Hydro-Mechanical Coupling in Saturated and Unsaturated Soils and its Consequences on the Electrical Behaviour

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Abstract: The consequences of hydro-mechanical coupling on the electrical conductivity of saturated and unsaturated soils are investigated experimentally and numerically. Simulations of the consolidation problem under vertical load for an elastic medium and of the coupled flow of two immiscible fluids have been performed in order to check the capability of electrical resistivity tomography to reconstruct the evolution of porosity and water content in a soil sample. The results are compared to experiments performed in the laboratory using a special oedometer.

Keywords: hydro-mechanical coupling, unsaturated soils, electrical characteristics.

1. Introduction

Soils are natural porous materials that can undergo a number of different physical processes. Geoenvironmental applications require the comprehension of their behaviour under the effect of various fields, such as mechanical, hydraulic, thermal, chemical and electrical. Depending on their physical and microstructural characteristics, a number of coupled processes can arise.

The present work deals with the hydro-mechanical coupled effects occurring in saturated and unsaturated soils and the consequences on the electric conductivity of the material. The relevance of this aspect is related to monitoring techniques based on geophysical methods widely adopted for site characterization. In this study the experimental results obtained under controlled conditions in laboratory tests are compared with numerical simulations aimed at improving the understanding of actual physical processes.

Specifically two classes of experiments are presented: uniaxial compression of fully saturated porous materials and wetting process in unsaturated soils. The experiments have been run in laterally confined conditions (oedometer), in an equipment specifically designed to allow for the reconstruction of the 3D distribution of the electrical conductivity within the sample using a tomographic approach.

For saturated materials, an experiment related to the consolidation of the soil under increasing loads has been reproduced to assess the consequences of the actual boundary conditions in the tomographic cell on the flow paths. The results of conventional external monitoring at different loads and for different time steps have been used to calibrate model parameters.

For unsaturated materials, the numerical simulation of the hydraulic problem has been conducted using simple equations in order to link the permeability, the electrical conductivity and the fluid pressure to the degree of saturation.

Balance equations considered are the mass balance of the water and air phases and the equilibrium of the porous medium. The mechanical behaviour has been described as linear elastic and the water flow by Darcy's law, where the hydraulic conductivity, for both water and air, has been set to depend on the saturation degree through water retention properties, assumed to be independent on the hydraulic history.

The spatial distribution of the saturation degree and porosity obtained from the simulation has then been used to estimate local values of the electrical conductivity at various times. Comparison with the distribution reconstructed through a tomographic technique during the physical experiments allowed for the calibration of the physical parameters introduced in the simulation.

2. Governing equations

2.1 Saturated porous medium

The general coupled system of equations describing the hydro-mechanical response of a saturated porous medium is composed by the mass balance equation for the liquid phase and the three equilibrium equations.

Assuming water as interstitial fluid, the mass balance equations states, neglecting source and sinks terms.

$$\frac{\partial(n\rho_w)}{\partial t}+\nabla\cdot(\rho_w\mathbf{q}_w)=0 \qquad (1)$$

where n is the porosity (volume of voids over the overall volume of the porous medium), ρ_w the density of water and $\mathbf{q}_w$ the specific discharge with respect to a fixed coordinate system.

Equilibrium equations, neglecting inertial contributes, can be written as

$$\nabla\cdot(\boldsymbol{\sigma})+\mathbf{b}=0 \qquad (2)$$

where $\boldsymbol{\sigma}$ is the second order stress tensor and $\mathbf{b}$ a body force vector. Coupling between hydraulics and mechanics rises from the link between volumetric deformation of the solid matrix and the effective stress $\boldsymbol{\sigma}$' (Biot, 1941). Assuming a linear elastic costituive law for the solid matrix, it follows that:

$$\frac{\partial\varepsilon_{vol}}{\partial t}=-\frac{1}{1-n_0}\frac{\partial n}{\partial t}=\frac{E}{3(1-2\nu)}\frac{\partial p'}{\partial t} \qquad (3)$$

where ε_{vol} is the volumetric strain, E and ν respectively the Young modulus and Poisson coefficient and p' the so-called "effective pressure", i.e. one third of the first invariant of the effective stress tensor, defined subtracting from the total stress tensor the isotropic contribution of pore water pressure u_w

$$\boldsymbol{\sigma}'=\boldsymbol{\sigma}-u_w\mathbf{I} \qquad (4)$$

In the previous equation the common compression-positive notation of geomechanics holds. The set of equations is completed by Darcy's law, expressing the linear relationship between the specific discharge and the gradient of the piezometric head h (defined as the sum of the elevation z and the pressure head), through the hydraulic conductivity tensor $\mathbf{k}_w$

