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COMSOL Implementation for Two-Phase Immiscible Flows in Layered Reservoir

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Outline

- Introduction
- Implementation
- Results
- Conclusion

Introduction

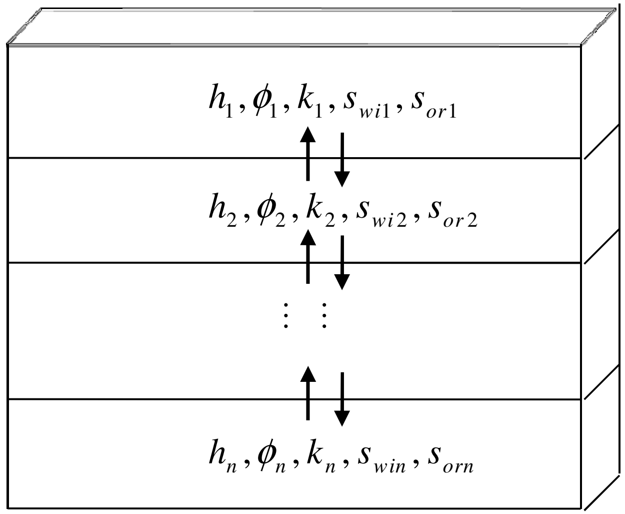
- Waterflooding
- Layered model of reservoir
- Non-/Communicating layers
- Anisotropy parameter

$$E = k_y x_0^2 / k_x y_0^2$$

x_0 :Length

y_0 :Height

k :Permeability



Scheme of layered reservoir

Introduction

- Mass conservation

$$\phi\frac{\partial s}{\partial T}+\frac{\partial U_X F(s)}{\partial X}+\frac{\partial U_Y F(s)}{\partial Y}=0$$

- Incompressible flow

$$\frac{\partial U_X}{\partial X}+\frac{\partial U_Y}{\partial Y}=0$$

U: total flow velosity

F: fractional flow of water

Φ: porosity

Sw: water saturation

- Darcy's law

$$U_X=-\Lambda_X\cdot\frac{\partial P}{\partial X}\qquad U_Y=-E\Lambda_Y\cdot\frac{\partial P}{\partial Y}$$

