Presented at the COMSOL Conference 2009 Milan

FSI Analysis of Microcantilevers Vibrating in Fluid Environment

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Outline

- ***Brief Presentation of Materials and Microsystems Laboratory***
- ***Fluid Structure Interaction Problem (FSI)***
  - ***- Analytical Model***
  - ***- FSI in Time Domain***
  - ***- FSI in Frequency Domain***
- ***Results***
- ***Conclusion and Future Works***
- ***References***

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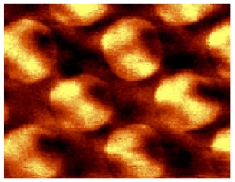
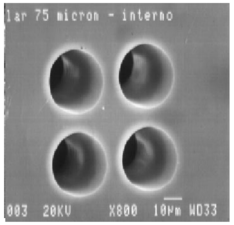
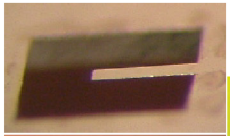
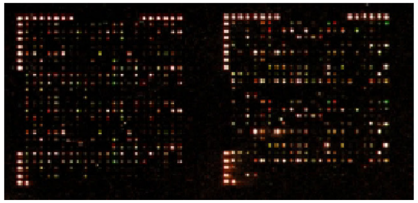
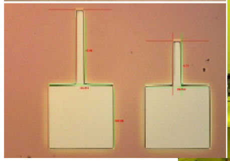



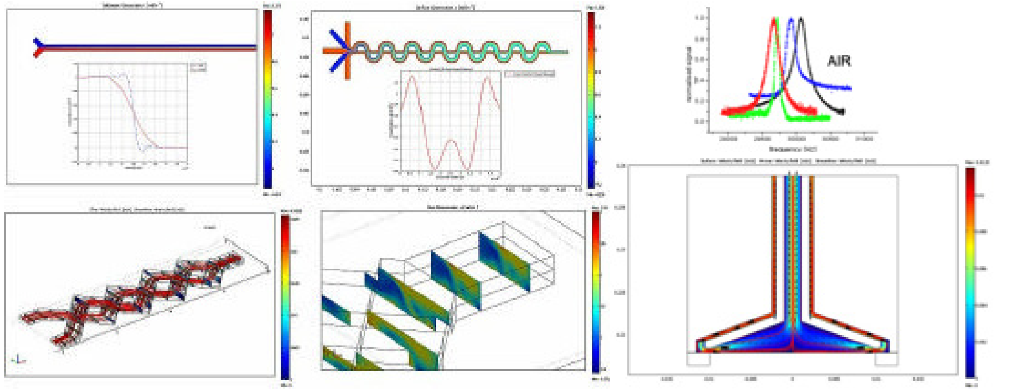
Materials and Microsystems Laboratory, *is managed by **Politecnico di Torino** and works on the design and realization of micro and nano systems prototypes with a specific focus on technological transfer.*



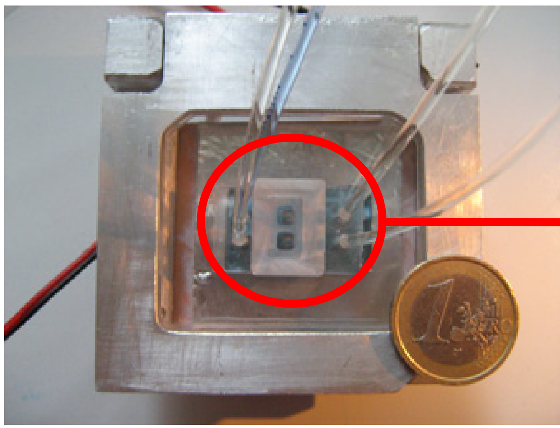
http://www.polito.it/micronanotech

MEMS *simulation activity is required for the design of microstructures or for their performance prevision.* ***F.E.M.*** *Simulations of microstructures behaviour is carried out by Comsol Multiphysics™*



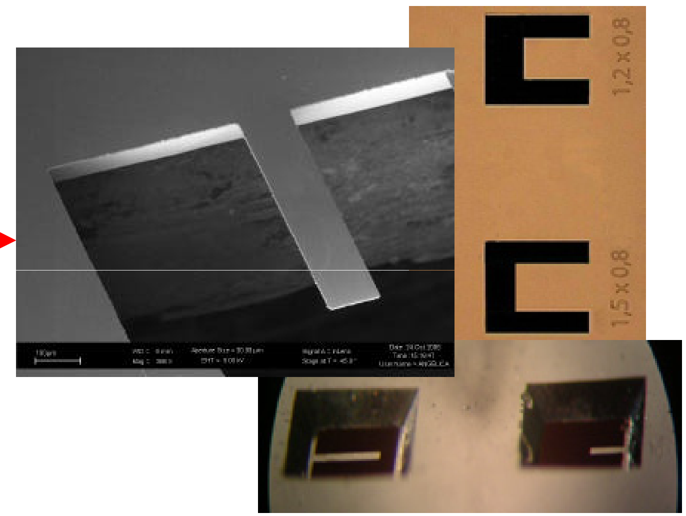
Fluid Structure Interaction (FSI)

of Microcantilivers Vibrating in Fluid Environment for Biosensing Applications



Lab–on-Chip (LOC)

for genomic and proteomic detection



Microcantilever based Bio-Sensor

Dynamic Measurement conducted evaluating

Q factors and Resonance Frequency

FSI Problem

Analytical Model

$$\frac{\omega_{R,n}}{\omega_{\mathrm{vac},n}} = \left[1 + \frac{\pi \rho b}{4 \rho_c h} \Gamma_r^f(\omega_{R,n}, n) \right]^{-1/2},$$

$$Q_n = \frac{(4\rho_c h / \pi \rho b) + \Gamma_r^f(\omega_{R,n}, n)}{\Gamma_i^f(\omega_{R,n}, n)},$$

