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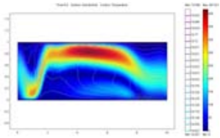

# FINITE ELEMENT ANALYSIS OF FERROFLUID COOLING OF HEAT-GENERATING DEVICES

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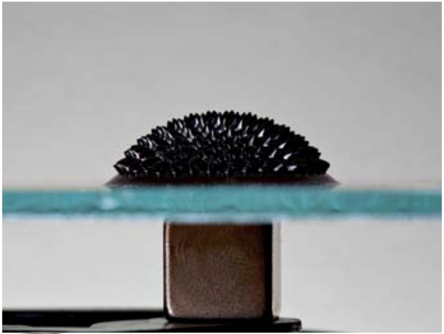
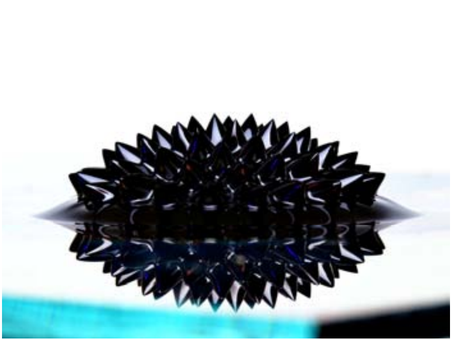
**CONTENTS**

# MAGNETORHEOLOGICAL FLUID 3D STRUCTURE



**THERMAL MANAGMENT** has become a critical aspect of today’s electronic systems, which often include many high-performance circuits that dissipate large amounts of heat. Many of these components require efficient cooling to prevent overheating.

This case examines the temperature field in the ferrofluid and in the electronic component with heat source. The ferrofluid transports heat by convection and conduction. Finally, to approximates the electronic component that requires cooling, the model uses a rectangular block with a given volume heat source. The electronic component transports thermal energy by pure conduction.

**ASSUMPTIONS**

- The fluid is assumed to be electrically nonconducting so that the ferrofluid flow does not induce any electromagnetic current in it.
- It is assumed also that there is no electric field effects.
- The variation in the magnetic field that could occur due to temperature gradients within the ferrofluid is negligible.
- Anisotropy in fluid properties are negligible.

## THE GOVERNING EQUATIONS

**The Maxwell's equations**, simplified for a non-conducting fluid with no displacement current, become,

$$\nabla \cdot \mathbf{B} = 0\,,\; \nabla \times \mathbf{H} = \mathbf{0} \tag{1}$$

where $\mathbf{B}$ is the magnetic induction, $\mathbf{M}$ is the magnetization vector and $\mathbf{H}$ is the magnetic field vector.
The constitutive relation

$$\mathbf{B} = \mu_0 \left( \mathbf{M} + \mathbf{H} \right) \tag{2}$$

where $\mu_0$ is a magnetic permeability in vacuum.

In the magnetostatic case where there are no currents present it is possible to define a magnetic scalar potential by the relation $\mathbf{H} = -\nabla V_m$. The scalar potential for the magnetic dipole is given by:

$$V_m(\mathbf{x}) = V_m(x_1, x_2) = \frac{\gamma}{2\pi} \frac{x_1 - a}{(x_1 - a)^2 + (x_2 - b)^2}\,, \tag{3}$$

where $\gamma$ is the magnetic field strength at the source (of the wire) and $(a,b)$ is the position where the source of the magnetic dipole is located (below the channel).

**The momentum equation** for magnetoconvective flow is modified typical natural convection equation by addition of a magnetic term

$$\rho_0\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right)=-\nabla p+\nabla\cdot\mathbf{S}+\alpha\rho_0 g\left(T-T_0\right)\mathbf{k}+\left(\mathbf{M}\cdot\nabla\right)\mathbf{B} \tag{4}$$

where $\rho_0$ is the density, $\mathbf{v}$ is the velocity vector, $p$ is the pressure, $T$ is the temperature of the fluid, $T_0$ is the reference temperature , $\mathbf{S}$ is the extra stress tensor, $\mathbf{k}$ is the unit vector of the gravity force and $\alpha$ is the thermal expansion coefficient of the fluid.

**The energy equation** for an incompressible fluid which obeys the modified Fourier's law is

$$\rho_0 c\left(\frac{\partial T}{\partial t}+\mathbf{v}\cdot\nabla T\right)=k\nabla^2 T+\eta\Phi-\mu_0 T\frac{\partial \mathbf{M}}{\partial T}\cdot\left(\left(\mathbf{v}\cdot\nabla\right)\mathbf{H}\right) \tag{5}$$

where $\eta$ is the viscosity and $\eta\Phi$ is the viscous dissipation

$$\Phi = \left( 2\left( \left(\frac{\partial v_1}{\partial x}\right)^2 + \left(\frac{\partial v_2}{\partial y}\right)^2 \right) + \left(\frac{\partial v_2}{\partial x} + \frac{\partial v_1}{\partial y}\right)^2 \right). \qquad ()$$

The last term in the energy equation represents the thermal power per unit volume due to the magnetocaloric effects.

**The heat equation -** the mathematical model for heat transfer in solid by conduction is:

$$\rho_s c_s \frac{\partial T}{\partial t} + \nabla \cdot \left(-k_s \nabla T\right) = Q_s \qquad (6)$$

Quickly review the variables and quantities in this equation: $T$ is the temperature, $\rho_S$ is the solid density, $c_s$ is the heat capacity, $k_s$ is the thermal conductivity and $Q_s$ is a heat source or heat sink.

The heat source describes heat generation within the domain. We can express heating and cooling with positive and negative values, respectively. Quantity $Q_s$ is defined as power per volume ($W / m^3$ in SI units).

## THE KELVIN BODY FORCE FOR THE MAGNETOCONVECTIVE FLOW

The last term in the momentum equation represents the Kelvin body force per unit volume

$$\mathbf{f} = (\mathbf{M} \cdot \nabla)\mathbf{B}, \tag{7}$$

which is the force that a magnetic fluid experiences in a spatially non-uniform magnetic field.

In a narrow temperature range, the magnetization vector $\mathbf{M}$ of the magnetic fluid can be expressed as a linearized function of $\mathbf{H}$ and the fluid temperature

$$\mathbf{M} = \chi_m \mathbf{H} \text{ where } \chi_m = \chi_m(T) = \frac{\chi_0}{1+\alpha(T-T_0)} \tag{8}$$

is the total magnetic susceptibility, $\chi_0$ is the differential magnetic susceptibility of the ferrofluid.

The magnetic induction vector can be written in the form $\mathbf{B} = \mu_0(1+\chi_m)\mathbf{H}$.