$$\mathbf{q}_w=-\mathbf{k}_w\nabla h=-\mathbf{k}_w\nabla\left(z+\frac{u_w}{\rho_w g}\right) \qquad (5)$$

and the constitutive equation for the liquid phase

$$\rho_w=f(u_w) \qquad (6)$$

2.1 Unsaturated porous medium

In an unsaturated porous medium, the void space is partly filled with air and partly by water. The relative quantity of water in a certain point of the porous medium is tipically indicated by the degree of saturation S_r, defined as the ratio between the volume of water in the pore space V_w and the volume of the overall pore space V_v

$$S_r=\frac{V_w}{V_v} \qquad (7)$$

Mass balance equations are to be written for both fluid phases. Therefore, being ρ_a and u_a respectively air density and air pressure and $\mathbf{q}_a$ air specific discharge

$$\begin{aligned}&\frac{\partial(nS_r\rho_w)}{\partial t}+\nabla\cdot(\rho_w\mathbf{q}_w)=0\\&\frac{\partial(n(1-S_r)\rho_a)}{\partial t}+\nabla\cdot(\rho_a\mathbf{q}_a)=0\end{aligned} \qquad (8)$$

Darcy-like laws holds also for unsaturated flow

$$\begin{aligned}\mathbf{q}_w&==-\mathbf{k}_w(S_r)\nabla\left(z+\frac{u_w}{\rho_w g}\right)\\\mathbf{q}_a&==-\mathbf{k}_a(S_r)\nabla\left(z+\frac{u_a}{\rho_a g}\right)\end{aligned} \qquad (9)$$

where $\mathbf{k}_a$ and $\mathbf{k}_w$ are now both functions of the degree of saturation. This dependence introduces a further coupling level, being the degree of saturation linked to the difference between u_a and u_w, by the means of the so-called water retention curve

$$S_r=g(u_a-u_w) \qquad (10)$$

For applied stress level sufficiently low to do not induce collapse of the solid matrix upon wetting, the constitutive relation for the porous medium can be considered, as a first approximation, linear and elastic, linking the strain tensor $\boldsymbol{\varepsilon}$ and the average skeleton stress $\boldsymbol{\sigma}$', defined for an unsaturated soil as

$$\boldsymbol{\sigma}'=\boldsymbol{\sigma}-u_a\mathbf{I}+S_r(u_a-u_w) \qquad (11)$$

Finally the constitutive equations for both fluid phases are to be specified

$$\begin{aligned}\rho_w&=f_w(u_w)\\\rho_a&=f_a(u_a)\end{aligned} \qquad (12)$$

3. Experimental set-up

An advanced oedometer cell, which allows for 3D electric tomography and seismic wave velocity measurements, has been used in the investigation. The execution of resistivity measurements for tomography requires the injection of current at two electrodes, while other pairs of electrodes are used to measure electric potential.

The loading scheme of the cell (see Comina et al., 2008 for further details) is the classical one of oedometers, in which a vertical load is applied to the soil sample under lateral confinement. Samples have a diameter of 130 mm and a maximum height of 60 mm. In this work, 40 mm height samples have been used. The inner walls of the oedometer are covered by an insulating and stiff plastic material, in order to allow the correct measurement of electric potentials. To avoid the presence of conductive elements that would cause electrical bridges on the cell top and bottom caps, and in order to allow the placement of electrodes, drainage is obtained by concentric rings having a tolerance of few micrometers between each of them (fig. 1). The equivalent hydraulic conductivity is about $6x10^{-6}$ m/s for both caps. For ERT measurements, 42 electrodes are hosted on the internal boundary of the cell (16 on the sidewall and 13 on each cap).

The equipment therefore allows for a mixed control of the mechanical boundary conditions (applied force on the upper cap, null displacements in radial direction on the lateral wall and null displacement at the bottom cap). From the hydraulic point of view, the equipment allows the imposition of a flux or a pressure for both fluid phases.