Assumptions

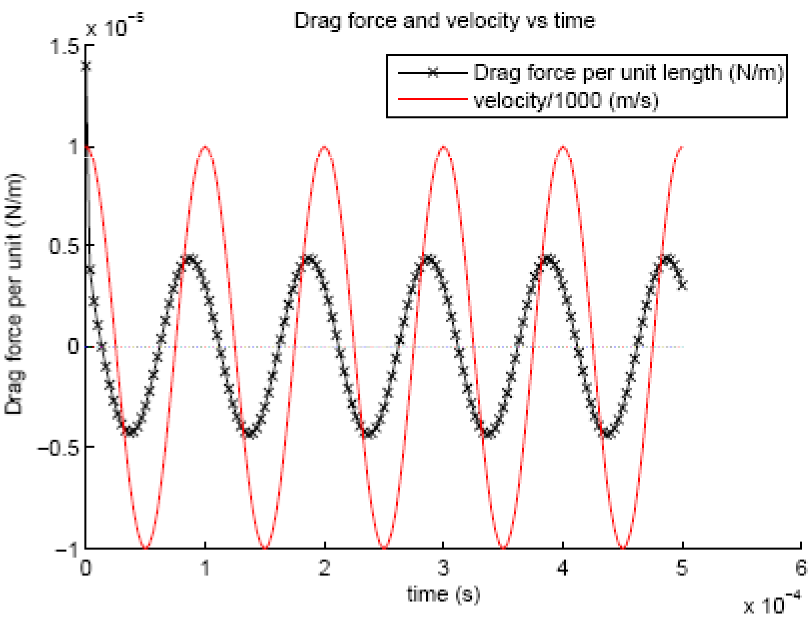
- *it is exact for a beam of infinite length vibrating in an incompressible viscous fluid*
- *thickness should be negligible compared to the length*
- *cantilever should have a constant cross section along the length*
- *modal cross talk is not taken in account*

Ref. C. A. Van Eysden, J. E. Sader, *Frequency response of cantilever beams immersed in viscous fluids with applications to the atomic force microscope: Arbitrary mode order*, J. Appl. Phys., 101, 044908 (2007).

FSI Problem in Time Domain

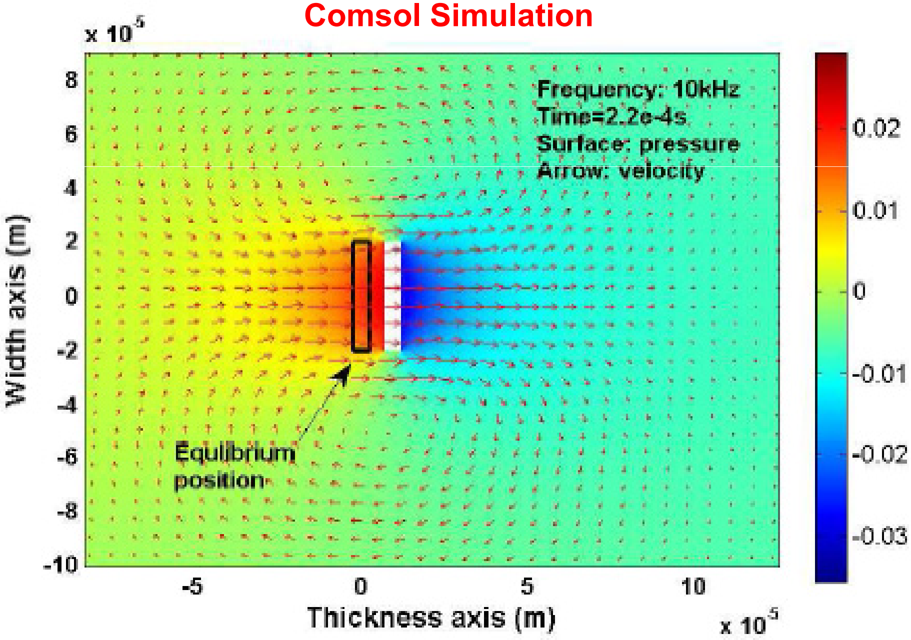
2D Models:

- *They hold only when length >> thickness and just for low mode numbers*
- *Time domain analysis*
- *Fitting step (possible source of inaccuracy)*



Ref. W. Zhang, M. Requa, K. Turner, *Determination of Frequency Dependent Fluid Damping of Micro and Nano Resonators for Different Cross-Sections*, Nanotech 2006, Boston, MA May (2006)

