Multphysics Approach to Sediment Transport in Shallow Water

Ekkehard Holzbecher, Ahmed Hadidi
German Univ. of Technology in Oman (GUtech)
E-mail: ekkehard.holzbecher@gutech.edu.om

Abstract: Sediment transport in shallow water is of concern in many hydro-ecological problems. Erosion or sedimentation processes are not only relevant in perennial systems as rivers, lakes, reservoirs and coastal regions, but also in ephemeral phenomena like gullies, inundations and floods.
Sediments are transported by water flow, and vice-versa sediments have an effect on flow, due to changes of the ground surface level. That two-way coupling has to be taken into account when sediment transport is simulated in computer models.
We present a multiphysics approach for such a coupled model. For flow the shallow water equations (SWE) are utilized, i.e. one equation for water height and two equations for the horizontal velocity components. The vertical component can be skipped due to the averaging process over the height of the water column, from which the SWE equations are derived. The flow equations are coupled with a transport equation for particulate matter and a bed-load equation. When using the CFD toolbox, turbulent mixing can be included in the transport equations. The bed-load equation is implemented using the coefficient pde formulation.
Using COMSOL Multiphysics software a set-up is presented that demonstrates the capability and feasibility of the approach. The non-linear system of five differential equations is solved simultaneously. For a test example of a cylindrical obstacle the model sensitivity to several physical and numerical parameters is discussed.

Introduction

Sediment transport in surface water bodies is a topic that is gaining increased relevance and scientific interest. With focus on sedimentation and re-suspension applied research is performed concerning rivers (Sibetheros *et al*. 2013, Zavattero *et al*. 2016), channels (Visescu *et al*. 2016), the coastal zone (Amoudry & Souza 2011, Aoki *et al.* 2015), reservoirs (Kondolf *et al.* 2014, Sumi & Hirose 2009) and floods (Eaton & Lapointe 2001, Berghout & Meddi 2016). In reservoirs deposition of sediments is a general problem: the water storage capacity may be reduced drastically. Groundwater recharge is reduced or completely stopped due to deposition of fine particles, which is a crucial issue at dams designed for recharge. In sediment-laden water bodies the deposition causes problems, if appearing at the wrong places. Navigation may become hindered or even impossible, for example. Similarly unwanted scours may emerge at the bottom and cause problems concerning the stability of bridge piers, for example.

In order to understand these processes models can play an important role. Places and amount of sedimentation and sediment re-location can be identified. Potential counter measures to avoid problems could be examined on a computer with a validated numerical model. However, simulation techniques and tools for the set-up and run of such models are still in development and not well established yet.

The physics of the situation in question is complicated. The first focus of modelling water bodies is the water flow itself. Models for water flow have to be extended to treat sediment transport in addition. If there is erosion or deposition at the bottom, the depth of the water column changes and thus the flow regime is altered. Thus there is a two-way coupling of flow and transport, which has to be reflected in the modelling approaches. Water flow and sediment transport have to be simulated simultaneously. This is a typical multiphysics problem.

Here we present a multiphysics approach, in which the coupling between flow and transport is taken into account. As a first approach we attempt to minimize the complexity and with it the required computational resources. For flow modelling we choose the shallow water equations (SWE), a system of two coupled differential equations. These constitute a minimalistic approach, as the vertical direction is not explicitly considered, and thus the problem setting is reduced to 2D, or even 1D in rivers and channels. As a minimal approach sediment transport is also described by two equations: one for particular load in the water column and one for bed-load. The latter is formulated as an expression for bottom elevation.

For a first check of the ability of such an analytical system of minimal complexity a test-case is simulated. The situation is simple, consisting of a circular obstacle placed into a uniform 2D flow field. It is shown that it is possible to capture both, erosion and deposition at different locations along the obstacle wall by a multiphysics modelling approach.