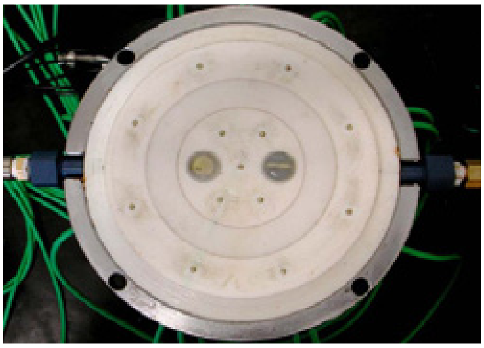

Figure 1. Drainage system on the bottom cap

4. Experimental results

4.1 Consolidation test

The first experimental results presented refer to an oedometer test run on a preconsolidated (100 kPa) kaolin sample. After every load increase a spatial reconstruction of the electrical conductivity has been carried out. The latter can be related to changes of porosity through Archie's law (1942):

$$\chi_s = \chi_w n^v S_r^q \qquad (13)$$

where χ_s is the overall electrical conductivity of the soil, χ_w is the electrical conductivity of the water phase, v and q are two exponents that account for the tortuosity and geometry of the porous medium. In this case v=1.5 and q was not taken into account due to the full saturation of the sample.

During compression under constant external load slight reduction of the averaged electrical conductivity has been observed (fig. 2), reflecting the reduced void space available for current flow, while the opposite occurred during swelling.

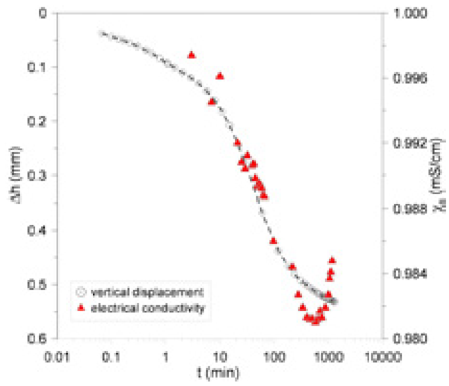

Figure 2. Time evolution of the vertical settlement (round symbols) and of the average electrical conductivity (red triangles) after application of a load increment of 25 kPa

4.2 Imbibition test

The second experimental test has been performed on a silty sand. Previously, the electrical and hydraulic behavior of the material in the unsaturated range has been investigated by the experimental determination of the relationships linking the electrical conductivity χ and the matric suction $s = u_a - u_w$ to the degree of saturation S_r. The first relation is necessary to interpret the electrical conductivity from the tomographic reconstruction, while the retention curve is a necessary information for the coupling between liquid and air phases.

As for the electrical behaviour, electrical conductivity has been measured in different homogeneous samples with increasing water contents at constant porosity. A solution 0.5 M of KCl has been used as interstitial fluid. Its conductivity is χ_w=6.86 x10^{-3} S/m. Experimental results expressed in terms of ratio between the conductivity measured in the different samples $\chi(S_r)$ and the value obtained for S_r = 1, $\chi(S_r = 1)$, allowed to estimate q = 2.0 for equation 13.

The link between degree of saturation and suction has been experimentally obtained by means of a suction controlled oedometer cell (Romero et al., 1995). Results of the wetting branch of the water retention curve have been interpreted through the van Genuchten relation (1980)

$$S_e = \frac{S_r - S_r^{RES}}{1 - S_r^{RES}} = \left(\frac{1}{1 + (\alpha s)^n} \right)^m \qquad (14)$$

where S_e and S_r^{RES} are respectively the effective and residual degree of saturation and α, n and m three fitting parameters. For the soil taken into consideration $\alpha = 4$ m^{-1}, n = 2.2 and m = 0.55.

The experimental results reported here for the silty sand regards a wetting stage at constant porosity. A cylindrical sample has been prepared by moist tamping at a dry density ρ_d = 1.49 g/cm^3 (that corresponds to a porosity n = 0.45) and a degree of saturation S_r = 0.2. After homogenization for about 12 h, the sample has been wetted by the inflow of a water volume of 90 cm^3 in 40s, applying a pressure of 50 kPa at the base. At the top of the cell a vertical stress of 2.84 kPa has been applied. At the end of the wetting stage, the bottom drainage system has been closed and the spatial distribution of electrical conductivity has been monitored for 3000 min with the ERT technique. Collected information has been used for the reconstruction of the conductivity map at selected time steps and therefore, by the means of Archie's law, the distribution of water inside the sample.

Figure 3 reports the tomographic reconstructions of a radial section of the sample orthogonal to the base for time t_0 = 0 min (before drainage opening), t_1 = 10 min, t_2 = 110 min and t_3=3000 min. All the tomographic reconstructions refer to the stage of the test performed with closed drainage systems and therefore don't allow for the reconstruction of the saturation front evolution. In figure 4 normalized conductivity values versus height along the axis of symmetry (r = 0 cm) of the cell are showed. The interpretation of the same results in terms of saturation is showed in figure 5.