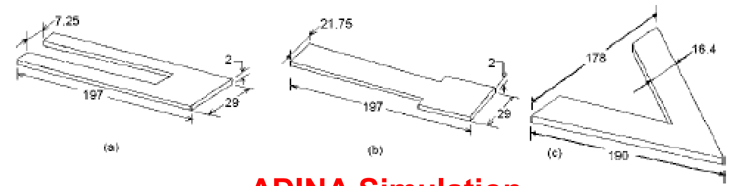
Comsol Simulation



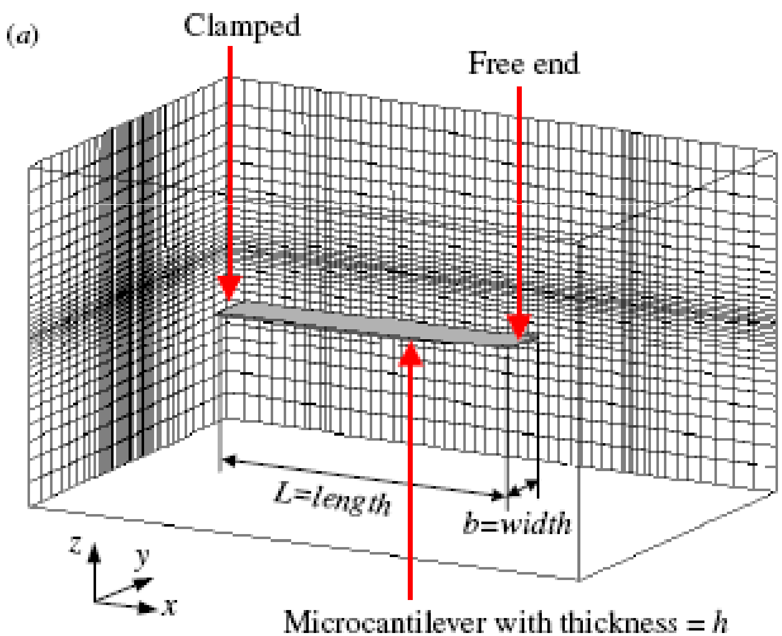
Ref. W. Zhang, K. Turner, *Frequency dependent fluid damping of micro/nano flexural resonators: Experiment, model and analysis*, Sens. Act. A, **134**, 594–599 (2007).

FSI Problem in Time Domain

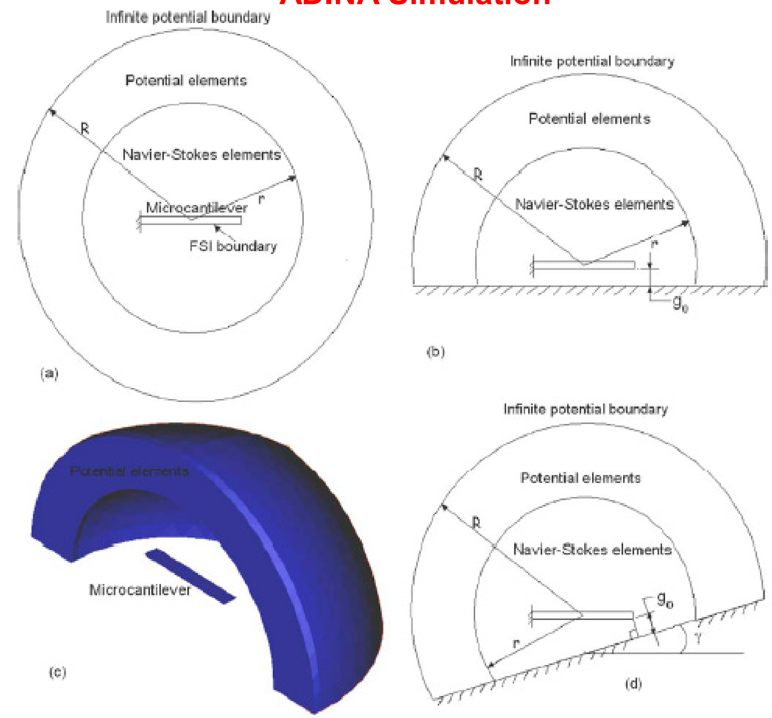
3D Models:



CFD-ACE+ Simulation



ADINA Simulation

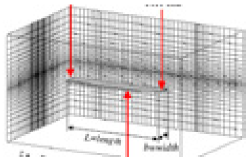


Ref. J. H. Lee, S. T. Lee, C. M. Yao, W. Fang, *Comments on the size effect on the microcantilever quality factor in free air space*, J. Micromech. Microeng., **17**, 139–146 (2007).

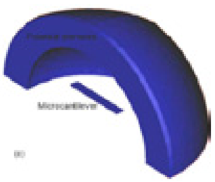
Ref. S. Basak, A. Raman, *Hydrodynamic loading of microcantilevers vibrating in viscous fluids*, J. Appl. Phys, **99**, 114906 (2006).

FSI Problem in Time Domain: drawbacks

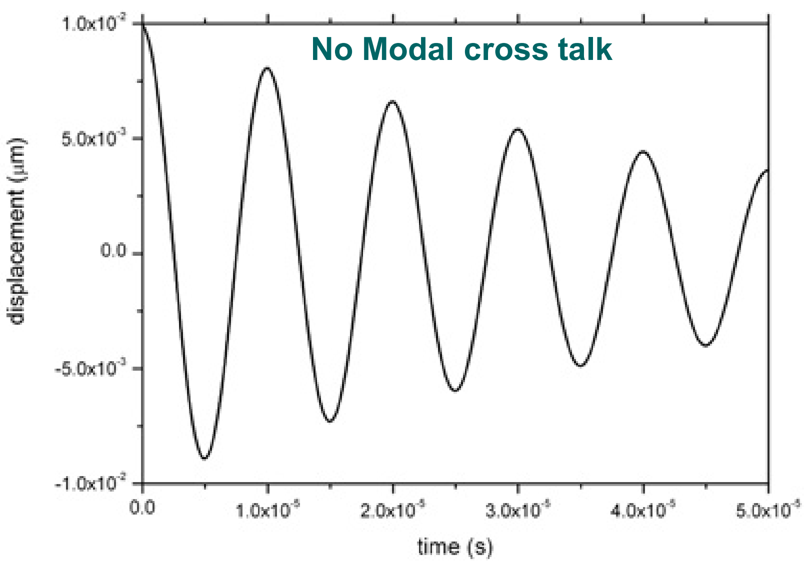
- *Initial displacement applied on the cantilever free end [Lee]*

