

Using COMSOL Multiphysics® for Benchmarking Problems in Cell Migration

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Introduction

A recently published conservative algorithm [1] for simulating reactions and transport in moving domains have been extended to models in which cell kinematics are coupled with intracellular dynamics. For this, the method that combines a Eulerian approach with tracking an explicit boundary was linked to FronTier, a robust front-tracking technique [2]. The extended algorithm was validated using a set of benchmark problems related to cell migration. COMSOL Multiphysics® was used extensively to generate precise reference solutions for test cases where no closed-form analytical solutions were available.

Expanding Circle Benchmark:

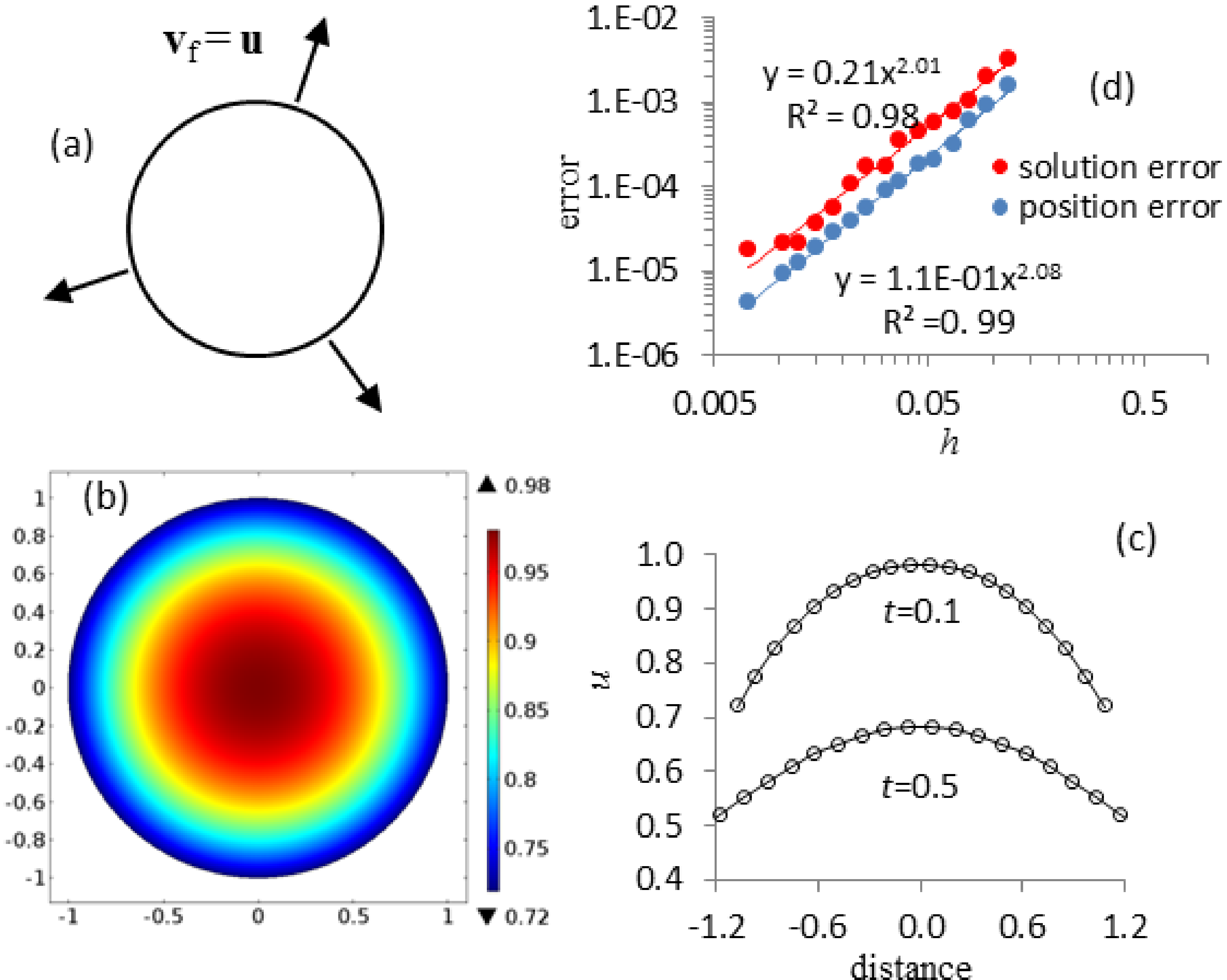


Figure 1. $\partial_t u = \Delta u$ is solved in an expanding circle (a), with boundary $(\nabla u + v_f u)n|_{\partial\Omega} = 0$ and initial condition $\Omega(0) = \{x, y \mid x^2 + y^2 < 1\}, u(x, y, 0) = 1$. The solution is tested against an equivalent problem solved in a fixed domain using COMSOL; a snapshot of the reference solution in (b) is shown for $t = 0.1$. Comparison of the results obtained with our method (circles) against reference solution (curves) is shown for the equator at specific times (c), and convergence of position and solution errors is illustrated in (d).