## DIMENSIONLESS VARIABLES AND NUMBERS

Non-dimensionalised variables and scales are defined as follow:

$$t' = \frac{\kappa}{h^2} t\,; \quad \mathbf{x}' = \frac{1}{h}\mathbf{x}\,; \quad \mathbf{v}' = \frac{\mathbf{v}}{v_r} = \frac{h}{\kappa}\mathbf{v}\,; \quad p' = \frac{p}{\rho\, v_r^2} = \frac{ph^2}{\rho\kappa^2}\,; \quad \mathbf{H}' = \frac{\mathbf{H}}{H_r}\,; \quad H' = \frac{H}{H_r}\,; \quad T' = \frac{T}{\delta T}\,;$$

$$\eta' = \frac{\eta}{\eta_0}\,,\; Q_s' = \frac{Q_s}{Q_0}\,.$$

Moreover some of the dimensionless ratios can be replaced with well-known parameters:

$$Pr = \frac{\eta_0}{\rho_0 \kappa}\,,\; Ra = \frac{\alpha\rho_0 g h^3 \delta T}{\eta_0 \kappa}\,,\; Ec = \frac{v_r^2}{c\delta T} = \frac{\kappa^2}{c\delta T h^2}\,,\; Re = \frac{h\rho_0 v_r}{\eta_0} = \frac{\rho_0 \kappa}{\eta_0}\,,$$
$$Mn = \frac{\mu_0 H_r^2}{\rho_0 v_r^2} = \frac{\mu_0 H_r^2 h^2}{\rho_0 \kappa^2}\,,\; Qn = \frac{Q_0 h^2}{\rho_s c_s \kappa_s \delta T}\,. \tag{9}$$

## DIMENSIONLESS EQUATIONS

The dimensionless form of Navier-Stokes and thermal diffusion equations:

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + Ra\, Pr \left( T - \frac{T_0}{\delta T} \right) \mathbf{k} + Pr\, \nabla \cdot \mathbf{S} + Mn\, \mathbf{f} \tag{10}$$

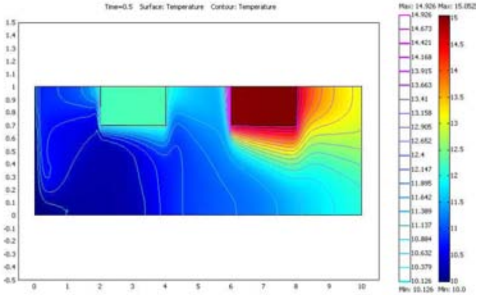
$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \nabla^2 T + Pr\, Ec\, \eta \Phi + Mn\, Ec\, T \frac{\partial (\chi_m \mathbf{H})}{\partial T} \cdot ((\mathbf{v} \cdot \nabla) \mathbf{H}) \tag{11}$$

$$\mathbf{f} = \frac{1}{2}\ \chi_m (1 + \chi_m) \nabla H^2 + \chi_m \mathbf{H} ((\mathbf{H} \cdot \nabla) \chi_m)\ \chi_m = \chi_m (T(\mathbf{x})) = \frac{\chi_0}{1 + (\alpha \delta T) \left( T(\mathbf{x}) - \frac{T_0}{\delta T} \right)} \tag{12}$$

$$\frac{\partial T}{\partial t} = \frac{\kappa_s}{\kappa} \nabla^2 T + Qn Q_s \tag{13}$$

## FORMULATION OF THE PROBLEM

The viscous, two-dimensional, incompressible and laminar ferromagnetic fluid flow is considered in this paper. Flow takes place in channel between two parallel flat plates. There are rectangular blocks (heat-generating devices) below the upper wall with a given volume heat source. The length of the channel is $L$ and distance between plates is $h$. Below the channel the magnetic dipole is located at point $(a,b)$.



The following boundary conditions for dimensionless variables are assumed:

- For upper wall: The velocity is 0 (no slip condition). Insulation condition for heat transfer by conduction (in solid domain) $\mathbf{n}\cdot\mathbf{q}=\mathbf{n}\cdot\left(-k_s\nabla T\right)=0$ and for heat transfer by conduction and convection (in fluid domain) $\mathbf{n}\cdot\mathbf{q}=\mathbf{n}\cdot\left(-k_f\nabla T+\rho_f c_f T\mathbf{u}\right)=0$ specifies where the domain is well insulated.
- For lower wall: The velocity is 0 (no slip condition). Insulation condition for heat transfer by conduction and convection (in fluid domain) $\mathbf{n}\cdot\mathbf{q}=\mathbf{n}\cdot\left(-k_f\nabla T+\rho_f c_f T\mathbf{u}\right)=0$.
- For inlet (left wall): The temperature is $\frac{T_l}{\delta T}$ where $\delta T=\left|T_u-T_l\right|$. At the inlet boundary there is a parabolic laminar flow profile given by equation $u_{in}=-4\frac{u_0}{u_r}y\left(y-1\right)$ for $y\in\left\langle 0,1\right\rangle$.
- For outlet (right wall): The convective flux is assumed for temperature, $\mathbf{n}\cdot\left(-k_s\nabla T\right)=0$. Pressure outlet is assumed, $\left(-p\mathbf{I}+\mathbf{S}\right)\mathbf{n}=-p_0\mathbf{n}$, where $p_0$ is the dimensionless atmospheric pressure.

- For solid-fluid interface: The velocity is 0 (no slip condition). Continuity equation for heat transfer equation $\mathbf{n}\cdot\left(\mathbf{q}_s - \mathbf{q}_f\right) = 0$ where $\mathbf{q}_s = -k_s \nabla T$ and $\mathbf{q}_f = -k_f \nabla T + \rho_f c_f T\mathbf{u}$.

The following initial conditions for dimensionless variables are assumed: fluid is motionless (velocity is zero), pressure is zero and temperature is $\frac{T_l}{\delta T}$ for whole domain (with fluid and solid).

Time-dependent flow is considered for time $t \in \left\langle 0{,}0.5 \right\rangle$. The problem is solved with finite element method COMSOL code using direct UMFPACK linear system solver. Relative and absolute tolerance used in calculations are 0.05 and 0.005, respectively.