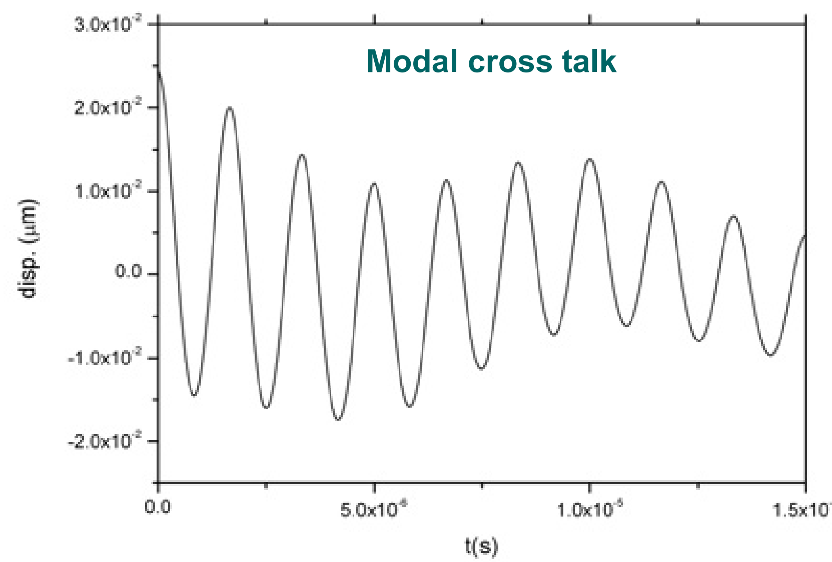


- *Vacuum eigenfrequency analysis results are the input for the time dependent analysis in fluid environment [Basak]*



- *Time dependent analysis*
- *Data fitting and filtering steps (Prony analysis) possible sources of inaccuracy*
- *Modal cross talk in not taken into account*





FSI Problem in Frequency Domain

Our method (*Comsol Multiphysics*) → Equations are written in frequency domain → *An eigenvalue problem is obtained*

Frequency Domain FSI Analysis vs Time Domain FSI Analysis

- ***On equal mesh density, an eigenfrequency analysis is certainly less time consuming than a time domain one***
- ***The convergence study regards just the mesh density and not, as in the time domain approach, both mesh density and time parameters***
- ***Mode shapes and frequency in fluid are directly calculated so that no curve fitting step is needed. A possible source of inaccuracy is therefore eliminated***

FSI Problem in Frequency Domain

Our method

(*Comsol Multiphysics*)

Equations are written in frequency domain

An eigenvalue problem is obtained

Equations in frequency domain $\Rightarrow$ $\frac{\partial}{\partial t} = j\omega$

Solid

$$\nabla \cdot \bar{\bar{\sigma}} = -\rho_s \omega^2 \bar{u}$$

Approximation 1

If the amplitude of structure vibration is far smaller then any other length scale the non-linear convective inertial term could be dropped

Fluid

$$-\nabla p + \mu \nabla^2 \bar{v} - \bar{v}\nabla\bar{v} = \rho_f j\omega\bar{v}$$

$$\nabla \cdot \bar{v} = 0$$

$$-\nabla p + \mu \nabla^2 \bar{v} - \underbrace{\cancel{\bar{v}\nabla\bar{v}}}_{=0} = \rho_f j\omega\bar{v}$$

FSI Problem in Frequency Domain

Our method

(*Comsol Multiphysics*)

Stokes equations are written in frequency domain

An eigenvalue problem is obtained

Approximation 2

Fluid vorticity, plays a significant role just in proximity of the vibrating structure; it is possible to further simplify Stokes equations in the region of fluid domain sufficiently far from the cantilever.

$$\nabla \times \bar{v} = 0 \quad \rightarrow \quad \nabla^2 \bar{v} = 0, \qquad \bar{v} = \nabla \phi$$

Φ = Scalar Velocity Potential

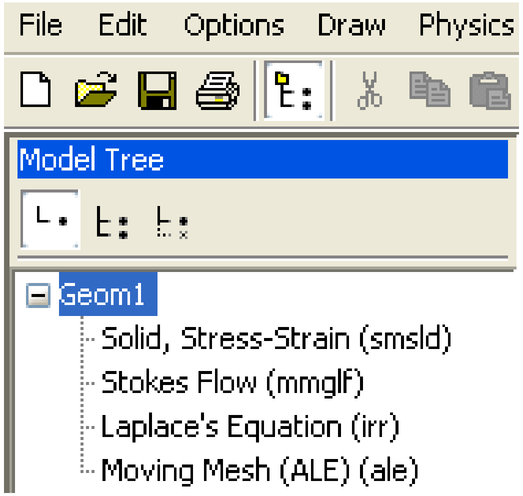
Equations in frequency domain

$$-\nabla p + \mu \cancel{\nabla^2} \bar{v} = \rho_f j\omega \bar{v} \qquad \text{= 0} \qquad \Rightarrow \qquad p\_irr = -\rho_f j\omega\phi$$

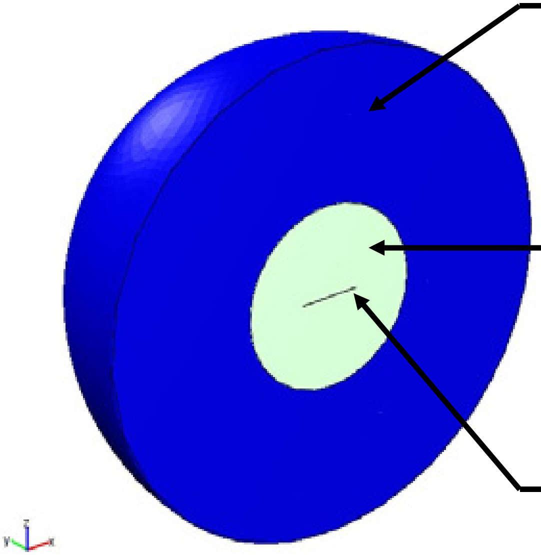
$$\nabla \cdot \bar{v} = 0 \qquad \Rightarrow \qquad \nabla^2 \phi = 0$$