Λ: mobility

$E=k_y x_0^2/k_x y_0^2$ - anisotropy ratio

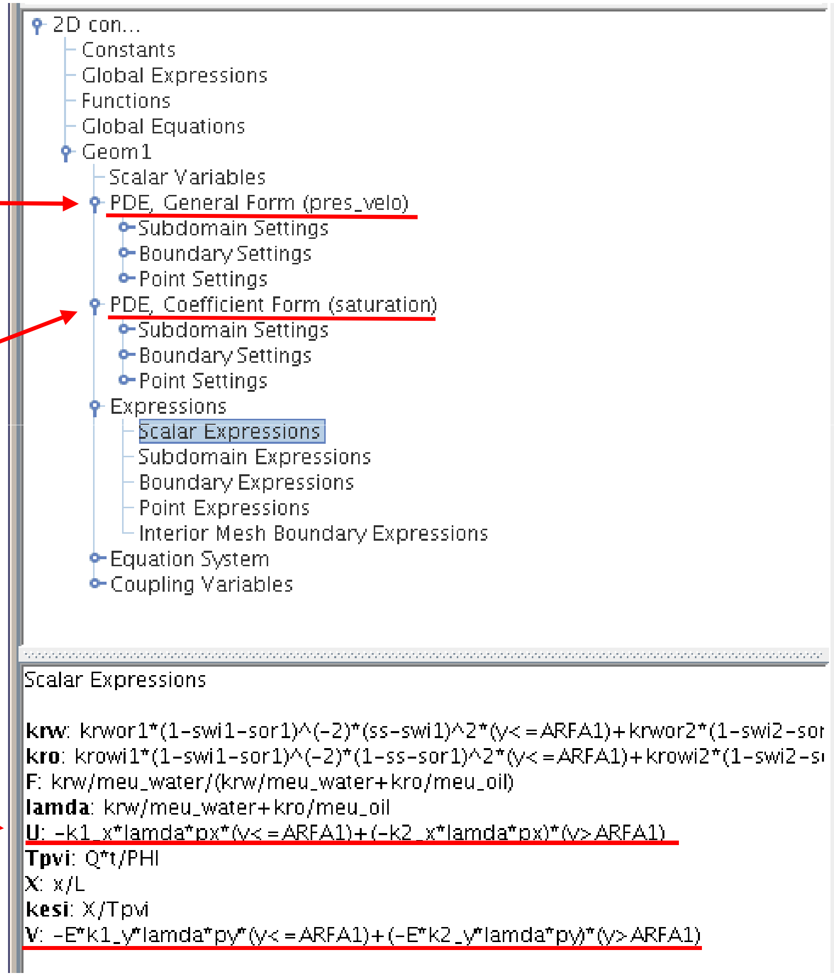
Implemention in COMSOL

$$\frac{\partial U_X}{\partial X} + \frac{\partial U_Y}{\partial Y} = 0$$

$$\phi \frac{\partial s}{\partial T} + \frac{\partial U_X F(s)}{\partial X} + \frac{\partial U_Y F(s)}{\partial Y} = 0$$

$$U_X = -\Lambda_X \cdot \frac{\partial P}{\partial X}$$

$$U_Y = -E\Lambda_Y \cdot \frac{\partial P}{\partial Y}$$



Implemention in COMSOL

- Initial conditions

$$s(t_0) = s_{wi} \qquad P(t_0) = P_0$$

- Boundary conditions

$$-n \cdot \Gamma = 0 \qquad -n \cdot \Gamma = 0$$

$$s_{in} = 1 - s_{or} \qquad s_{out} = s_{wi}$$

$$-n \cdot \Gamma = Q \qquad P_{out} = P_t$$

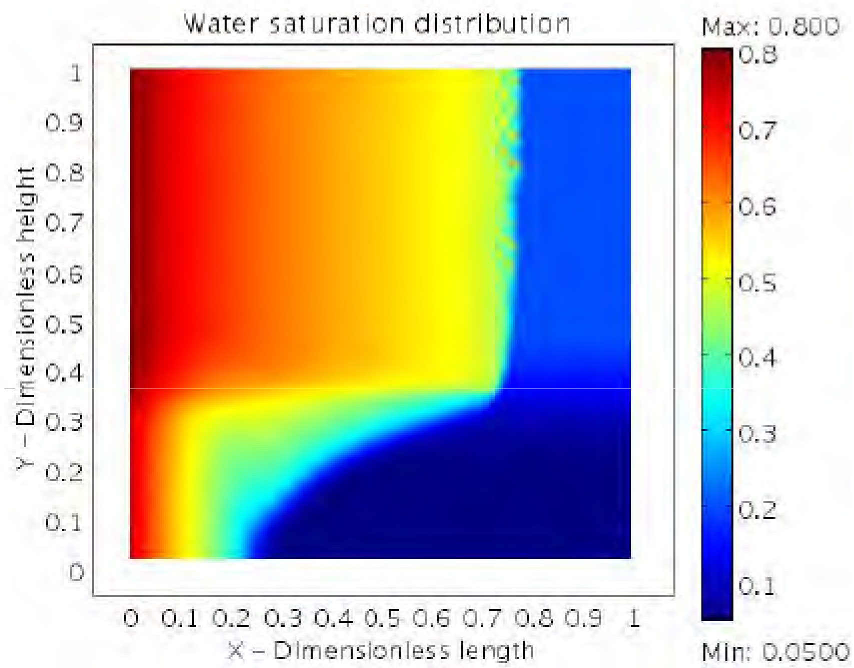
$$-n \cdot \Gamma = 0 \qquad -n \cdot \Gamma = 0$$