Table 1. Fluid

| Quantity | Variable | Unit | Value |
|---|---|---|---|
| Density | $\rho_0$ | $\left[\frac{kg}{m^3}\right]$ | 1180.0 |
| Viscosity | $\eta_0$ | $\left[\frac{kg}{m \cdot s}\right]$ | 0.08 |
| Thermal conductivity | $k$ | $\left[\frac{J}{m \cdot s \cdot K}\right]$ | 0.06 |
| Heat capacity | $c$ | $\left[\frac{J}{kg \cdot K}\right]$ | 4200 |
| Thermal diffusivity (diffusion coefficient) | $\kappa = \frac{k}{\rho_0 c}$ | $\left[\frac{m^2}{s}\right]$ | 1.210654e-7 |
| Thermal expansion coefficient | $\alpha$ | $\left[\frac{1}{K}\right]$ | 5.6e-3 |
| Magnetic susceptibility | $\chi_0$ | - | 6e-2 |

Table 2. Solid

| Quantity | Variable | Unit | Value |
|---|---|---|---|
| Density | $\rho_s$ | $\left[\frac{kg}{m^3}\right]$ | 8960 |
| Thermal conductivity | $k_s$ | $\left[\frac{J}{m \cdot s \cdot K}\right]$ | 401 |
| Heat capacity | $c_s$ | $\left[\frac{J}{kg \cdot K}\right]$ | 384 |
| Thermal diffusivity (diffusion coefficient) | $\kappa_s = \frac{k_s}{\rho_s c_s}$ | $\left[\frac{m^2}{s}\right]$ | 1.165481e-4 |
| Heat source | $Q$ | $\left[\frac{J}{s \cdot m^3}\right]$ | 8.0e+8 |

Table 3. Flow

| Quantity | Variable | Unit | Value |
|---|---|---|---|
| Velocity | $u_0$ | $[m/s]$ | 5.0e-3 |
| Characteristic velocity | $v_r$ | $[m/s]$ | 1.210654e-7 |
| Magnetic permeability of a vacuum | $\mu_0 = 4\pi \cdot 10^{-7}$ | $[N/A^2]$ | 1.2566e-6 |
| Difference of temperatures | $\delta T$ | $[K]$ | 30 |
| Temperature | $T_0$ | $[K]$ | 300 |
| Temp. of upper wall | $T_u = T_0 + \delta T$ | $[K]$ | 330 |
| Temp. of lower wall | $T_l = T_0$ | $[K]$ | 300 |
| High , length | $h$ , $L$ | $[m]$ | 1e-3, 1e-2 |
| Centre of magnetic wire | $(a,b)$ | $[m]$ | (2e-3, -3e-3) |
| Magnetic field strength at the source | $\gamma$ | $[A \cdot m]$ | 10 |
| Ratio of thermal diffusivities | $\kappa_{ratio} = \frac{\kappa_s}{\kappa}$ | - | 962.687174 |

Table . Flow 1

| Quantity | Flow 1A | Flow 1B | Flow 1C | Flow 1D |
|---|---|---|---|---|
| $a$<br>$b$ | 2<br>-3 | 2<br>-2.5 | 2<br>-2 | 2<br>-1.5 |
| $H_r$ | 1.768388e+5 | 2.546479e+5 | 3.978874e5 | 7.073553e+5 |
| $Pr$ | 560 | | | |
| $Ra$ | 200.7938 | | | |
| $Ec$ | 1.16324e-13 | | | |
| $Re$ | 0.001786 | | | |
| $Mn$ | 2.272182e+9 | 4.711597e+9 | 1.150292e+10 | 3.635492e+10 |
| $Qn$ | 64.019097 | | | |
| $V_{avg}$ (fluid) | 292.980369 | 309.281869 | 331.905089 | 371.38104 |
| $T_{avg}$ (fluid) | 101.330192 | 101.314674 | 101.096441 | 100.801837 |
| $T_{avg}$ (all) | 118.521428 | 118.261882 | 117.760089 | 117.17941 |