Differential equations

Fluid flow modelling is based on the mathematical analytical formulation in differential equations. The Saint-Venant equations, also known as shallow water equations for depth averaged flow in one or two space dimensions can be written as:

$$\frac{\partial}{\partial t}(H-d)+\nabla\cdot\left(H\mathbf{u}\right)=0 \qquad (1)$$

$$\frac{\partial \mathbf{u}}{\partial t}+\left(\mathbf{u}\cdot\nabla\right)\mathbf{u}+g\nabla H-\mathbf{F}=0 \qquad (2)$$

with total water depth H, water depth d below a reference level, velocity vector $\mathbf{u}$, acceleration and due to gravity g (for example: Takase *et al*. 2010). In the vector $\mathbf{F}$ the contributions of all other forces are gathered. The equations are derived from the volume and momentum conservation principles, formulated using depth-averaged velocities. The system of equations (1) and (2) is nonlinear. The derivation is based on several assumptions: (1) the fluid is incompressible, (2) in the vertical direction there is hydrostatic pressure distribution, (3) depth-averaged values can be used for all properties and velocities, (4) the bottom slopes are small, (5) there are no density effects from variable fluid density or fluid viscosity, (6) the eddy viscosity is much larger than molecular viscosity, (6) atmospheric pressure gradient can be ignored. Despite of the numerous assumptions, the validity of the SWEs for many application cases is widely accepted.

Friction at the walls, i.e. at the interfaces between fluid and solid, can be taken into account by an additional term in equation (2) (Brufau & García-Navarro 2000, Duran 2015):

$$\frac{\partial \mathbf{u}}{\partial t}+\left(\mathbf{u}\cdot\nabla\right)\mathbf{u}+g\nabla H+g\eta n^2\frac{|\mathbf{u}|}{\eta^{4/3}}\mathbf{u}-\mathbf{F}=0 \qquad (3)$$

with water height above reference height η and Manning coefficient n. In 1D, i.e. for rivers, channels and channel systems equations (1) and (3) constitute a coupled system for H and u. In 2D the equations determine three variables, H and two components of $\mathbf{u}$.

For sediment transport we choose the concentration of suspended material c as dependent variable. Following Li & Duffy (2011) the differential equation reads:

$$\frac{\partial Hc}{\partial t}+\nabla\cdot\left(Hc\mathbf{u}\right)-E+D=0 \qquad (4)$$

E and D denote the erosion and deposition terms, which will be outlined below. Using the product rule equation (4) can be re-written as:

$$H\frac{\partial c}{\partial t}+c\frac{\partial H}{\partial t}+H\nabla\cdot\left(c\mathbf{u}\right)+c\nabla\cdot\left(H\mathbf{u}\right)-E+D=0 \qquad (5)$$

Both terms with leading factor c cancel out because of equation (1). The remainder can be written as:

$$\frac{\partial c}{\partial t}+\nabla\cdot\left(c\mathbf{u}\right)-\frac{1}{H}\left(E-D\right)=0 \qquad (6)$$

The corresponding conservative form is given by

$$\frac{\partial c}{\partial t}+\left(\mathbf{u}\cdot\nabla\right)c-\frac{1}{H}\left(E-D\right)=0 \qquad (7)$$

Note that the sediment load is represented as a concentration with mass/volume unit. In the formulation (7) the diffusion is not considered. In analogy to mass transport diffusive processes are taken into account by an additional term:

$$\frac{\partial c}{\partial t}-\nabla\left(\mathbf{D}\nabla c\right)+\left(\mathbf{u}\cdot\nabla\right)c-\frac{1}{H}\left(E-D\right)=0 \qquad (8)$$

Here $\mathbf{D}$ denotes the dispersion tensor, in which all types of diffusive processes are gathered (Rowinski & Kalinowska 2006). In the following we consider turbulent diffusivity as most relevant part:

$$\mathbf{D}=\frac{\upsilon}{Sc}\mathbf{I} \qquad (9)$$

with turbulent viscosity υ and turbulent Schmidt number Sc. $\mathbf{I}$ denotes the 2D unit matrix.

Except from the consideration of diffusion the presented approach is similar to Cao *et al*. (2004), Li & Duffy (2011) and Rowan & Seaid (2017). In analogy to the cited references the change of the bed due to settling and re-suspension is thus governed by the formula:

$$\frac{\partial d}{\partial t}-\frac{1}{1-\theta}\left(E-D\right)=0 \qquad (10)$$

where θ denotes the bed-load porosity.

The system of equations (1), (3), (7) and (10) is a coupled multi-physics approach. H and $\mathbf{u}$ appear in equation (7), forming the link between flow and transport processes. As the next section shows there are further dependencies in the exchange terms D and E, constituting a coupling between equations (7) and (10). The back-coupling is given as the depth d appears in equation (1).