FSI Problem in Frequency Domain

Use of Comsol Multiphysics



Fluid Domain can be subdivided in:



Fluid Far Field

$$\begin{cases} \nabla^2 \phi = 0 \\ p = -jomega\_smsld \cdot \rho_f \phi \\ \bar{v} = \nabla \phi \end{cases}$$

Φ is the only dependent variable of this subdomain

Fluid Near Field

$$\begin{cases} -\nabla p + \mu \nabla^2 \bar{v} = -jomega\_smsld \cdot \rho_f \bar{v} \\ \nabla \cdot \bar{v} = 0 \end{cases}$$

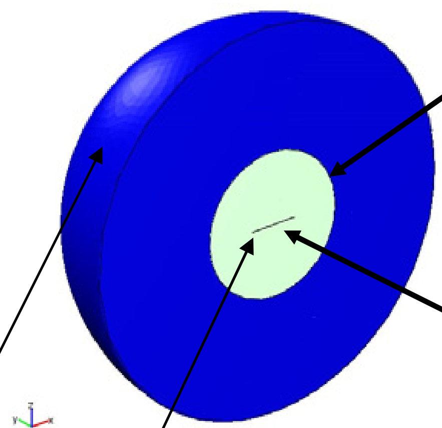
Solid - Cantilever

$$\nabla \cdot \bar{\bar{\sigma}} = (jomega\_smsld)^2 \cdot \rho_s \bar{u}$$

FSI Problem in Frequency Domain

Boundary Conditions

The Model contains two type of interfaces



Near Field – Far Field

$$\begin{cases} p = p\_irr \\ \dfrac{\partial \phi}{\partial n} = \bar{v} \cdot \bar{n} \end{cases}$$

Solid – Near Field

$$\begin{cases} \bar{v} = jomega\_smsld \cdot \bar{u} \\ f^{ext}\_smsld = \bar{T}\_mmglf \end{cases}$$

$$\bar{u} = 0$$

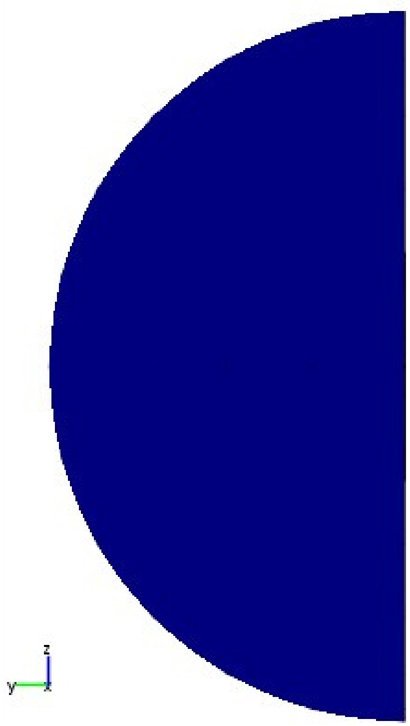
Clamp boundary condition is assigned to the fixed end of the cantilever

Dirichlet boundary condition is assigned external surface of the fluid far field to mimics an "open" condition since the value of the pressure is constrained to zero.

FSI Problem in Frequency Domain

Symmetry Conditions

Since the model is symmetrical with respect to xz plane, symmetry conditions are required both for the solid and the fluid domains



$$u_y = 0$$

Solid - Cantilever

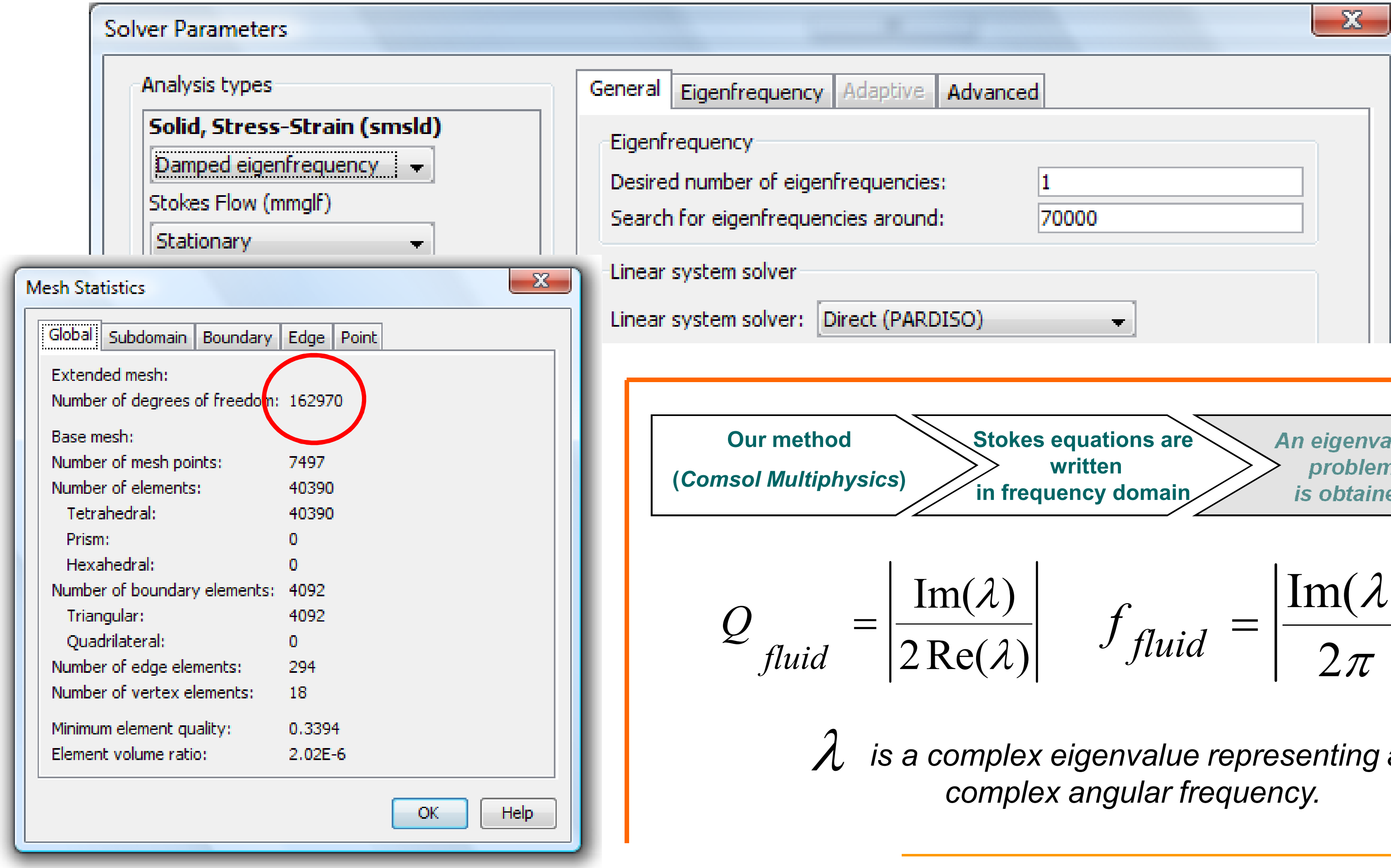
$$\begin{cases} v_y = 0 \\ T_{(x,z)} = 0 \end{cases}$$