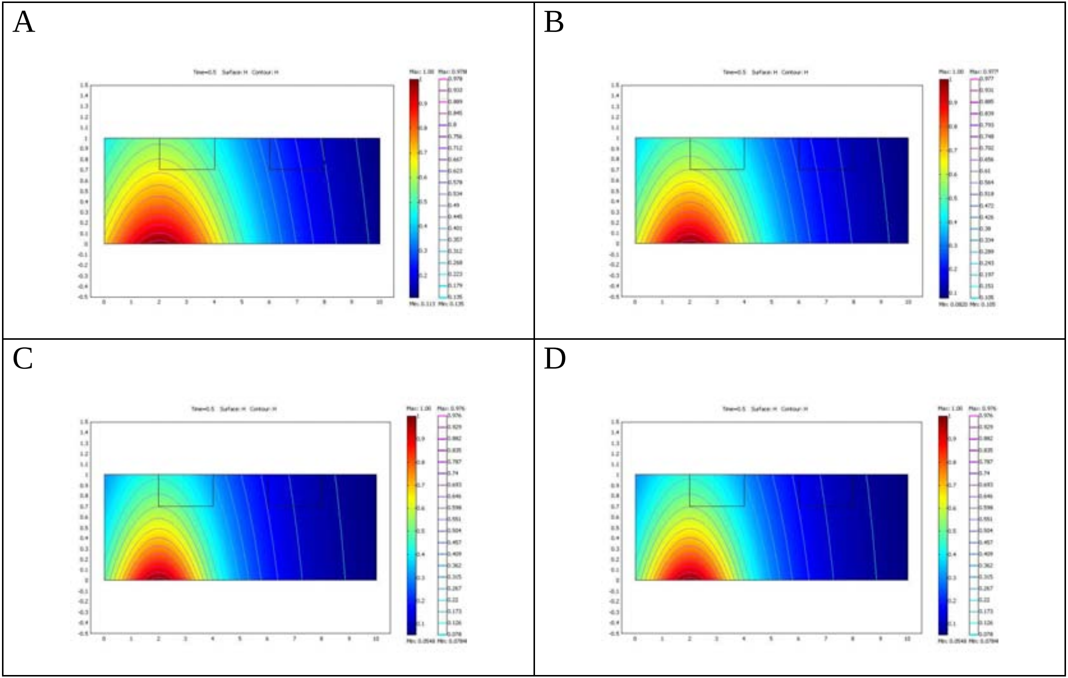



Figure 1.1. Intensity of magnetic field. Flow 1A, 1B, 1C, 1D

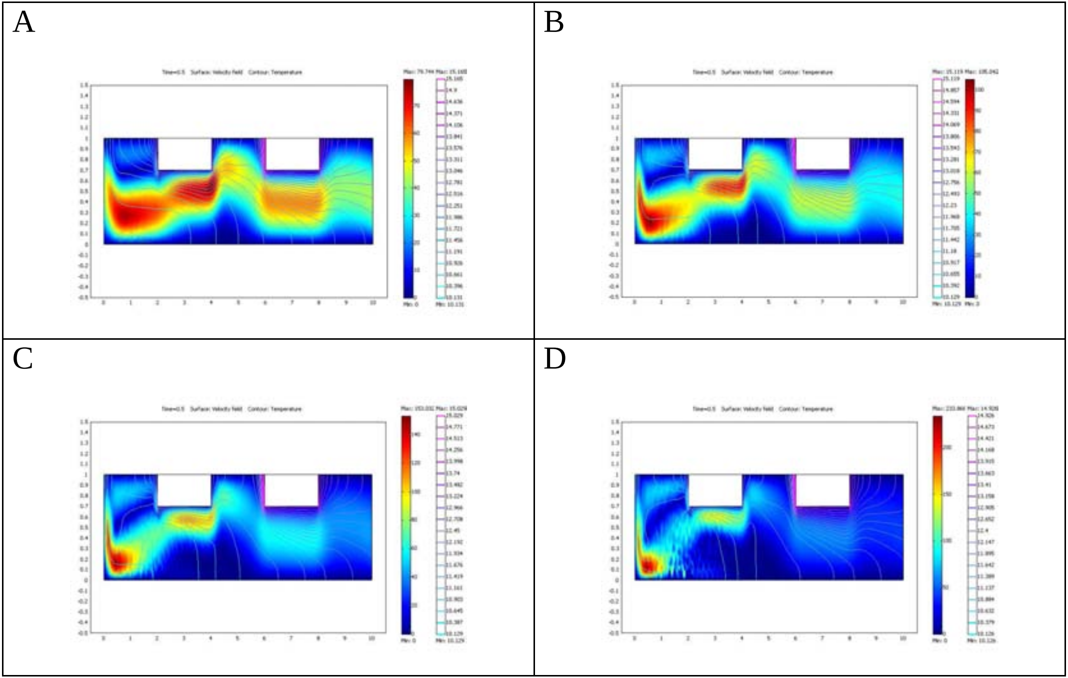



Figure 1.2. Velocity. Flow 1A, 1B, 1C, 1D.

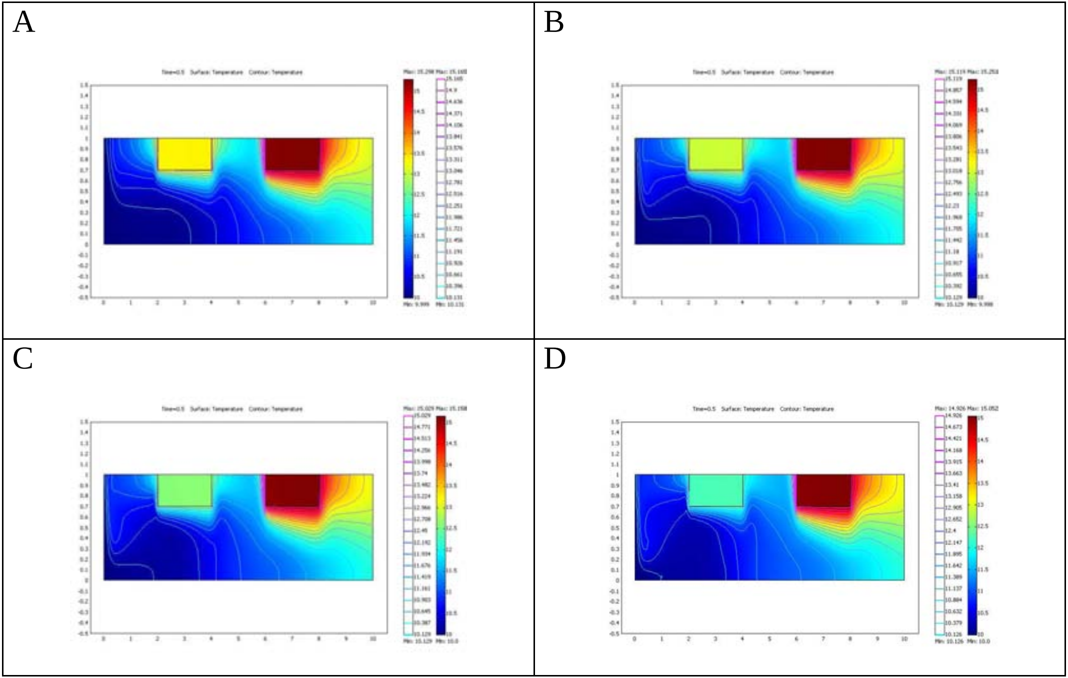



Figure 1.3. Temperature. Flow 1A, 1B, 1C, 1D.

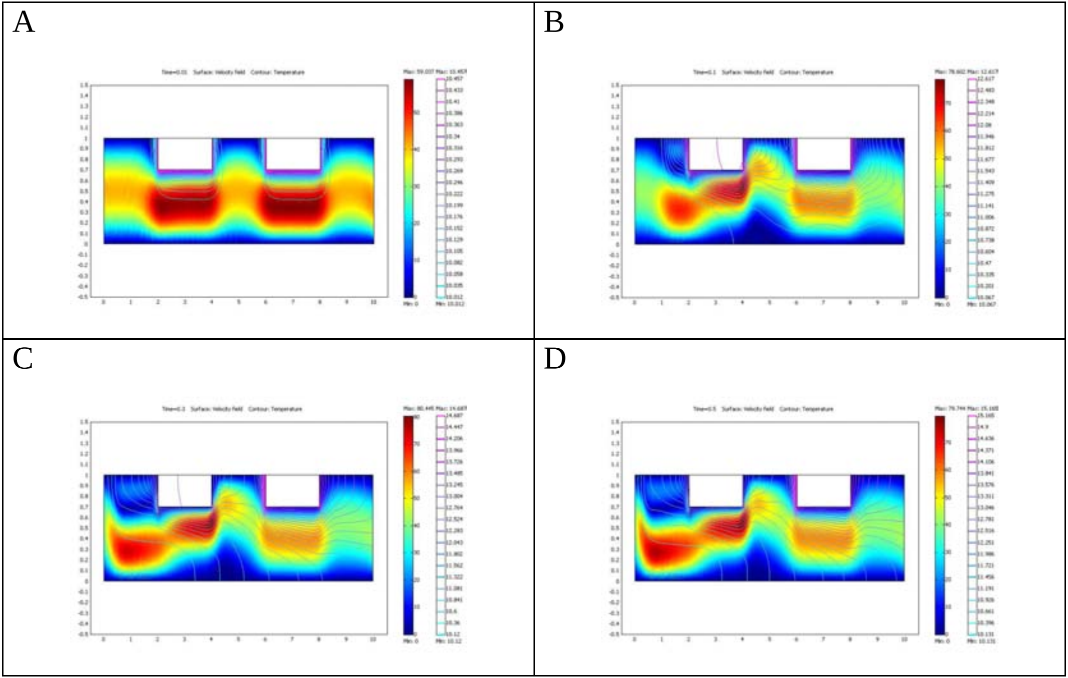



Figure 1.4. Velocity. Flow 1A for time=0.01 (A), 0.1 (B), 0.3(C). 0.5 (D).