Sedimentation and Erosion Approaches

For the settling and re-suspension terms several approaches can be found in current literature. For D Li & Duffy (2011) propose:

$$D = \beta v_s c \qquad (11)$$

with the settling velocity v_s. The amount of settling material is proportional t the settling velocity and the concentration of the suspension. The β factor thus has the dimension of a length^{-1}. It is the mean travel length in vertical direction. In our first approach we use an expression, in which β is set to $H/2$, the mean settling depth:

$$D / H = 2 v_s c / H \qquad (12)$$

The model allows working with a constant settling velocity, but also more complex approaches can be utilized. For example the more general approach

$$D = v_s c_s \left(1 - c_s\right)^m \qquad (13)$$

proposed by Cao *et al.* (2004) can be included easily. The sediment concentration near to the bed c_s is proportional to the sediment concentration with a proportionality factor α greater than 1: $c_s = \alpha c$. Rowan & Seaid (2017) suggest formula (13) with power m=1.4 for non-cohesive material. Dealing as well with 1D settings in channels and channel networks Zhang *et al.* (2014) make D dependent on the carrying capacity c_*

$$D = v_s \alpha \left(c - c_*\right) \qquad (14)$$

with

$$c_* = K \left(u^3 \big/ gRv_s \right)^m \qquad (15)$$

depending on the hydraulic radius R and parameters K and m. Li & Duffy (2011) extend the 1D approach for 2D, using the following expressions for β and v_s

$$\beta = \min\left\{2, (1-\theta)/c\right\} \qquad (16)$$

$$v_s = \sqrt{\left(13.95 \nu / \delta\right)^2 + 1.09 g \delta \left(\rho_s / \rho_f - 1\right)} - 13.95 \nu / \delta \qquad (17)$$

with particle diameter δ, kinematic viscosity ν, and particle and fluid densities ρ_s and ρ_f. For re-suspension Li & Duffy (2011) propose:

$$E = \alpha \left(\Theta - \Theta_c\right) H \left|\mathbf{u}\right| \qquad (18)$$

with coefficient α and Shields parameters Θ and Θ_c, defined by $\Theta = u_*^2 / sg\delta$, and

$$u_* = \sqrt{gh\left(S_{fx}^2 + S_{fy}^2\right)} \qquad (19)$$

$$S_{fx} = n u_x \left|\mathbf{u}\right| / h^{4/3} \qquad S_{fy} = n u_y \left|\mathbf{u}\right| / h^{4/3} \qquad (20)$$

$$s = \rho_s / \rho_f - 1 \qquad (21)$$

Θ_c denotes the critical Shields parameter, which has to be exceeded by Θ for re-suspension (erosion) to become active. Cao *et al*. (2004) use a similar relation, and consider the dependence of α on parameters δ, s, Θ_c and on fluid velocity.

In the presented approaches the terms are dependent on particle size and density. For the general modelling approach the heterogeneous sediment has to be partitioned in several classes of different size and weight, similar to SISYPHE (TELEMAC 2017). The numerical calculation then has to be performed for the different sediment classes. In the presented numerical approach this can be included easily. For each class a differential equation as formulated in equation (8) has to be added. In equation (10) the contributions of the various sediment fractions have to be added. For the first demonstration of the approach we restrict our simulations to a single sediment type.

Numerical Demonstration Model

For the numerical simulations we apply the COMSOL Multiphysics (2017) software, a versatile and flexible code for Finite Element solutions of coupled partial differential equations. The software can be used for all kinds of (multi)-physical set-ups. It is equipped with a graphical user interface that allows easy handling and coupling of different physics modes. The entire system can be implemented in COMSOL Multiphysics using pde-modes. We utilized a physics mode for the SWE (Schlegel 2012), i.e. equations (1) and (2). Particulate load in the water column, following equation (7), is modelled by the solute transport mode. For simplicity only one sediment class is considered. The settling velocity is considered in a loss term, according to equation(12). Finally the coefficient form pde of COMSOL Multiphysics is utilized to include the bed-load equation (10) in the model.

The model geometry is given by a square cavity that is open on two opposite sides, and closed at the other sides. In the centre a circular obstacle is located. The situation is sketched in Figure 1.