Fluid Near Field

$$\frac{\partial \phi}{\partial n} = 0$$

Fluid Far Field

FSI Problem in Frequency Domain

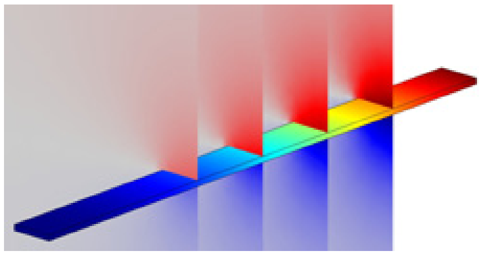


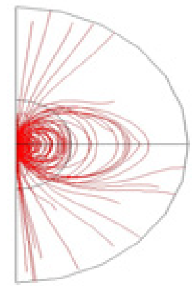
Our method (*Comsol Multiphysics*) → **Stokes equations are written in frequency domain** → ***An eigenvalue problem is obtained***

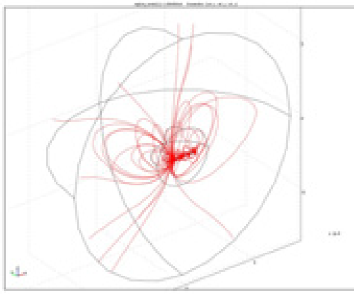
$$Q_{fluid} = \left| \frac{\mathrm{Im}(\lambda)}{2\,\mathrm{Re}(\lambda)} \right| \qquad f_{fluid} = \left| \frac{\mathrm{Im}(\lambda)}{2\pi} \right|$$

λ *is a complex eigenvalue representing a complex angular frequency.*

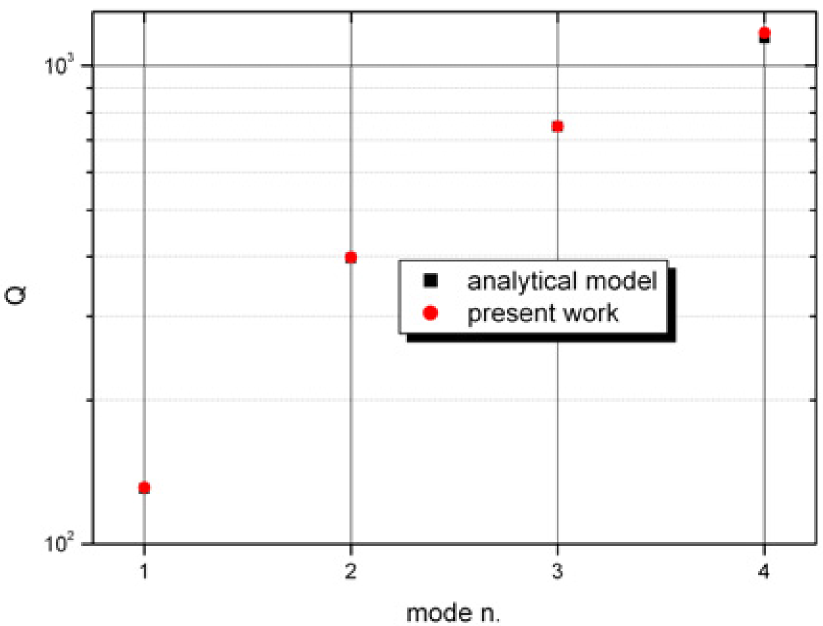
Results: benchmark with the analytical model