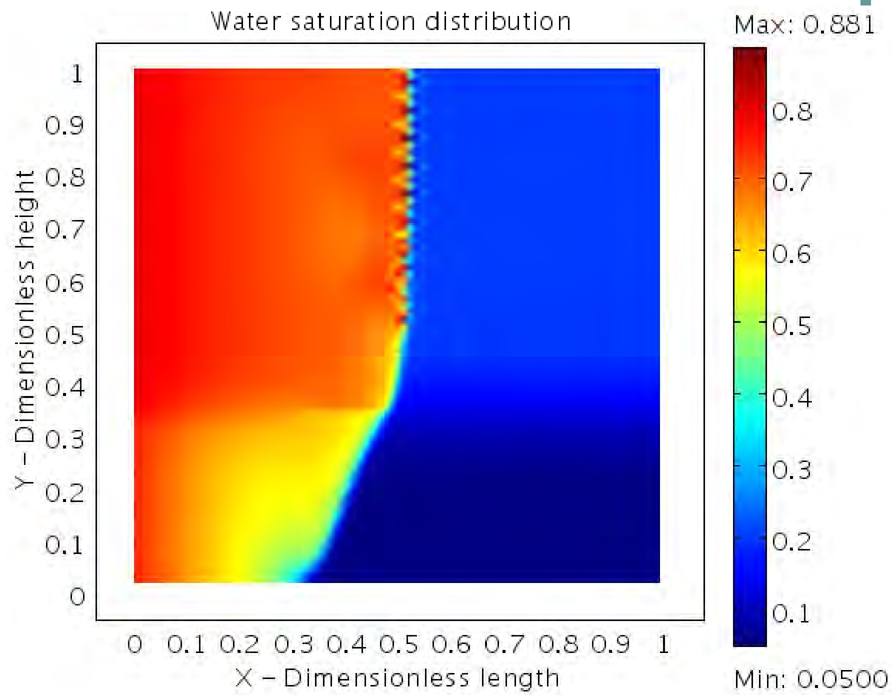
Results-2 layers

Dimensionless parameters	Layer 1	Layer 2
Fraction of thickness H	0.33	0.67
Irreducible water saturation s_{wi}	0.05	0.2
Residual oil saturation s_{or}	0.25	0.2
Relative water permeability at residual oil saturation kr_{wor}	0.8 (0.4)	0.8 (0.4)
Relative oil permeability at irreducible water saturation kr_{owi}	0.8	0.8
Dimensionless permeability in X-direction K_X	0.33	0.67
Dimensionless permeability in Y-direction K_Y	0.33	0.67
Dimensionless porosity Φ	1	
Viscosity ratio of water to oil μ_w/μ_o	1:3 (1:1.5)	
Anisotropy ratio E	1000	
Dimensionless injection rate Q	1	