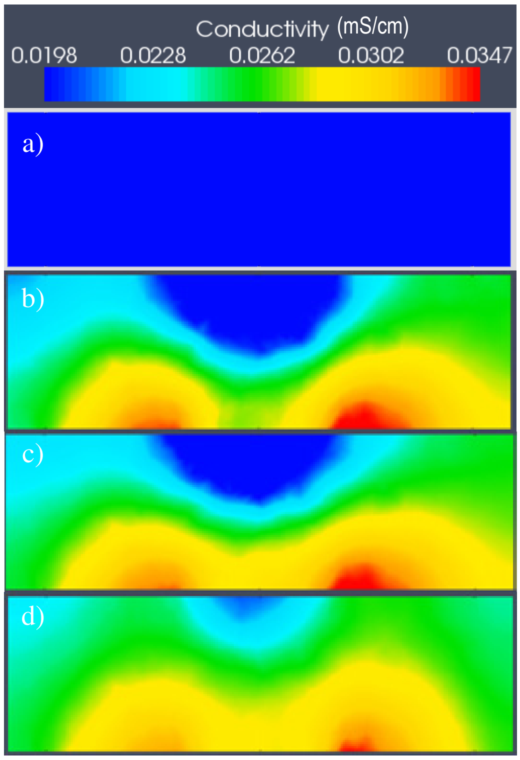

Figure 3. Time evolution of tomographic reconstructions. a) t=0, b) t=10 min, c) t=110 min, d) t=3000 min

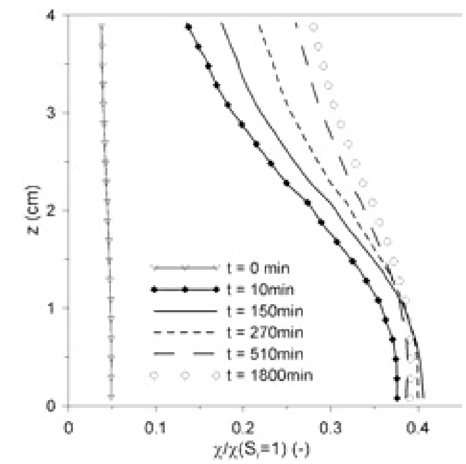

Figure 4. Normalized conductivity versus depth along the vertical axis of the cylindrical sample

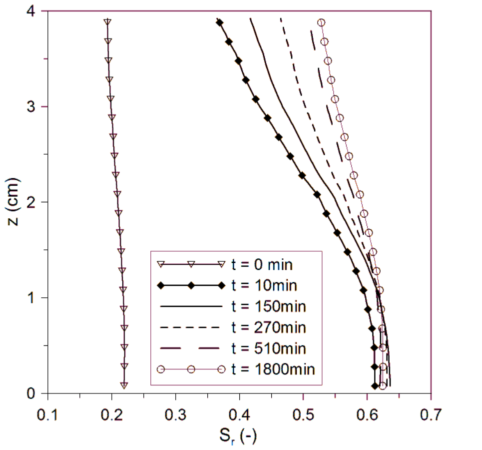

Figure 5. Distribution of local water saturation along the vertical axis of symmetry

5. Simulations

The configuration of constraints and the geometry of the equipment allow to analyze the problem in axis-symmetric conditions.

Simulations of both experimental tests have been conducted using the "Earth Science Module" of COMSOL MULTIPHYSICS.

In the first case, the poroelastic model has been used to describe the coupled interaction between fluid flow and deformation in a porous medium. Comparison between simulations and experimental data are reported in the following.

Figure 6 shows the time evolution of the vertical displacement due to the application of a load increment of 25 kPa.

In figure 7, a comparison between the time evolution of conductivity measurements and simulated values are presented. In both cases a good agreement between experimental data and the simulation is obtained.

The second test has been simulated as a coupled two-fluid flow problem and by means of the Richard's equation (assuming air pressure constant and equal to the atmospheric pressure). In both cases the wetting process has been simulated through a flux of water from the bottom of the cell trough the real draining surface. Initial suction has been imposed starting from the known initial water saturation (S_{r0}=0.2) and calculated by the means of the water retention curve (equation 14). During the analysis water and air densities have been considered constant.