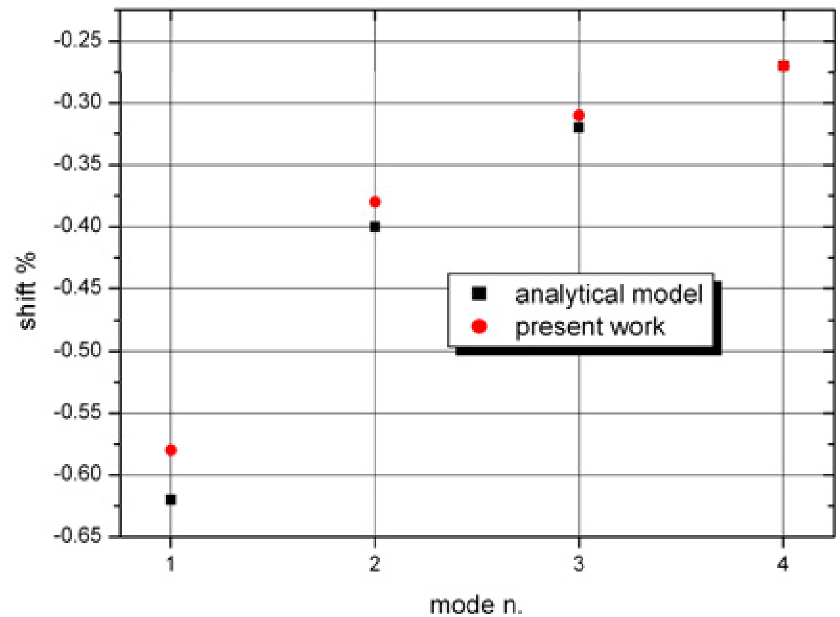






Streamlines





Ref. C. A. Van Eysden, J. E. Sader, *Frequency response of cantilever beams immersed in viscous fluids with applications to the atomic force microscope: Arbitrary mode order*, J. Appl. Phys., 101, 044908 (2007).

Results: model validation

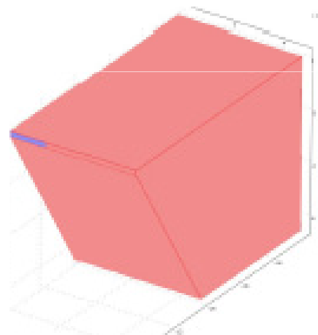
Experimental Data

Computational Data 1

Computational Data 2

mode n.	f^e		f^a		f^B		f^{pw1}		f^{pw2}	
	data (KHz)	err%	data (KHz)	err%	data (KHz)	err%	data (KHz)	err%	data (KHz)	err%
1	69.87	-	70.61	1.06	70.49	0.89	70.49	0.89	69.52	-0.50
2	438.5	-	443.50	1.14	441.6	0.71	442.45	0.90	436.23	-0.52

Table 1a. Comparison between experimental (e) [9, 14], analytical (a) [16] and computational (B "Basak et al. [9]", pw1, 2 "present work") results about the first two mode resonance frequencies in air environment of cantilever C2 [9, 14].



mode n.	Q^e		Q^a		Q^B		Q^{pw1}		Q^{pw2}	
	data	err%	data	err%	data	err%	data	err%	data	err%
1	136	-	130.7	-3.89	144.8	6.47	131.4	-3.38	130.4	-4.12
2	395	-	396.8	0.45	367	-7.09	397.7	0.68	394.4	-0.15

Table 1b. Comparison between experimental (e) [9, 14], analytical (a) [16] and computational (B "Basak et al. [9]", pw1, 2 "present work") results about the first two mode Q factors in air environment of cantilever C2 [9, 14].

Results: squeeze film damping simulation

Experimental Data

Present Work

Film Damping Model

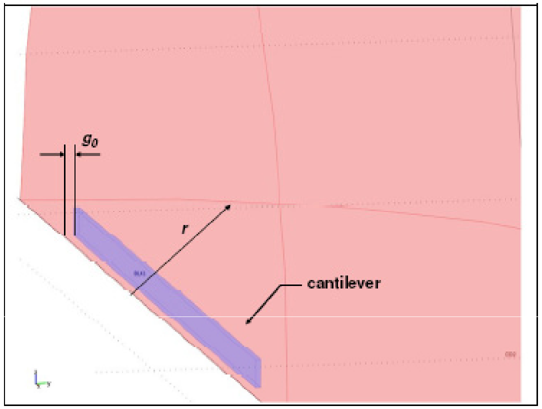
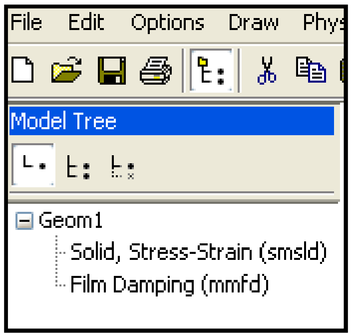


Figure 9. Detail of the 3D FSI model about a cantilever vibrating near a surface at distance g_0 [15]. Only the near field is showed.



Q^{e}_{air}	Q^{a}_{air}		Q^{s1}_{air}		Q^{s2}_{air}	
	data	err %	data	err %	data	err %
5.7	6.0	5.1	5.5	-3.0	12.7	123.81

Table 2b. Comparison between experimental (e) [15], analytical (a) [18] and computational results (subdivided in the ones calculates through the full 3D FSI model, "s1" superscript, and those obtained by the "Solid, stress-strain with film Damping" application mode, "s2" superscript) about the first mode Q factor of cantilever A [15].

f^{e}_{vac}	$shift^{e}_{air}$	f^{a}_{vac}		$shift^{a}_{air}$		f^{s1}_{vac}		$shift^{s1}_{air}$		$shift^{s2}_{air}$	
data (KHz)	data	data (KHz)	err %	data	err %	data (KHz)	err %	data	err %	data	err %
18.33	-2.10	18.45	0.68	-0.74	-64.91	18.54	1.16	-1.00	-52.42	-0.07	-96.67

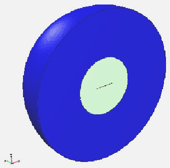
Table 2a. Comparison between experimental (e) [15], analytical (a) [18] and computational results (subdivided in the ones calculates through the full 3D FSI model, "s1" superscript, and those obtained by the "Solid, stress-strain with film Damping" application mode, "s2" superscript) about the first mode resonance frequency of cantilever A [15].

