Multiphysics Approach to Sediment Transport in Shallow Water

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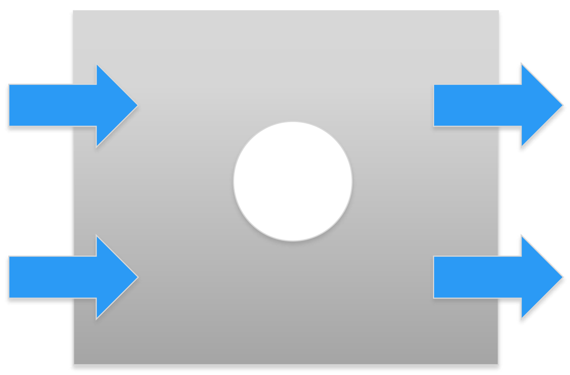
INTRODUCTION: Sediment transport in shallow water is of concern in many hydro-ecological problems. Erosion or sedimentation processes are not only relevant in perennial systems as rivers, lakes, reservoirs and coastal regions, but also in ephemeral phenomena like gullies, inundations and floods.

Sediments are transported by water flow, and vice-versa sediments have an effect on flow, due to changes of the ground surface level. That two-way coupling has to be taken into account when sediment transport is simulated in computer models

We present a multiphysics approach of such a coupled model. The shallow water equations (SWE) for water height and velocity are coupled with transport for particulate matter and a bedload equation. The non-linear system of five differential equations is solved simultaneously. Using COMSOL Multiphysics® software a set-up is presented that demonstrates the feasibility of the approach.

Figure 1. Sketch of flood domain with a cylindrical obstacle



COMPUTATIONAL METHODS: We couple the Shallow Water Equations (SWE) (1) with equations for particular and bedload transport (2):

$$\frac{\partial h}{\partial t} + \nabla \cdot \left(H\mathbf{u}\right) = 0$$
$$\frac{\partial \mathbf{u}}{\partial t} + \left(\mathbf{u}\cdot\nabla\right)\mathbf{u} + g\nabla H + ghn^2\frac{|\mathbf{u}|}{h^{4/3}}\mathbf{u} = 0 \qquad (1)$$

$$\frac{\partial c}{\partial t} + \left(\mathbf{u}\cdot\nabla\right)c - \frac{1}{H}\left(E - D\right) = 0$$
$$\frac{\partial d}{\partial t} - \frac{1}{1-q}\left(E - D\right) = 0 \qquad (2)$$

with total water depth H, water height above reference height η, velocity vector $\mathbf{u}$, acceleration due to gravity g, Manning parameter n, particle concentration c, depth below reference d=H-η, sedimentation E, re-mobilization D, and bedload porosity θ. Terms D and E are specified in terms of settling velocity, Shields parameter, particle diameter and density [1].