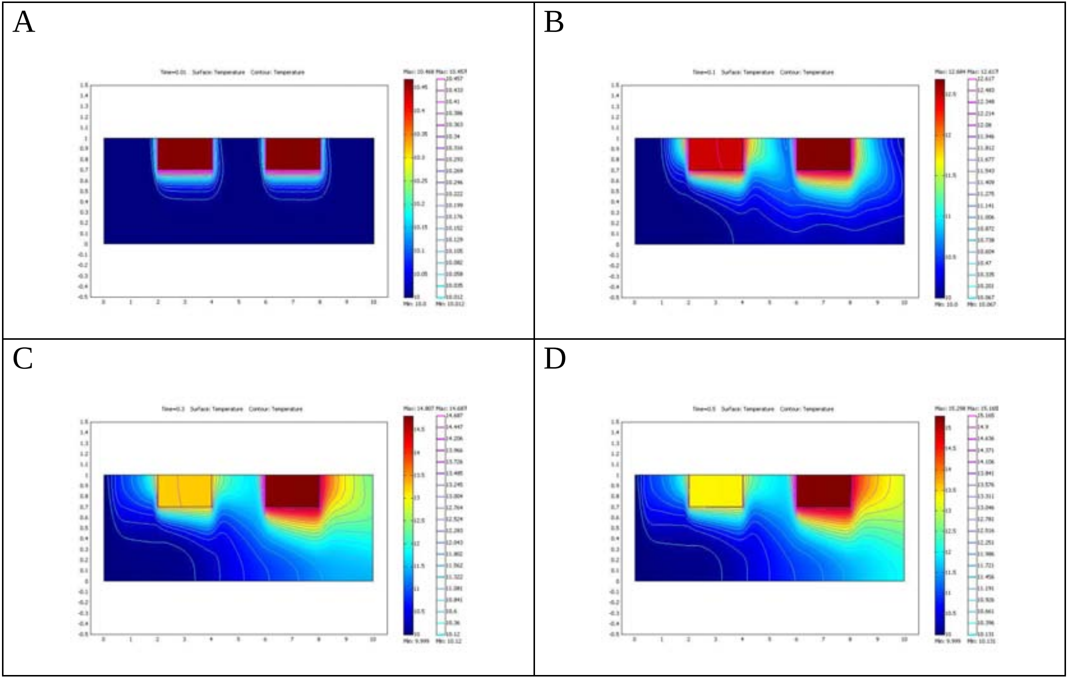



Figure 1.5. Temperature. Flow 1A for time=0.01 (A), 0.1 (B), 0.3(C). 0.5 (D).

Table . Flow 2

| Quantity | Flow 2A | Flow 2B | Flow 2C | Flow 2D |
|---|---|---|---|---|
| $a$<br>$b$ | 4<br>-3 | 4<br>-2.5 | 4<br>-2 | 4<br>-1.5 |
| $H_r$ | 1.768388e+5 | 2.546479e+5 | 3.978874e5 | 7.073553e+5 |
| $Pr$ | 560 | | | |
| $Ra$ | 200.7938 | | | |
| $Ec$ | 1.16324e-13 | | | |
| $Re$ | 0.001786 | | | |
| $Mn$ | 2.272182e+9 | 4.711597e+9 | 1.150292e+10 | 3.635492e+10 |
| $Qn$ | 64.019097 | | | |
| $V_{avg}$ (fluid) | 300.949913 | 331.078973 | 375.238349 | 445.972581 |
| $T_{avg}$ (fluid) | 101.964051 | 102.076129 | 102.080147 | 101.443362 |
| $T_{avg}$ (all) | 119.429217 | 119.129908 | 118.70548 | 117.611141 |

