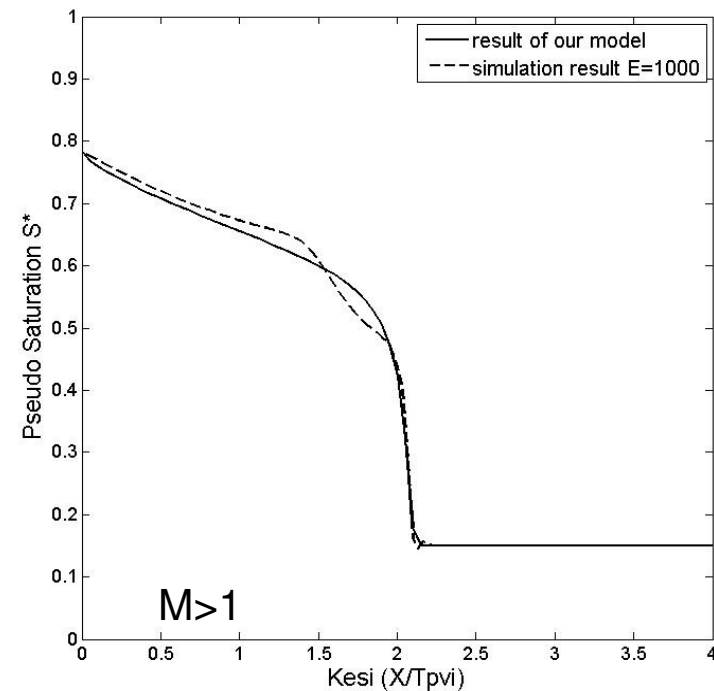
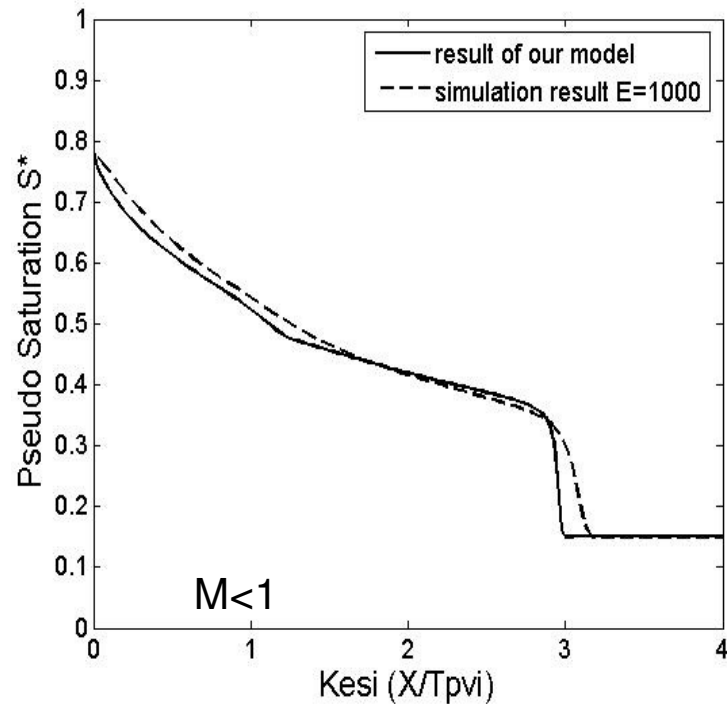
# Results- 2 layers



Water saturation profile of 2D waterflooding simulation in COMSOL, at time=0.25 PVI (Pore Volume Injected). Left:  $M < 1$ , right:  $M > 1$



# Comparison with analytical solution



Average water saturation profile of 2D waterflooding, at time=0.25 PVI (Pore Volume Injected). Left:  $M < 1$ , right:  $M > 1$

# Analytical derivation

## Anisotropy parameter $E$

- Small  $E$  – Poorly communicating layers
- Large  $E$  – Well communicating layers

Asymptotic approximation – assumption for perfectly communicating layers:

**$E$  tends to infinity !**

Consequence: *Pressure gradient across the layers is negligibly small!*

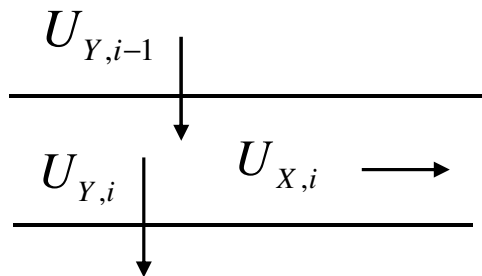
# Analytical derivation

- Pressure P is constant in y-direction



For the ith layer

$$\Phi_i \frac{\partial s_i}{\partial T} + \frac{\partial F_i \bar{U}_{Xi}}{\partial X} + \frac{(F\bar{U}_Y)_{i-1} - (F\bar{U}_Y)_i}{\alpha_i} = 0$$



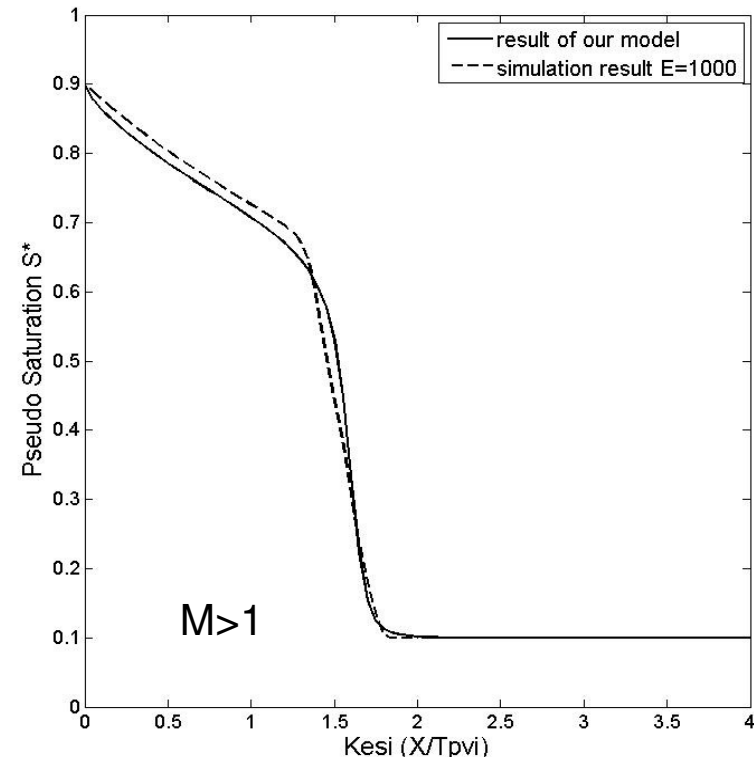
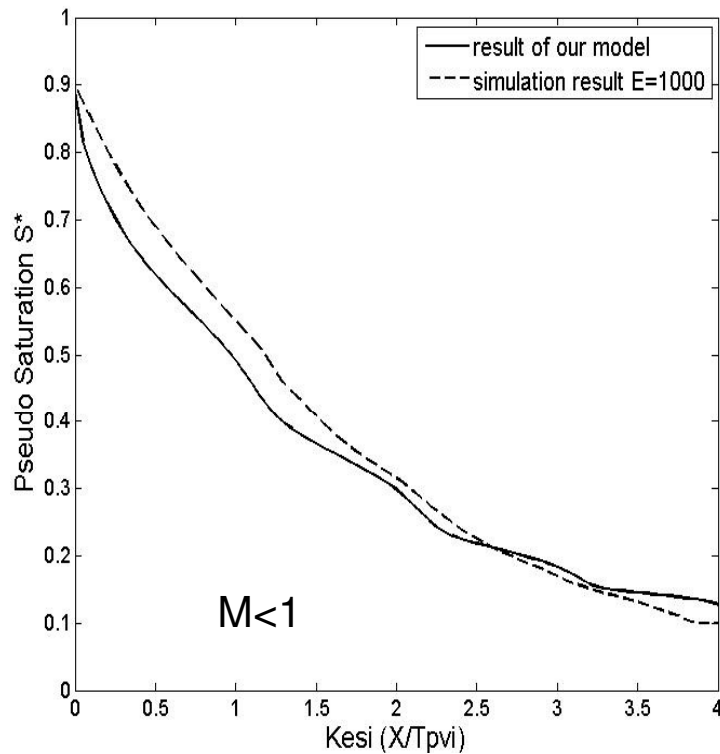
$$U_{X,i} = \frac{\Lambda_{X,i}}{\int_0^1 \Lambda_X dY} Q$$