From the solution of the finite element problem in terms of air and water pressure, degree of saturation and electrical conductivity have been obtained by equations 14 and 13. Figure 8 shows, for different time steps, a comparison between the results of the simulations in terms of evolution of degree of saturation along the height of the sample for r = 3 cm. For t<40 s (dashed lines) water inflow is allowed and induces a quick saturation of the elements closer to the draining surface. After drainage closing (t>40 s) the progressive reduction of pressure gradients induced by the wetting stage is appreciable by the spatial homogenization of the degree of saturation and, thus, of the electrical conductivity (figure 9). For the problem and the material taken into account, it is evident that the difference between the results given by the single fluid model and the coupled analysis is negligible.

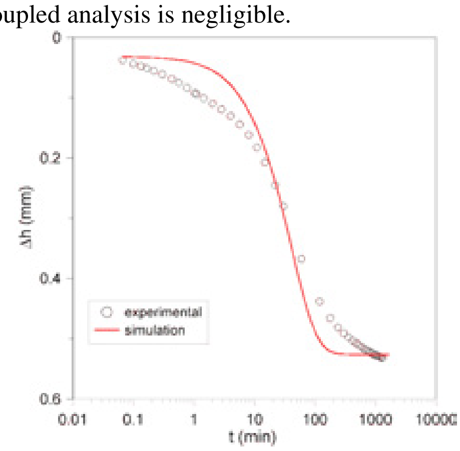

Figure 6. Time evolution of vertical displacement during consolidation stage in oedometer

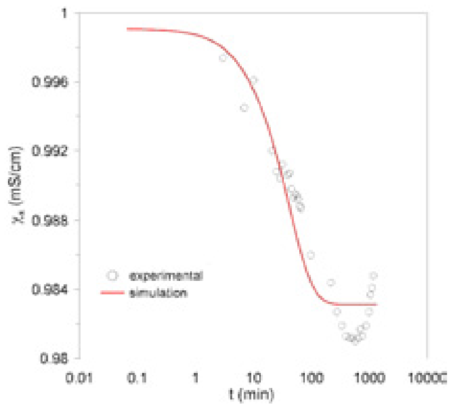

Figure 7. Time evolution of the normalized electrical conductivity after application of a load increment

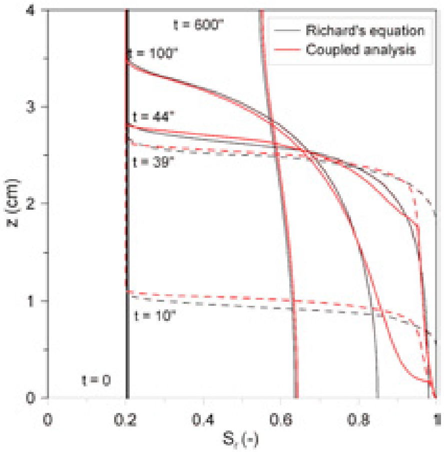

Figure 8. Time evolution of the degree of saturation along the vertical corresponding to r = 3cm. Dashed lines: open drainage, unbroken line: closed drainage.

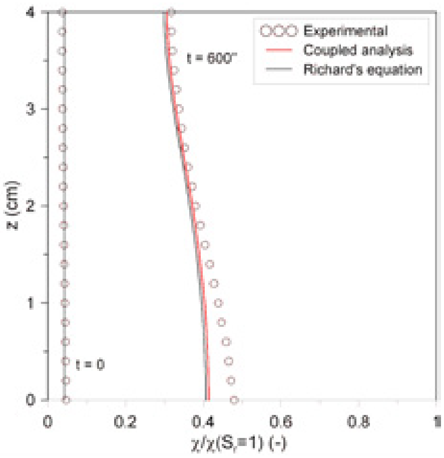

Figure 9. Comparison between experimental data, single fluid simulation and coupled analysis in terms of normalized electrical conductivity along a vertical (r = 3cm).

7. Conclusions

In the present work some hydro-mechanical coupled effects occurring in saturated and unsaturated soils together with their consequences on the electric conductivity of the material have been presented. Numerical simulations have been performed with the aim of checking the capabilities of tomographic reconstruction to deal with phenomena involving variation of porosity and water content in both saturated and unsaturated soils. Comparison between experimental data and numerical simulations gives promising results for the potential applications of the tomographic technique. On the other hand, COMSOL MULTIPHYSICS has allowed for a hierarchical solution of the investigated problems, helping to understand physical phenomena that have to be taken into account for a satisfactory prediction. Further investigations will concentrate on hydro-mechanical coupling in unsaturated conditions and the influence of temperature.

8. References

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