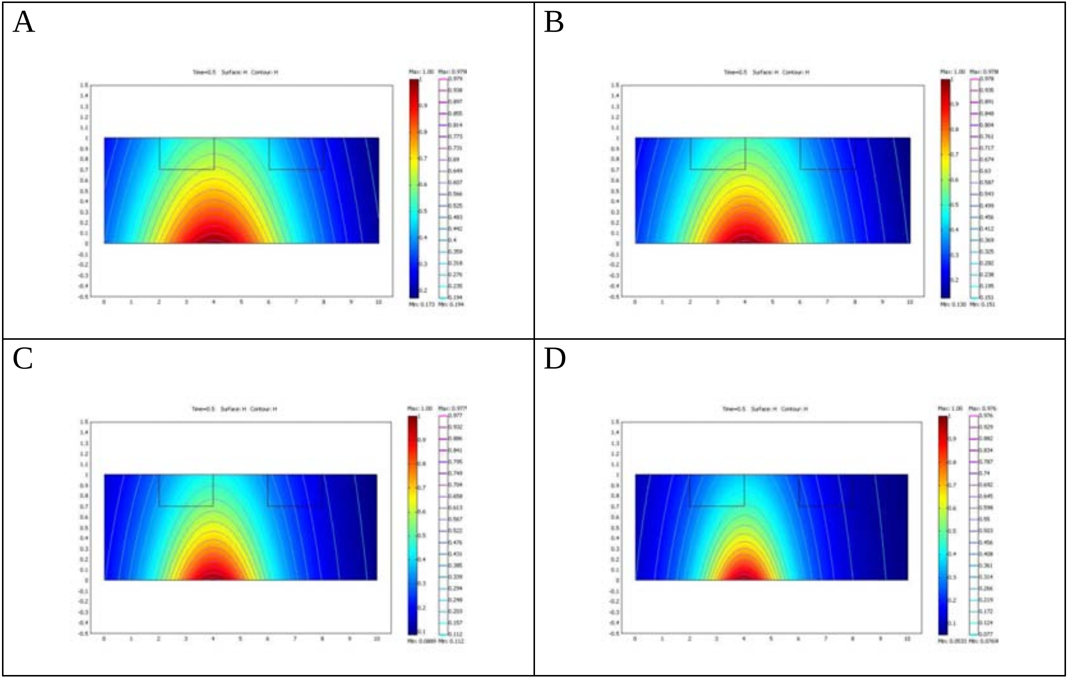



Figure 2.1. Intensity of magnetic field. Flow 2A, 2B, 2C, 2D

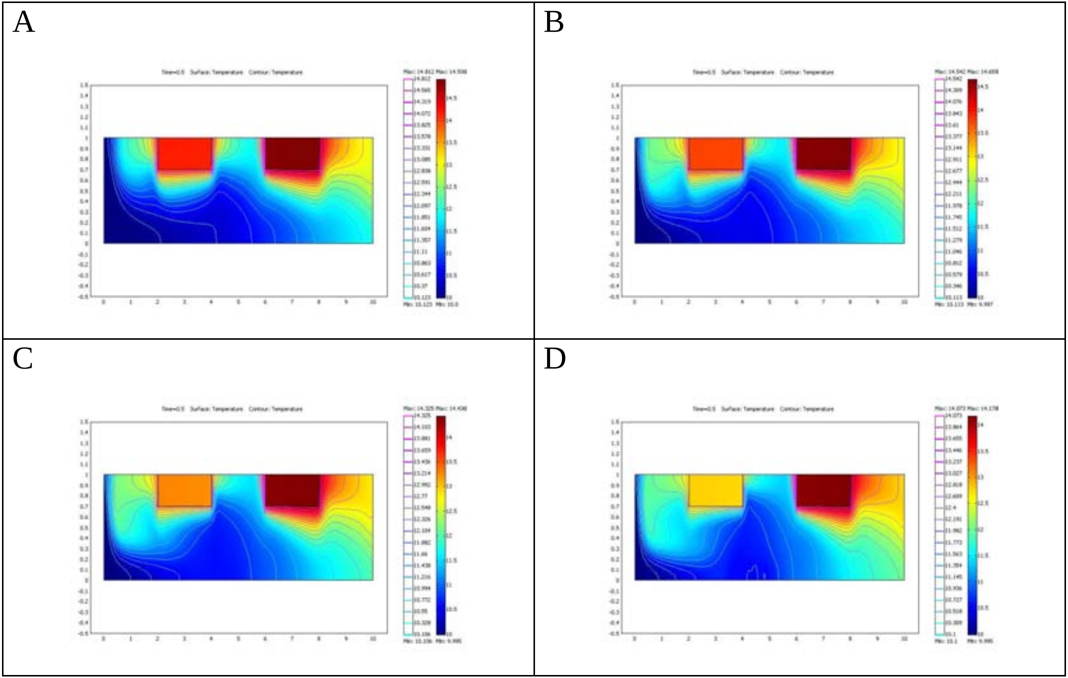



Figure 2.3. Temperature. Flow 2A, 2B, 2C, 2D.

Table . Flow 3

| Quantity | Flow 3A | Flow 3B | Flow 3C | Flow 3D |
|---|---|---|---|---|
| $a$<br>$b$ | 4<br>-3 | 4<br>-2.5 | 4<br>-2 | 4<br>-1.5 |
| $H_r$ | 1.768388e+5 | 2.546479e+5 | 3.978874e5 | 7.073553e+5 |
| $Pr$ | 560 | | | |
| $Ra$ | 200.7938 | | | |
| $Ec$ | 1.16324e-13 | | | |
| $Re$ | 0.001786 | | | |
| $Mn$ | 2.272182e+9 | 4.711597e+9 | 1.150292e+10 | 3.635492e+10 |
| $Qn$ | 64.019097 | | | |
| $V_{avg}$ (fluid) | 293.987901 | 323.793528 | 364.54859 | 425.701388 |
| $T_{avg}$ (fluid) | 102.016613 | 102.391017 | 102.540234 | 101.932843 |
| $T_{avg}$ (all) | 119.892233 | 119.877038 | 119.491615 | 118.356234 |

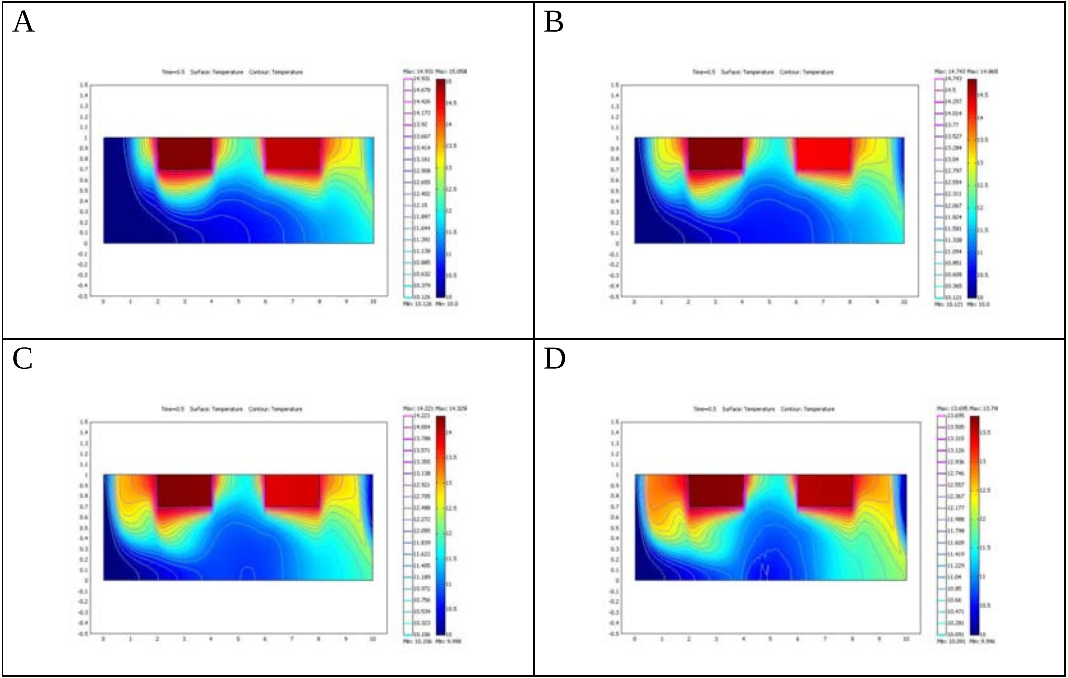



Figure 3.3. Temperature. Flow 3A, 3B, 3C, 3D.