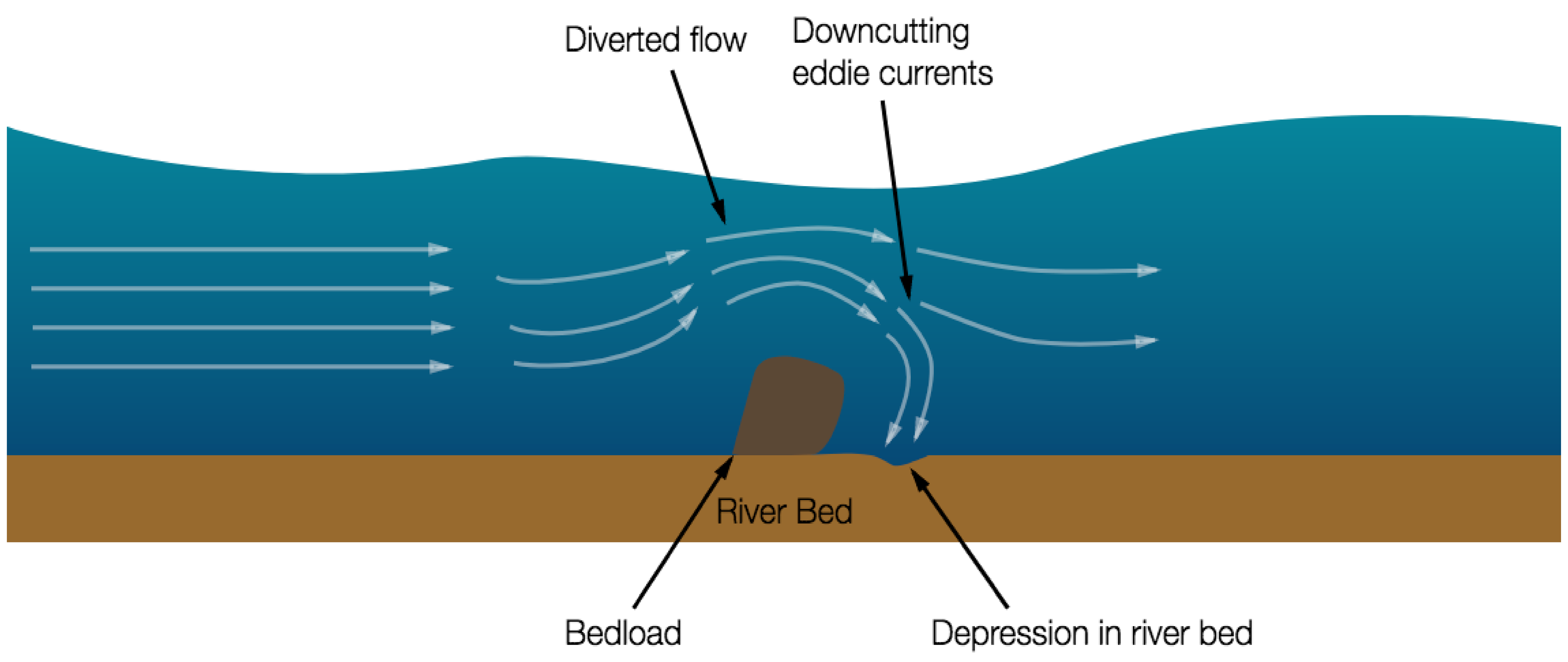


Figure 2. Depression formation behind an obstacle [2]

RESULTS: The situation sketched in Figure 1 was simulated using numerical methods solving the coupled system of eq.s (1) & (2). The Finite Element mesh was refined at the obstacle walls, as depicted in Figure 3. See Table 1 for parameters.

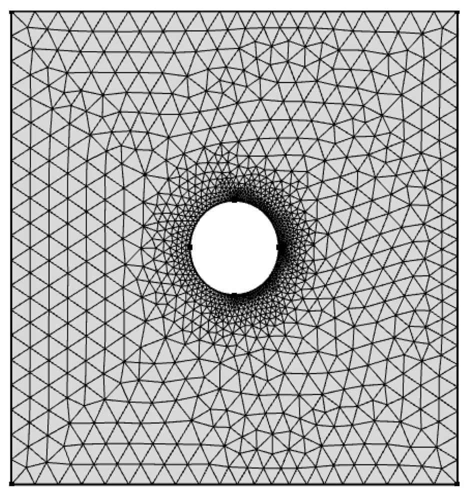


Figure 3. FE Mesh

Parameter, Symbol	Value, Unit	Parameter, Symbol	Value, Unit
Length	1 m	Settling velocity	0. 1 m/s
Width	1 m	Diffusivity	10^{-9} m²/s
Bed below reference	0.5 m	Manning coefficient	0.03 s/m$^{1/3}$
Initial water table above reference	0.75 m	Re-suspension parameter α	8.5 10^{-6}
Inflow water table above reference	1 m	Critical Shields parameter	0.4
Velocity at outlet	1 m/s	Specific gravity ρ_s/ρ_f	2.65

Table 1. List of parameters

Results of the numerical simulation are depicted in Figure 4.