Q: injection rate

$$U_{Y,i} = -\frac{Q}{E} \frac{\partial}{\partial X} \left( \frac{\int_0^{Y(i)} \Lambda_X dY}{\int_0^1 \Lambda_X dY} \right)$$

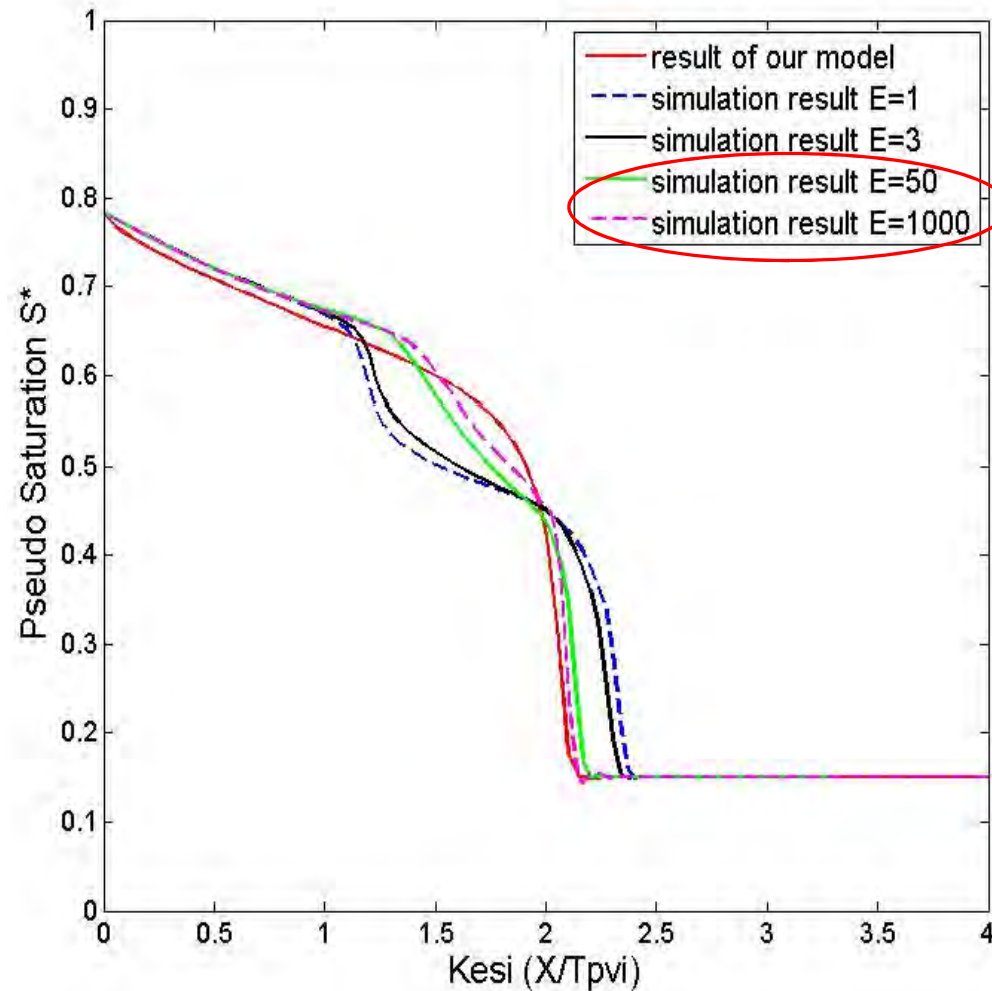
Y(i): height from top to ith layer

# Results: log-normal distributed permeability



Average water saturation profile of 2D waterflooding, at time=0.25 PVI (Pore Volume Injected). Left:  $M < 1$ , right:  $M > 1$