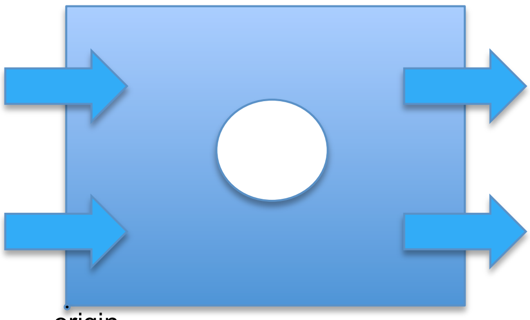

Figure 1: Sketch of flood domain with cylindrical obstacle

It is assumed that there is a sudden increase of the water column height at the inflow boundary. At the outlet we assume a constant outflow velocity. The inflowing water has a constant load of particulate matter. The initial values for the water table and depth are constants. The initial particulate load concentration is zero, also the velocity field. All input parameters are gathered in Table 1.

Parameter, Symbol	Value, Unit	Parameter, Symbol	Value, Unit
Length	1 m	Initial particulate load	0
Width	1 m	Settling velocity v_s	0.01 m/s

Obstacle radius	0.1 m	Particle diffusivity	10^{-9} m²/s
Initial bed below reference	0.5 m	Turbulent viscosity υ	0.0025 m²/s
Initial water table above reference	0.5 m	Turbulent Schmidt number Sc	0.71
Inflow water table above reference	1 m	Critical Shields parameter Θ_c	0.4
Velocity at outlet	1 m/s	Particle diameter δ	0.0001 m
Manning parameter n	0.03 s/m$^{1/3}$	Re-suspension parameter α	5 10^{-4}
Froude number	0.26	Specific gravity ρ_s/ρ_f	2.65

Table 1: Parameter list

For the SWE and the transport equation linear elements are used, for the bed equation quadratic elements. The Finite Element mesh is refined at the obstacle boundaries. It. Constructing the mesh a maximum element side length of only 0.01 m at the upstream side, and of 0.005 m at the downstream side was allowed. The resulting mesh, consisting of 2476 elements, is depicted in Figure 2. The discretization of the entire system of coupled differential equations has 10468 degrees of freedom.

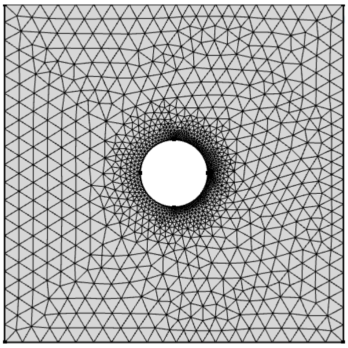

Figure 2: Finite Element mesh

It is well known that the numerical solution of the advection-diffusion transport equation may suffer from severe instabilities. Straightforward modelling, either using finite differences or finite element techniques, leads to spurious oscillations. The numerical solution of the SWEs (1) and (2) leads to the same problem. For the application of COMSOL Multiphysics the case was examined by Holzbecher & Hadidi (2017). In order to suppress instabilities various stabilization schemes have been proposed.

For the transport equation the most basic stabilization method is the introduction of an artificial diffusivity (Quarteroni 2017). In CFD implementations an artificial viscosity ν can be introduced, which appears in an additional term on the left side of equations (2) (Chen *et al.* 2013):

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} + g\nabla H - \nu\nabla^2\mathbf{u} - \mathbf{F} = 0 \qquad (22)$$

Using basic stabilization methods non-physical terms, like artificial diffusivity and viscosity, are introduced, which may lead to increased smoothing of steep gradients. For that reason more sophisticated schemes have been proposed. For the presented demonstration case, in order to avoid any stabilization problems various schemes are utilized: streamline stabilization, shock wave capturing and artificial kinematic viscosity for the SWE, streamline and crosswind diffusion for the transport equation.

Results

Figure 3 depicts the outcome of the numerical model, outlined above. The sub-figures show the water table and bottom elevation at four different time instants. For better visualization the water table is shifted by -0.6 m. At time t=0.1 s the wave that is initiated by the elevated water table at the inlet, is moving into the domain, the wave front reaching the obstacle. The bottom is still almost flat as in the initial state. At time t=0.3 s the wave trough has passed the obstacle. The water depth is increased at the upstream edge of the obstacle and at the outlet; it decreased behind the obstacle. The bottom surface begins to show slight changes from the initial constant state.