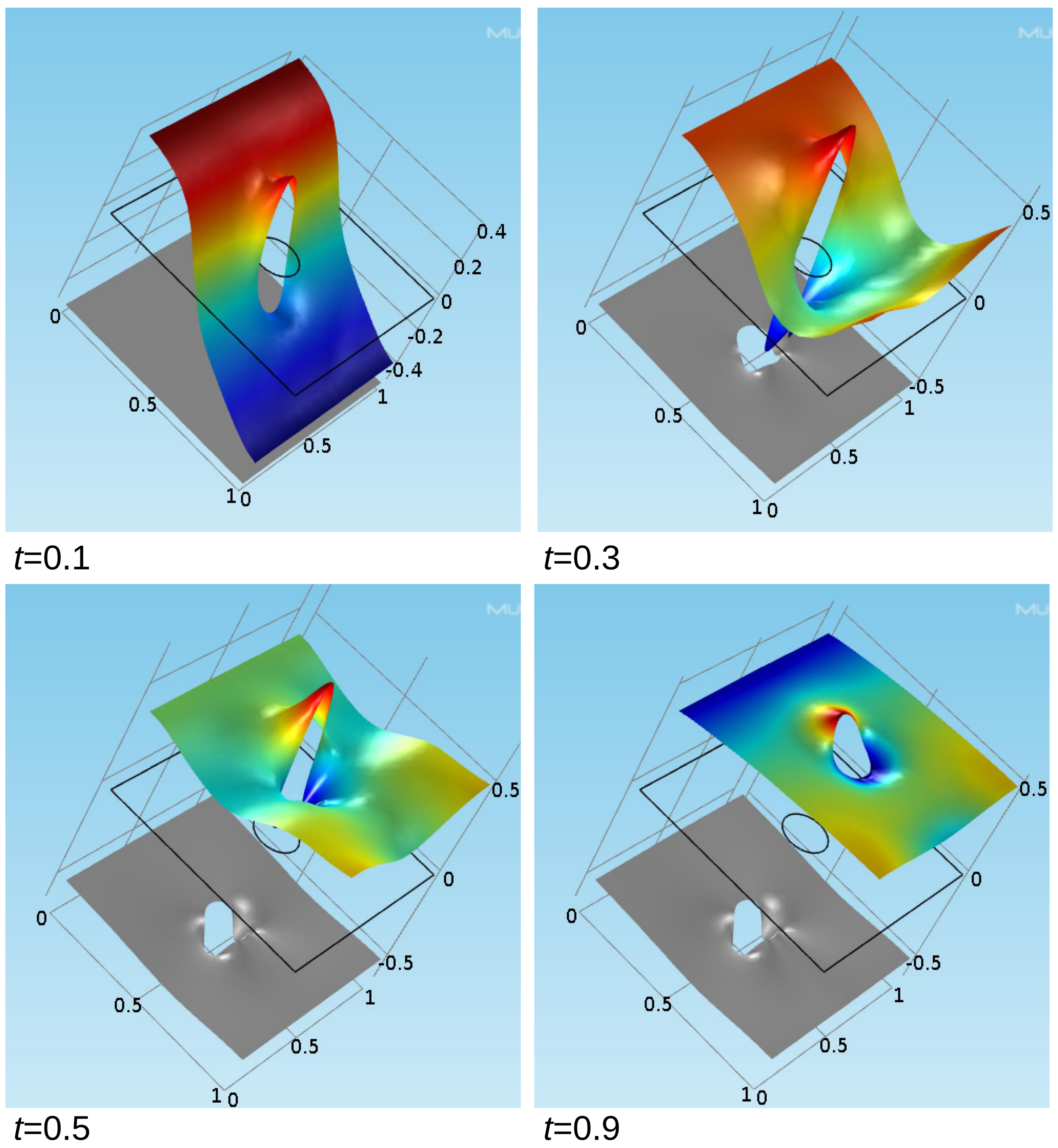


Figure 4. Water table and ground surface at times t = 0.1, 0.3, 0.5 & 0.9 s

CONCLUSIONS: The example shows that basic phenomena of sediment transport like deposition and depression creation (Figure 5) in flooding water can be captured by a 2D coupled modelling approach. The set-up could become a benchmark for coupled flow and transport pro-cesses in rivers and channels (1D), and during flooding events (2D).

Figure 5. Depression at obstacle, Wadi Abyad

REFERENCES:

1. Li S. and Duffy C.J., Fully coupled approach to modeling shallow water flow, sediment transport, and bed evolution in rivers, WATER RESOURCES RESEARCH, VOL. 47 (2011)
2. https://geographyas.info/rivers/river-landforms/

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