Table . Flow 4

| Quantity | Flow 4A | Flow 4B | Flow 4C | Flow 4D |
|---|---|---|---|---|
| $\gamma \ [A \cdot m]$ | 10 | 8 | 5 | 2 |
| $H_r$ | 1.768388e+5 | 1.414711e+5 | 8.8419412e+4 | 3.5367765+4 |
| $Pr$ | 560 | | | |
| $Ra$ | 200.7938 | | | |
| $Ec$ | 1.16324e-13 | | | |
| $Re$ | 0.001786 | | | |
| $Mn$ | 2.272182e+9 | 1.454197e+9 | 5.680456e+8 | 9.088773e+7 |
| $Qn$ | 64.019097 | | | |
| $V_{avg}$ (fluid) | 292.980369 | 284.941996 | 278.417311 | 277.342162 |
| $T_{avg}$ (fluid) | 101.330192 | 101.518341 | 101.634182 | 101.525145 |
| $T_{avg}$ (all) | 118.521428 | 118.912839 | 119.279439 | 119.198103 |

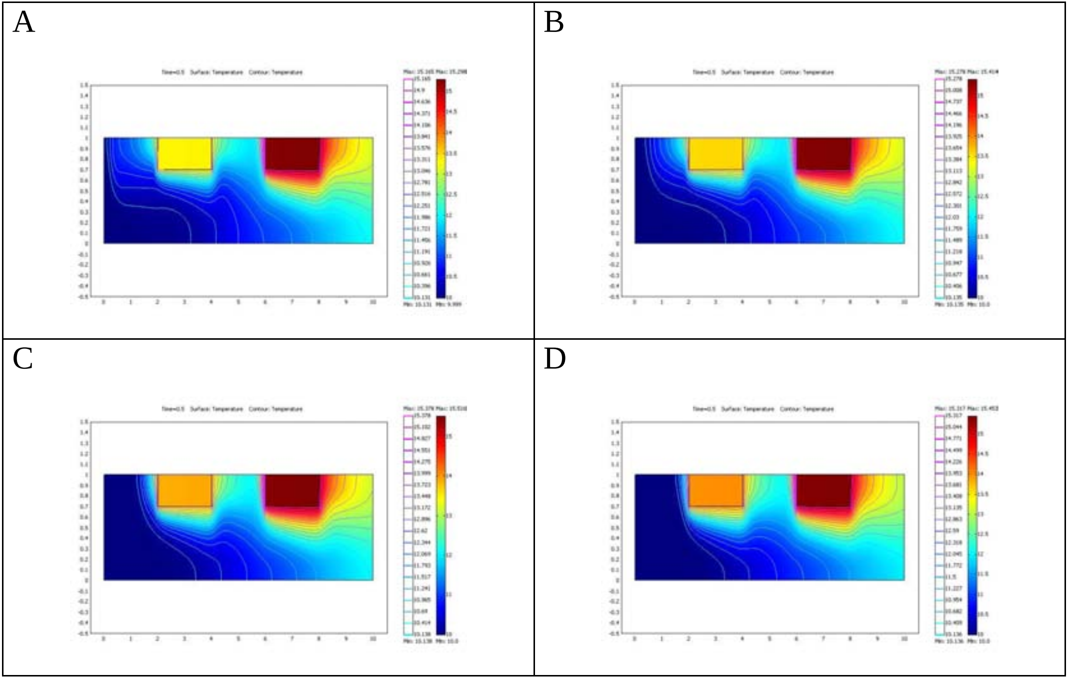



Figure 4.2. Temperature. Flow 4A, 4B, 4C, 4D.

Table . Flow 5

| Quantity | Flow 5A | Flow 5B | Flow 5C | Flow 5D |
|---|---|---|---|---|
| $\chi_0$ | 0.5 | 0.1 | 0.06 | 0.04 |
| $H_r$ | 1.768388e+5 | | | |
| $Pr$ | 560 | | | |
| $Ra$ | 200.7938 | | | |
| $Ec$ | 1.16324e-13 | | | |
| $Re$ | 0.001786 | | | |
| $Mn$ | 2.272182e+9 | | | |
| $Qn$ | 64.019097 | | | |
| $V_{avg}$ (fluid) | 411.490459 | 306.564373 | 292.980369 | 284.589693 |
| $T_{avg}$ (fluid) | 100.741705 | 101.246393 | 101.330192 | 101.453763 |
| $T_{avg}$ (all) | 116.618815 | 118.171175 | 118.521428 | 118.846783 |

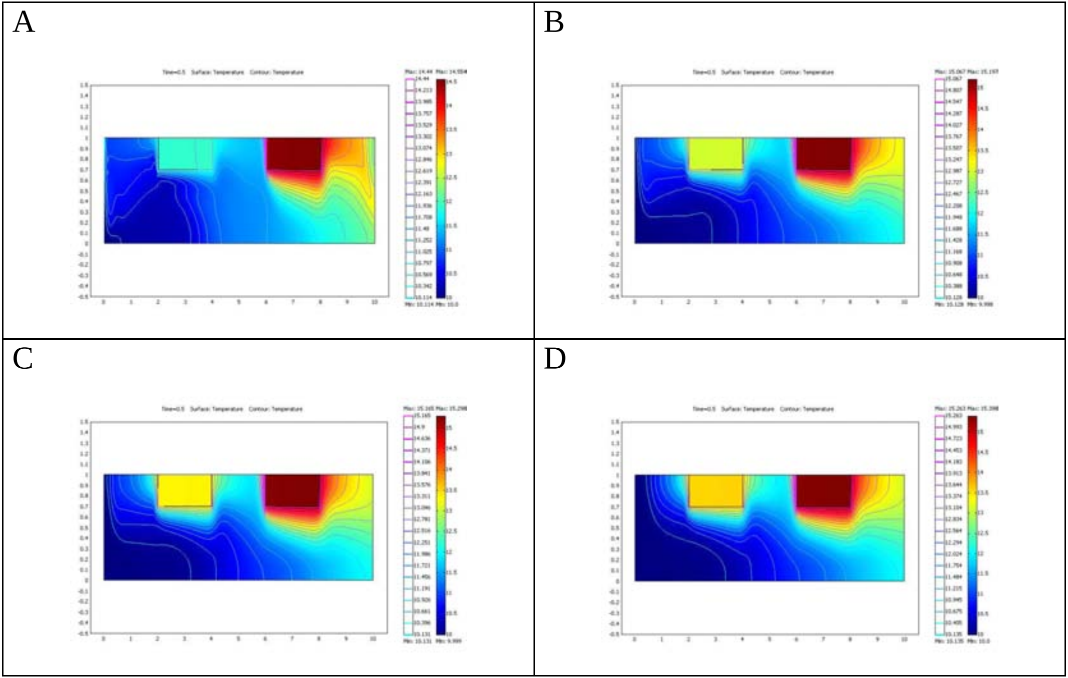



Figure 5.2. Temperature. Flow 5A, 5B, 5C, 5D.