Conclusions

Frequency Domain Approach

$$\frac{\partial}{\partial t} = j\omega$$

Subdivision of the fluid domain in a Near and a Far Field



Strong reduction of the computation time

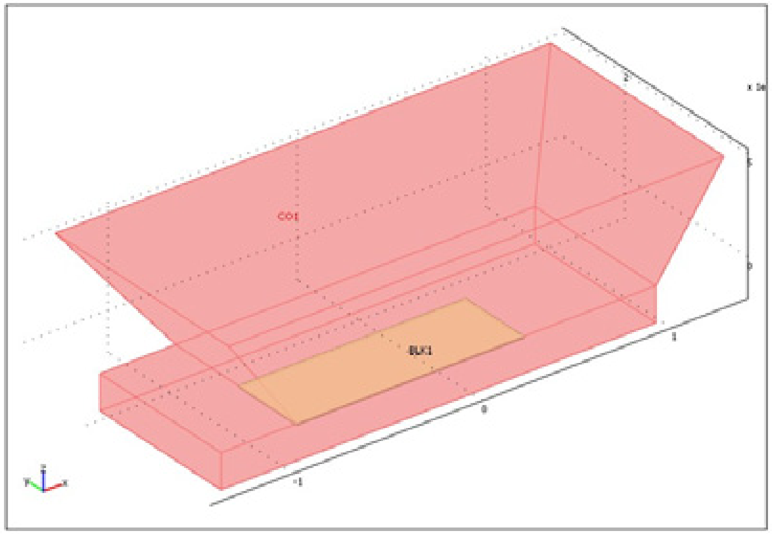
+

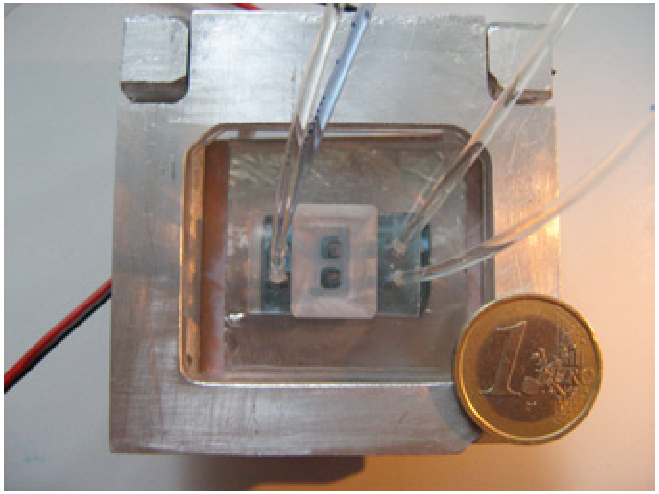
high degree of accuracy of results

Future Works

Design and optimization of a fluid cell containing a vibrating <u>Cantilever Plate</u>

- *Eigenfrequency Analyses in Fluid Environment*
- *Frequency Response Analyses in Fluid Environment with Magnetic Excitation*





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7. W. Zhang, K. Turner, *Frequency dependent fluid damping of micro/nano flexural resonators: Experiment, model and analysis*, Sens. Act. A, **134**, 594–599 (2007).
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15. Cocuzza et al., *Silicon laterally resonant microcantilevers for absolute pressure measurement with integrated actuation and readout*, J. Vac. Sci. Technol. B, **26**, 1071-1023 (2008).
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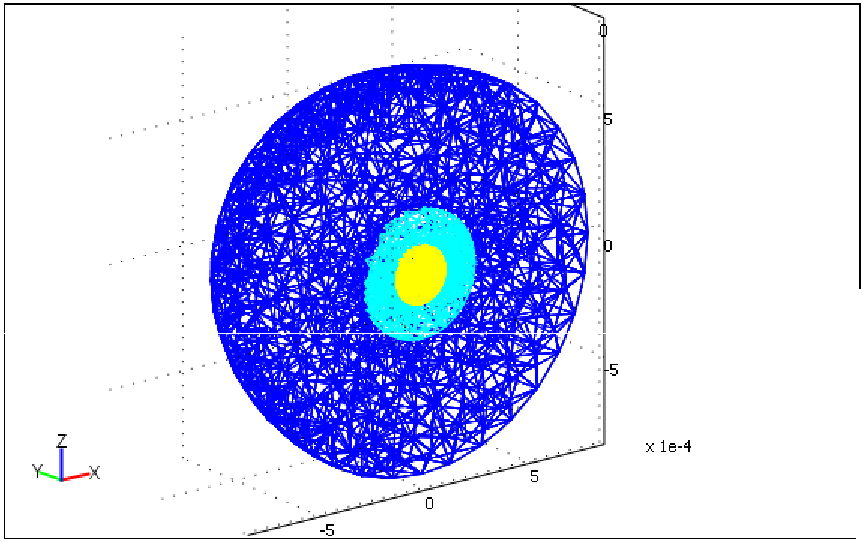
18. C. P. Green, J. E. Sader, *Frequency response of cantilever beams immersed in viscous fluids near a solid surface with applications to the atomic force microscope*, J. Appl. Phys. **98**, 114913 (2005).

Thanks For Your Attention

FSI Problem in Frequency Domain

Domain Optimization



Near Field

Near Field