$$M = \frac{kr_{owi}}{kr_{wor}} \frac{\mu_w}{\mu_o}$$

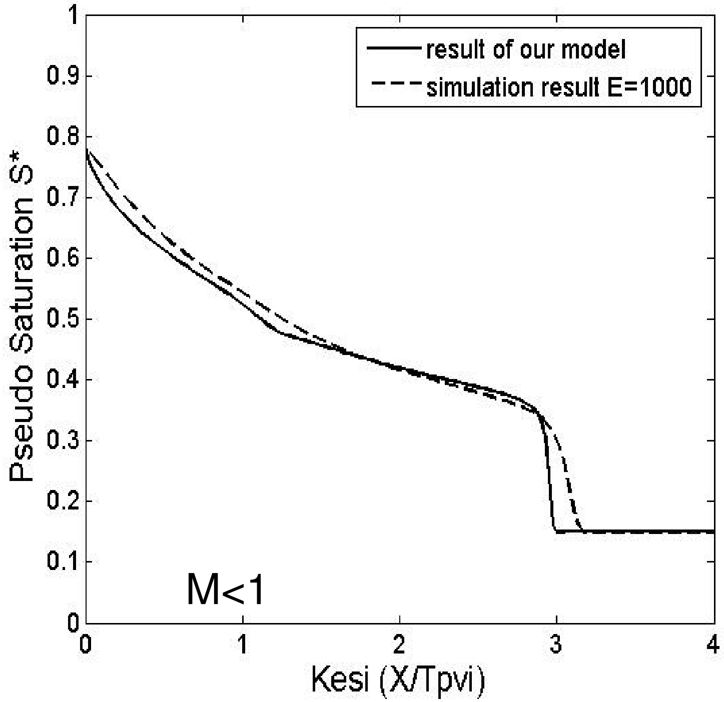
Results- 2 layers

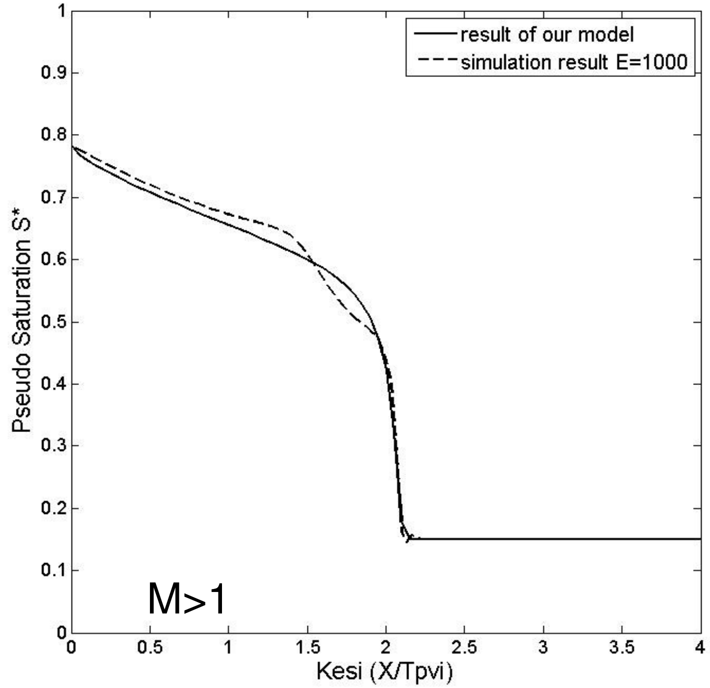




Water saturation profile of 2D waterflooding simulation in COMSOL, at time=0.25 PVI (Pore Volume Injected). Left: M<1, right: M>1

Comparision with analytical solution





Average water saturation profile of 2D waterflooding, at time=0.25 PVI (Pore Volume Injected). Left: M<1, right: M>1

Analytical derivation

Anisotropy parameter E

- Small E – Poorly communicating layers
- Large E – Well communicating layers

Asymptotic approximation – assumption for perfectly communicating layers:

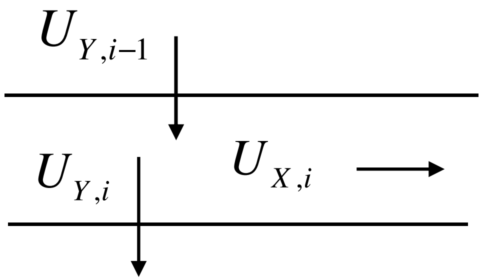
E tends to infinity !

Consequence: *Pressure gradient across the layers is negligebly small!*

Analytical derivation

- Pressure P is constant in y-direction

For the ith layer
$$\Phi_i \frac{\partial s_i}{\partial T} + \frac{\partial F_i \bar{U}_{Xi}}{\partial X} + \frac{(F\bar{U}_Y)_{i-1} - (F\bar{U}_Y)_i}{\alpha_i} = 0$$