# Results: Levels of communication between layers

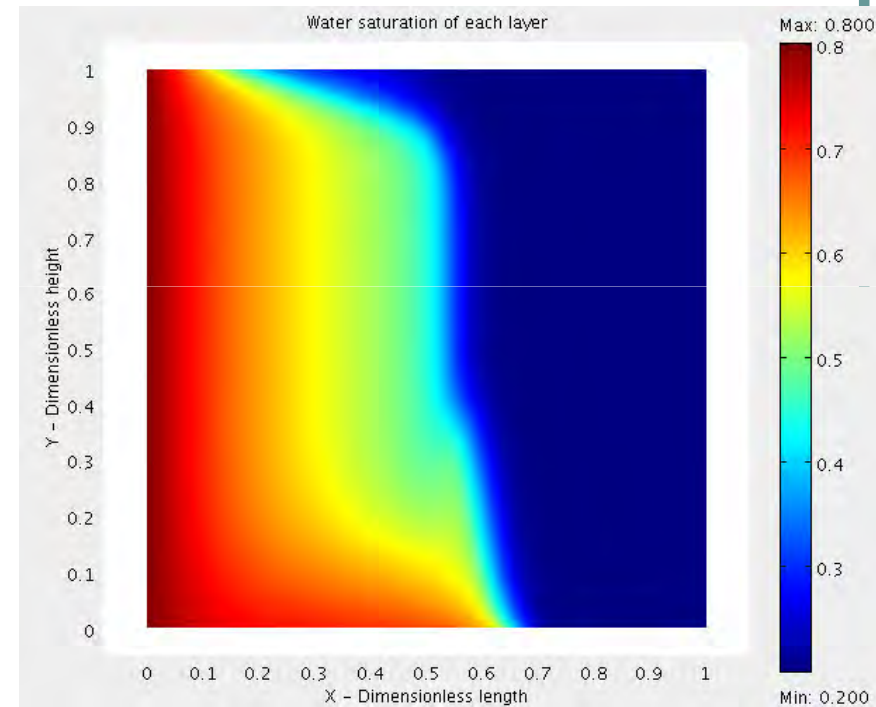
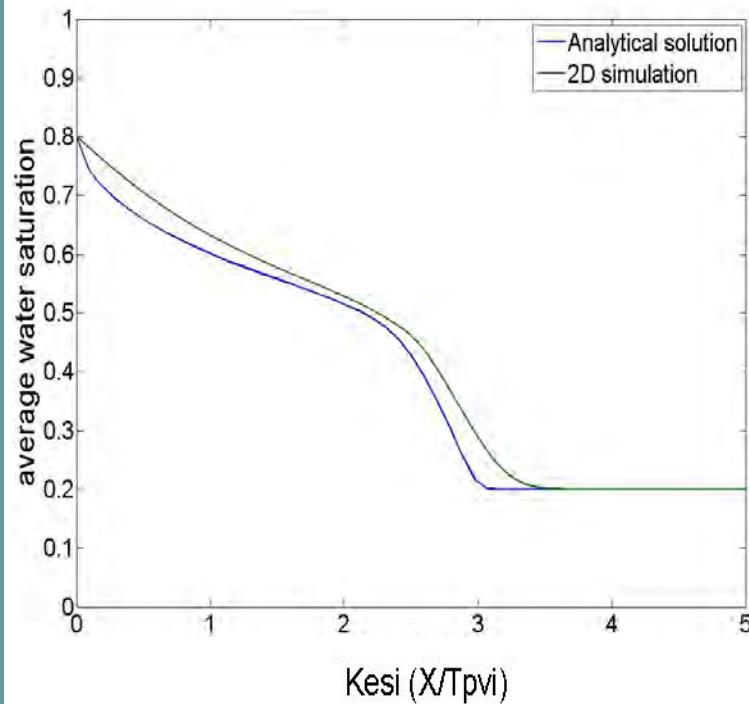


# Results: Involving gravity

$$\bar{U}_Y = -E\Lambda_Y \frac{\partial P}{\partial Y} - EG(\Lambda_Y - A\Lambda_{oY})$$

$G$ : Gravity ratio

$A$ : Density ratio



Average water saturation profile of 2D waterflooding, at time=0.2 PVI (Pore Volume Injected). 1 layer.  $G=0.02$ ,  $A=0.8$ .

# Conclusions

- 2D simulation of waterflooding in oil recovery
- Show the effect of crossflow
- When  $E$  increases, inter-layer communication increases
- Gravity and capillary can be added easily
- Artificial diffusion is needed