At t=0.5 the water depth distant from the obstacle fluctuates slightly around the same value; at the outlet it is slightly higher than at the inlet, while a slight wave trough is seen in between. At the upstream edge of the obstacle the water is still higher, and it is lower behind the downstream edge. Changes at the bottom elevation have become more pronounced: at the flanks digging of scours can be observed, while the bottom is elevated in the wake of the obstacle.

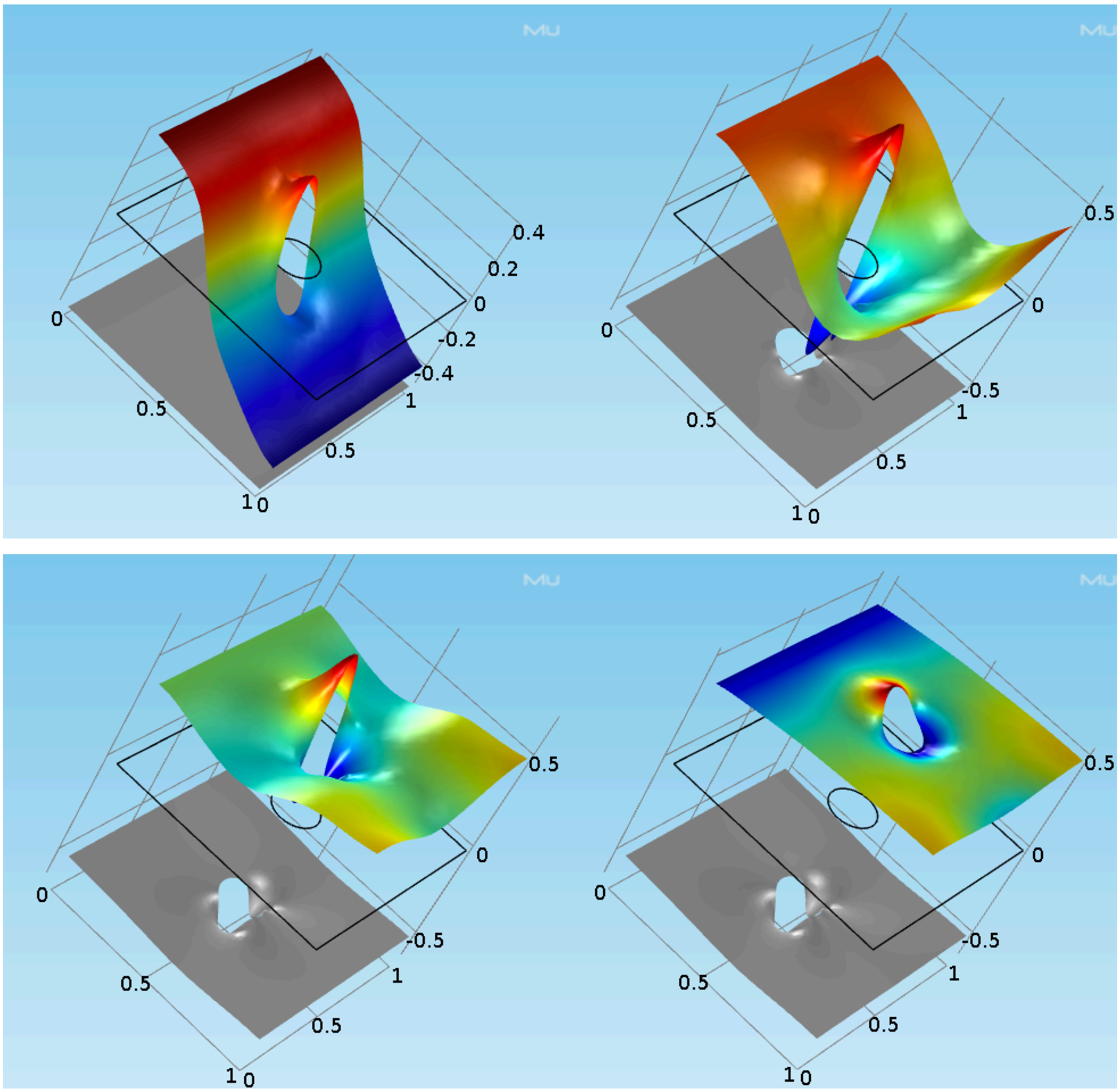

Figure 3: Water table (colour) and ground surface (grey) at different time instants t = 0.1 (top left), 0.3 (top right), 0.5 (bottom left) & 0.9 (bottom right) s; all units in m

In the sub-figure for t=0.9 s the general observation is still the same. The water table deviations from a constant value decrease. Still the depth extremes are at the obstacle boundaries: highest value appears upstream, the lowest value downstream. Changes of the bottom elevation increase further: trough building at the flanks and sedimentation in the wake.