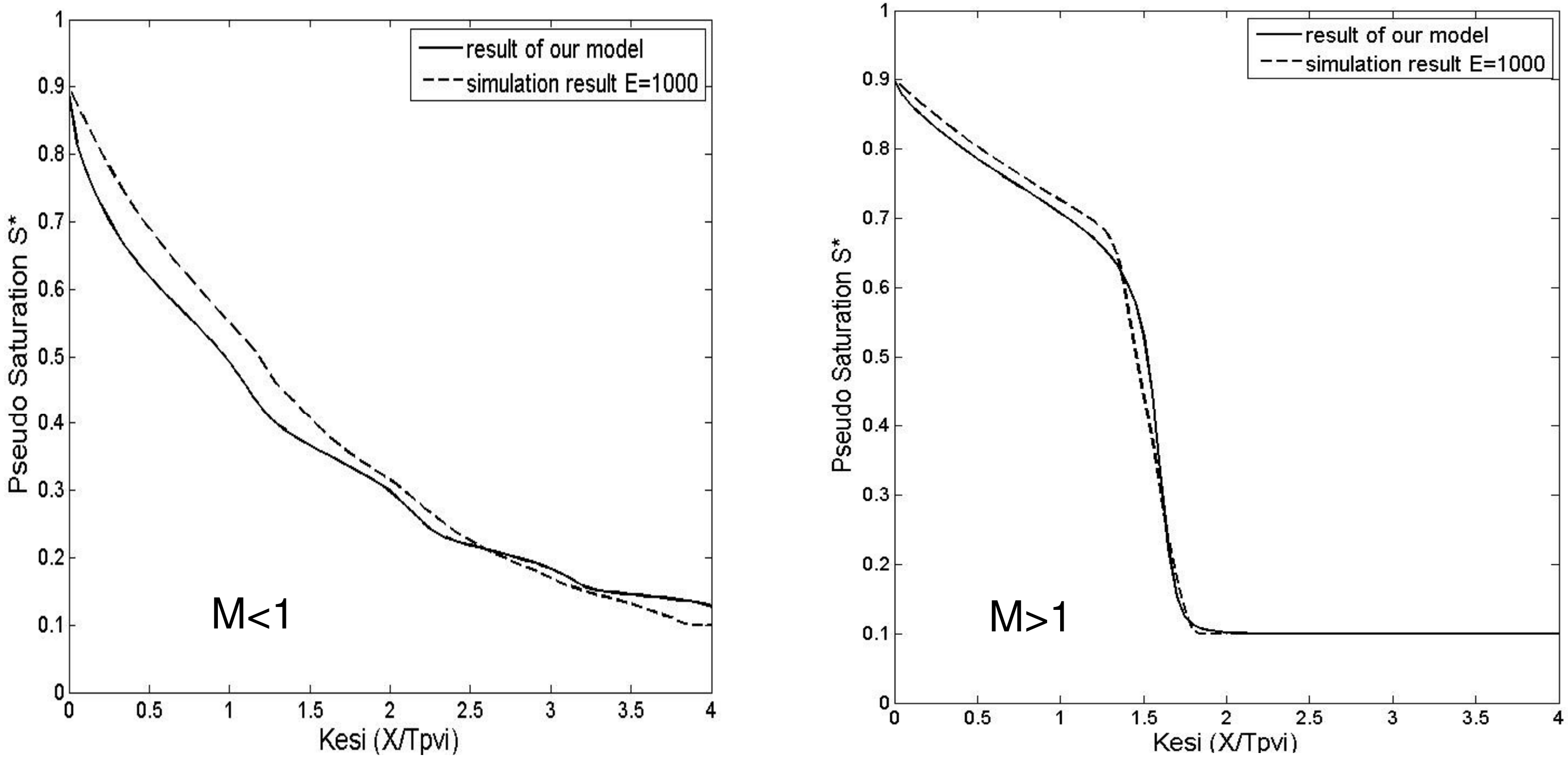
$$U_{X,i} = \frac{\Lambda_{X,i}}{\int_0^1 \Lambda_X dY} Q$$

Q: injection rate

$$U_{Y,i} = -\frac{Q}{E} \frac{\partial}{\partial X} \left(\frac{\int_0^{Y(i)} \Lambda_X dY}{\int_0^1 \Lambda_X dY} \right)$$