Table . Flow 6

| Quantity | Flow 6A | Flow 6B | Flow 6C | Flow 6D |
|---|---|---|---|---|
| $\alpha\ [1/K]$ | 1e-2 | 5e-3 | 1e-3 | 1e-4 |
| $H_r$ | 1.768388e+5 | | | |
| $Pr$ | 560 | | | |
| $Ra$ | 358.560405 | 179.280203 | 35.85604 | 3.585604 |
| $Ec$ | 1.16324e-13 | | | |
| $Re$ | 0.001786 | | | |
| $Mn$ | 2.272182e+9 | | | |
| $Qn$ | 64.019097 | | | |
| $V_{avg}$ (fluid) | 301.700686 | 291.26183 | 278.372856 | 277.394904 |
| $T_{avg}$ (fluid) | 101.321273 | 101.394785 | 101.594218 | 101.621808 |
| $T_{avg}$ (all) | 118.401486 | 118.626348 | 119.220403 | 119.311931 |

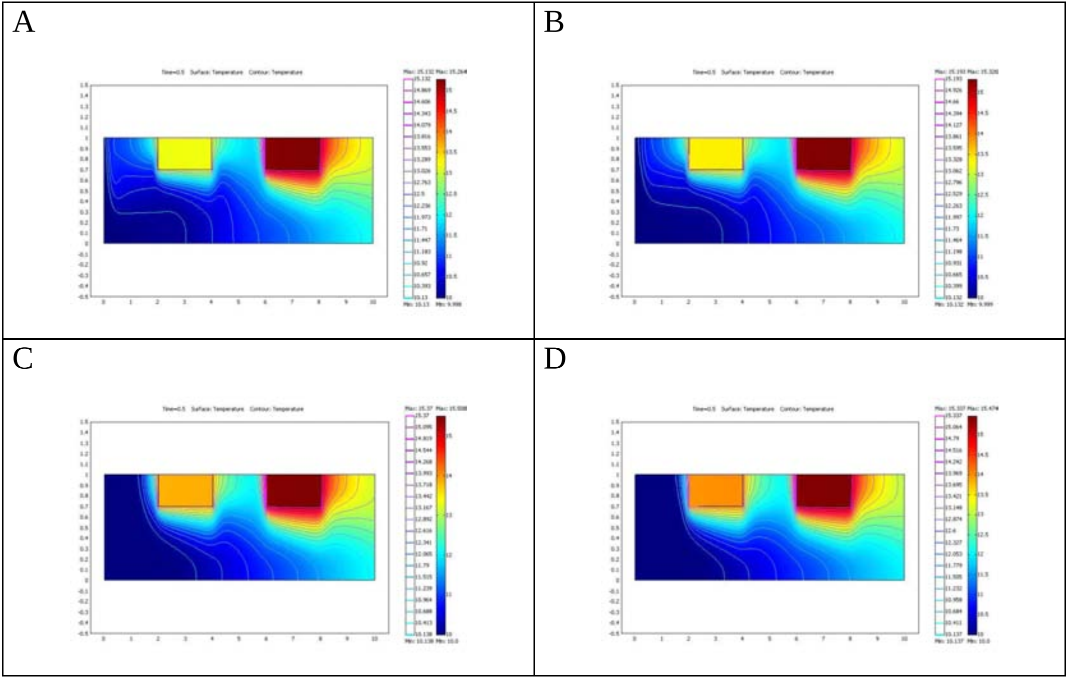



Figure 6.2. Temperature. Flow 6A, 6B, 6C, 6D.

Table . Flow 7

| Quantity | Flow 7A | Flow 7B | Flow 7C | Flow 7D |
|---|---|---|---|---|
| $H_r$ | 1.768388e+5 | | | |
| $Pr$ | 560 | | | |
| $Ra$ | 200.7938 | | | |
| $Ec$ | 1.16324e-13 | | | |
| $Re$ | 0.001786 | | | |
| $Mn$ | 2.272182e+9 | | | |
| $Qn$ | 0.800239 | 4.001194 | 8.002387 | 40.01194 |
| $V_{avg}$ (fluid) | 277.384394 | 277.522085 | 277.876352 | 286.028446 |
| $T_{avg}$ (fluid) | 88.171168 | 88.853008 | 89.702394 | 96.368224 |
| $T_{avg}$ (all) | 100.242442 | 101.208415 | 102.4098 | 111.69463 |

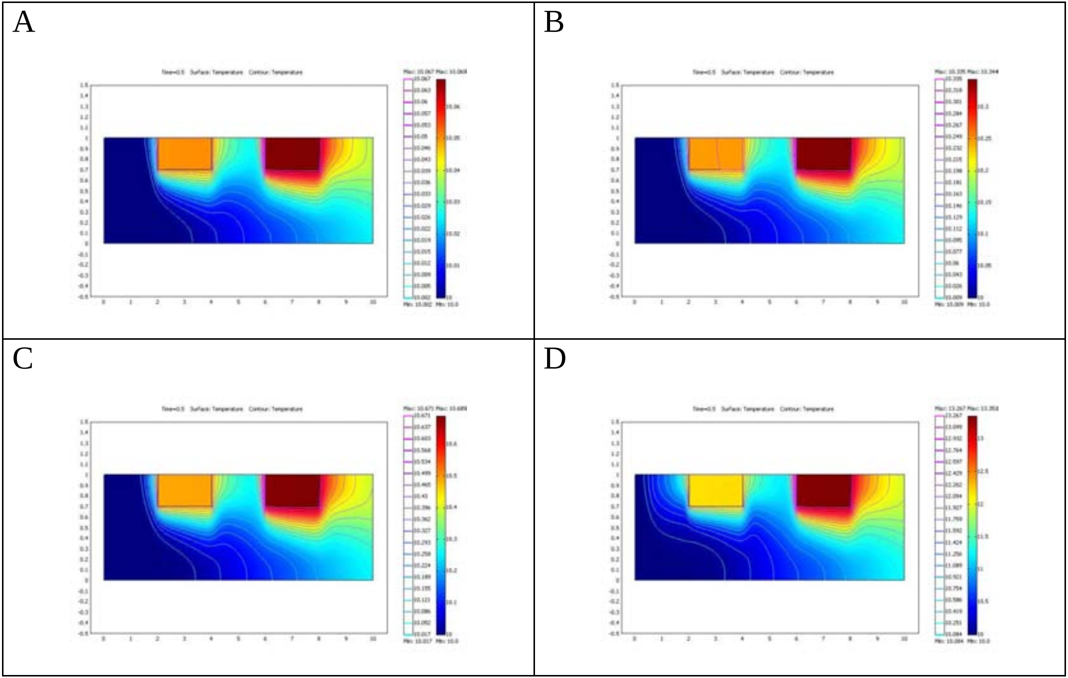



Figure 7.3. Temperature. Flow 7A, 7B, 7C, 7D.

## CONCLUSIONS

- The flow was relatively uninfluenced by the magnetic field until its strength was large enough for the Kelvin body force to overcome the viscous force.
- It can be observed that the cooler ferrofluid flows in the direction of the magnetic field gradient and displaced hotter ferrofluid.
- This effect is similar to natural convection where cooler, more dense material flows towards the source of gravitational force.
- Ferrofluids have promising potential for the heat transfer applications because a ferrofluid flow can be controlled by using an external magnetic field.

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