These findings are highlighted in Figure 4, which depicts water table and bottom changes at two positions as functions of time. The flank position is located directly at the obstacle boundary at the most transverse point relative to the main flow axis. The downstream position is located slightly beyond the obstacle, downstream near to the flow axis. The graphs show clearly the deepening of the bottom at the flanks, down to almost 10 cm, where the value nearly stabilizes after 0.6 s. The increase in the wake amounts to few centimeters only, but is still increasing at the end of the simulated period.

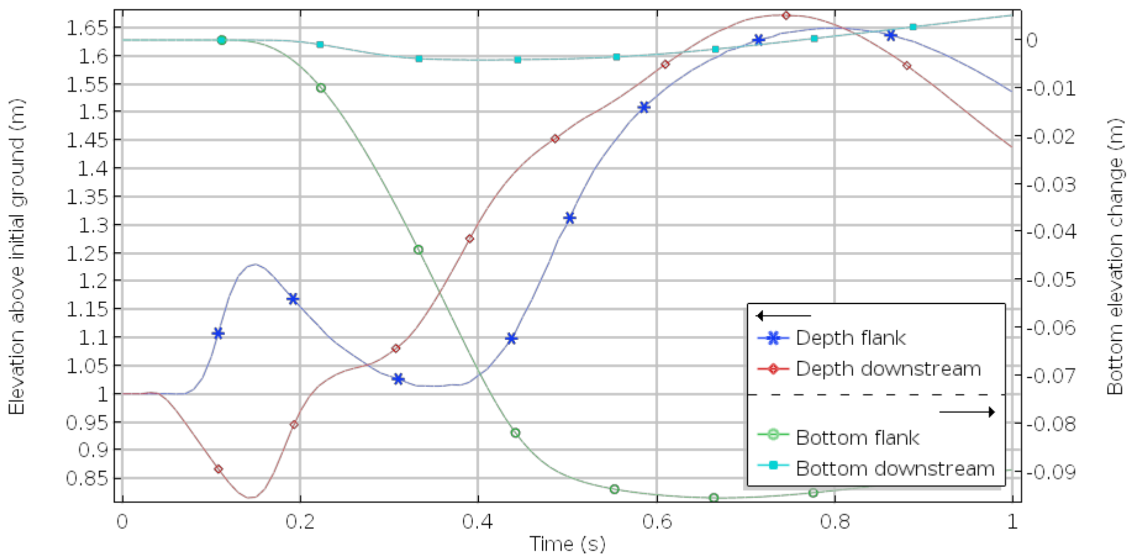

Figure 4: Water table and bottom changes at selected locations

Conclusions

Sedimentation processes have to be described by coupled models. The flow determines the transport of particular sediments in the water column, and due to the settling process also the sedimentation. Also the re-mobilization of bed-load depends on flow velocities. Together these processes determine the link from flow to transport. Vice versa the flow depends on the depth of the water table, which at the bottom is determined by the amount of bed-load. Thus there is a 2-way coupling between flow and transport, as it is characteristic for multiphysics modelling.

A minimalistic approach is proposed that is based on the two SWE equations for the flow simulation and two transport equations for particulate load and bed-load. This is an approach of minimal complexity.

A simple test case with an obstacle was set up in order to examine if the approach is able to cover basic processes concerning change at the bottom of the water body. Results, shown in Figure 3, clearly identify scours at the flanks of the obstacle and sedimentation downstream behind the obstacle. In fact this behaviour coincides with observations that can be made in the field. Figure 5 shows an upstream view in a wadi with a stone obstacle. Clearly the water filled scours at the sides of the stone can be identified, while in the backwater (front in the photo) sediment is deposited.

The study demonstrates that basic phenomena of sediment transport like scour creation and sediment deposition can be captured by a 2D coupled multiphysics approach. Further experimental and numerical research is needed to examine the real capabilities and limits of the approach. In order to improve simulations, the proposed minimal approach offers lots of options for the consideration of additional dependencies and processes.

Figure 5: Depression and sedimentation around obstacle, view upstream in Wadi Abyad, Oman

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