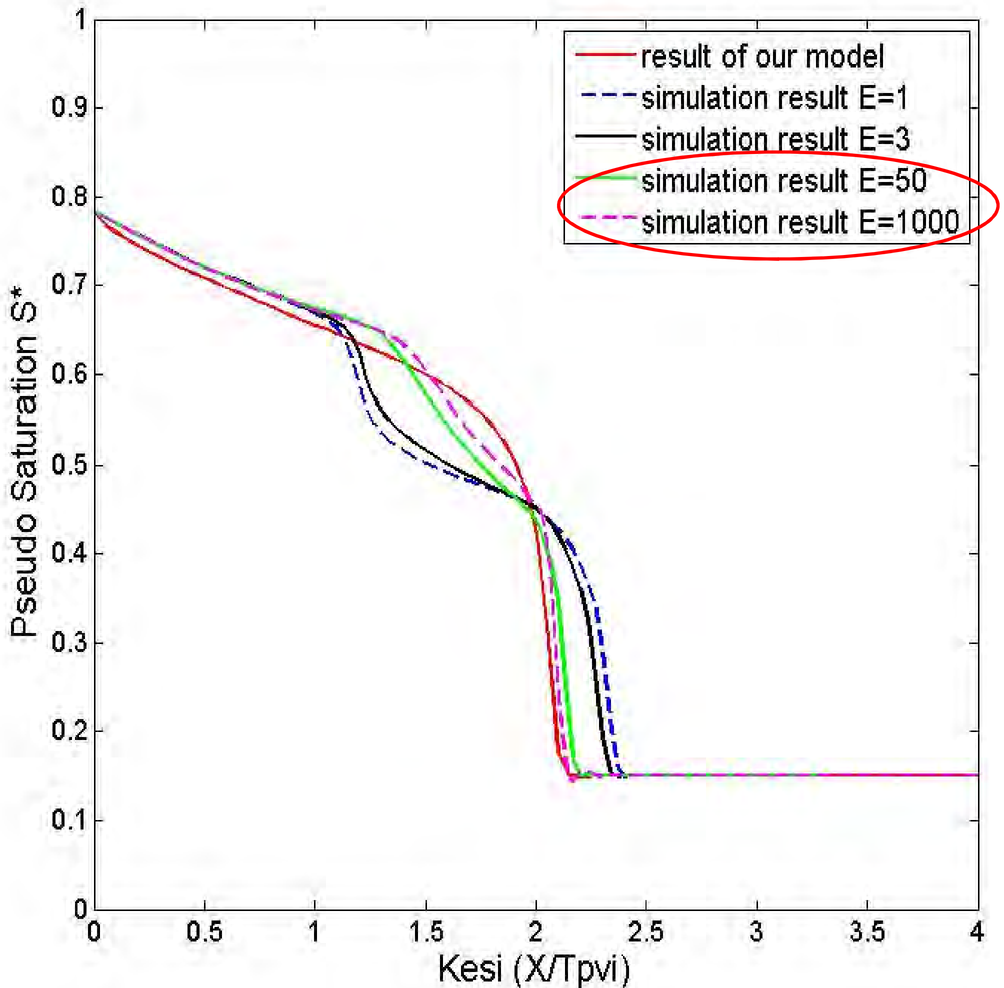
Y(i): height from top to ith layer

Results: log-normal distributed permeability



Average water saturation profile of 2D waterflooding, at time=0.25 PVI (Pore Volume Injected). Left: M<1, right: M>1