Actin-based Cell Motility Benchmark:

Advection-diffusion of myosin

$$\partial_t M = D\Delta M - \kappa \nabla \cdot (UM)$$

Viscoelastic actin velocity

$$\partial_t U = \mu \Delta U + \nabla \cdot (MI) - \xi U$$

Kinematic condition

$$n \cdot v_f|_{\partial\Omega} = \frac{V_0}{M_0 + M|_{\partial\Omega}} - K|\Omega|$$

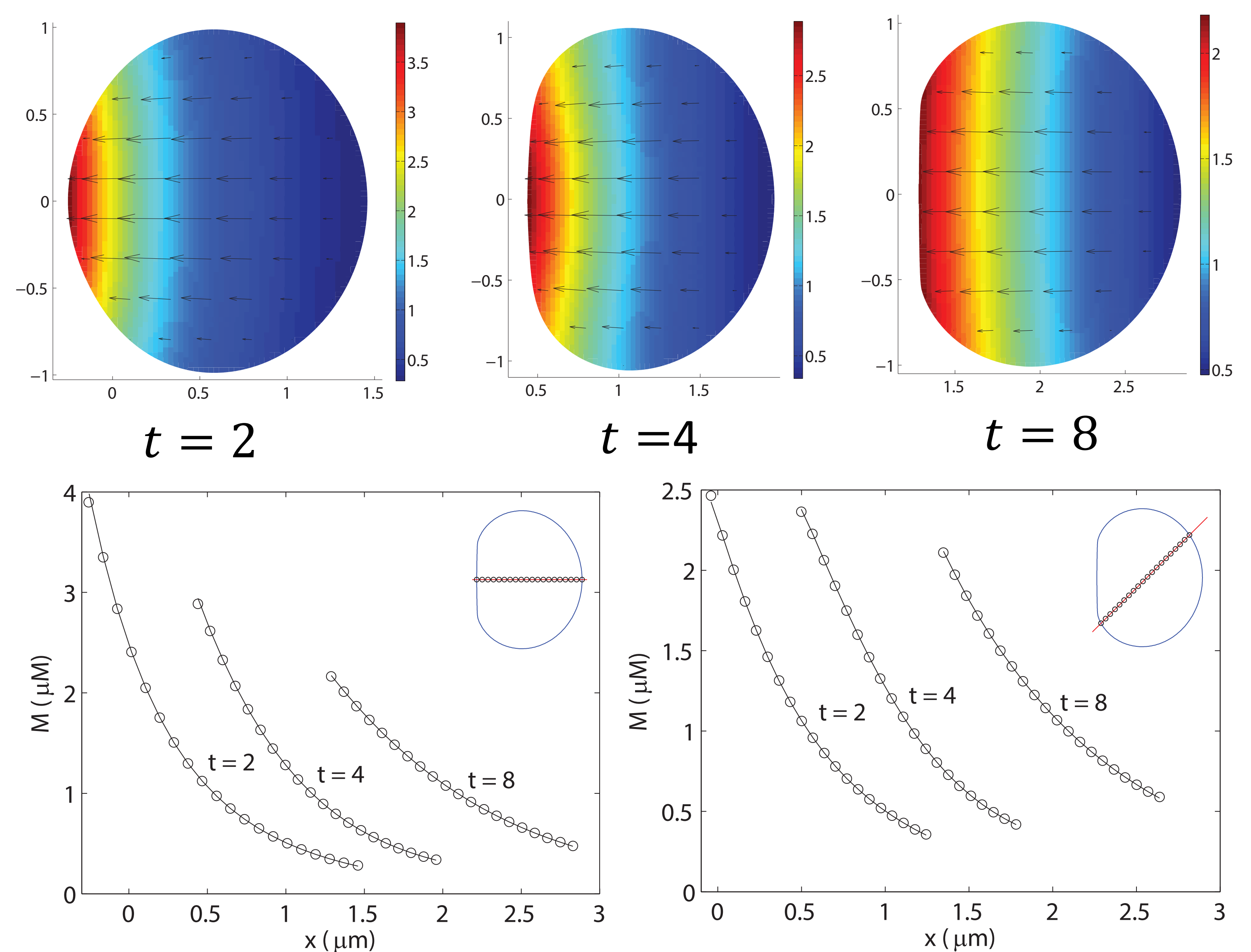


Figure 2. Cell initially has a circular shape, a slight linear myosin gradient, and no actin flow. Impermeable and zero-velocity boundary conditions are applied for Myosin and Actin velocity equations, respectively. (top row) myosin distribution in pseudocolor and actin velocity vectors at different times; (bottom row) comparison of our solutions (circles) and the ALE method of COMSOL (solid lines). Insets show the cut-line at $t = 8$.

Conclusions: COMSOL Multiphysics proved to be an essential tool for the development and analysis of numerical methods in cell migration.

References:

1. I.L. Novak and B.M. Slepchenko, A conservative algorithm for parabolic problems in domains with moving boundaries, J. Comp. Phys., 270:203-213, 2014.
2. J. Du et al., A simple package for front tracking, J. Comp. Phys. 213:613-628, 2006.