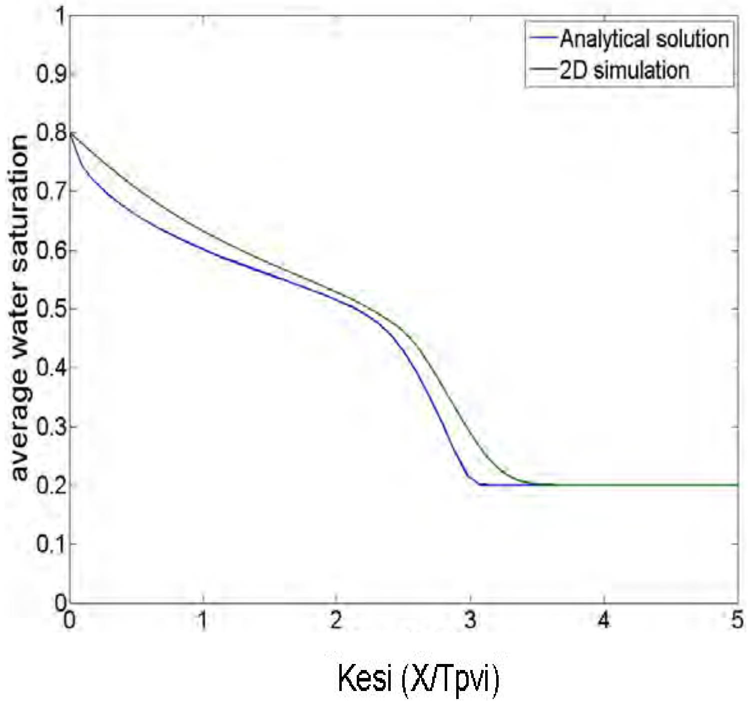
Results: Levels of communication between layers

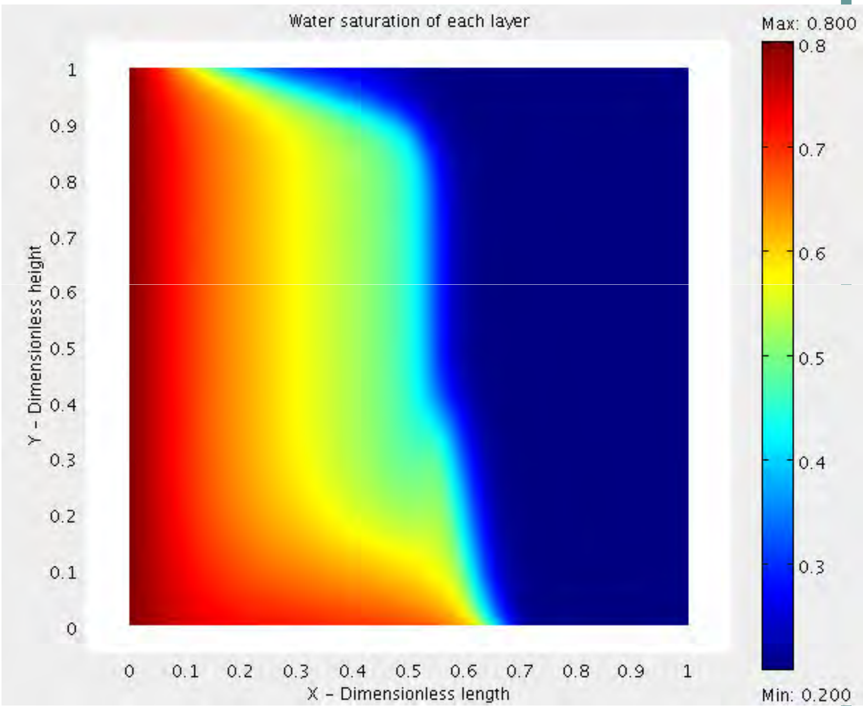


Results: Involving gravity

$$\bar{U}_Y = -E\Lambda_Y \frac{\partial P}{\partial Y} - EG\left(\Lambda_Y - A\Lambda_{oY}\right)$$

G: Gravity ratio $\quad$ A: Density ratio





Average water saturation profile of 2D waterflooding, at time=0.2 PVI (Pore Volume Injected). 1 layer. G=0.02, A=0.8.

Conclusions

- 2D simulation of waterflooding in oil recovery
- Show the effect of crossflow
- When E increases, inter-layer communication increases
- Gravity and cappilary can be added easily
- Artificial diffusion is needed