Determination of the "Sweet Spot" of a Cricket Bat using COMSOL Multiphysics®

Y. Mulchand[1], A. Pooransingh[1] and R. Latchman[1]
[1]The University of the West Indies – St. Augustine Campus, St. Augustine, Trinidad and Tobago
yogeshwarmulchand@hotmail.com, akash.pooransingh@sta.uwi.edu, richielatchman@hotmail.com

Abstract: The aim of this paper is to determine the location of the "sweet spot" for a selected cricket bat commonly used in the sport. Knowledge of the "sweet spot" is important in delivering a shot that utilizes the optimal zone of the bat that corresponds to the maximum power of the stroke. A COMSOL model of the cricket bat was constructed in the structural mechanics module and utilized the Eigenfrequency analysis which aided in determining the "sweet spot" of the cricket bat. The COMSOL model determined the mode shapes and the Eigenfrequencies at which they occur. Simulated results showed that the "sweet spot" was located in the region of 10 cm to 15 cm from the toe of the bat at the center.

Keywords: Sweet spot, cricket bat, Eigenfrequency, mode shape

1. Introduction

Over the years, the game of cricket has tremendously evolved and is now one of the most commercialized sports in the world. In cricket, there is an increasing trend to monitor the performance of the athlete. Currently there is increasing research on monitoring the performance of the batsman (Busch and James 2007) and the bowler (Portus, et al. 2004). These studies focus on using signal processing technology in perfecting the elite athlete. Cricket is a traditional "bat and ball" sport like baseball and tennis. Some studies, (Latchman and Pooransingh 2015), (Fontes 2014), scientifically explored the limits of the motion of the ball. However limited studies have focused on the bat (Boochie 2015) (D. A. Russell 2006) with less specifically on the cricket bat (Hariharan and Srinivasan 2012) (John and Bang Li 2002) (S. Knowles 1996) (Sarkar, et al. 2012).

The aim of this paper is to determine the "sweet spot" of a cricket bat. The "sweet spot" can be defined as the position of the bat where the maximum energy is conveyed with the least vibration (D. A. Russell 2015). Knowledge of the location of the "sweet spot" on the bat aids the batsman in improving their batting ability and delivering optimal shots in attaining maximum power with the least effort. Additionally, this can also aid coaches during coaching sessions.

Russell (2015) utilized a special force hammer and an accelerometer. The accelerometer measures the vibrations produced by the baseball bat when it is struck by the force hammer to determine the "sweet spot". All the results obtained were then analyzed offline to show the vibrations and the frequency response. Cheng and Ku (2015) built upon Cross (1998) who proposed two methods utilizing triaxial accelerometers. For the first method, the accelerometer was mounted onto a hand-held stationary cricket bat. The bat was then tapped in the upper, middle and lower regions along its length. The second method aimed to identify the "sweet spot" of five players performing defensive drives along the ground which analyzed cricket shots by using accelerometers.

2. Theory

The main vibrational modes of the performance of a bat are the bending modes of a bat. A freely supported bat exhibits several bending modes of vibrations when compared to a hand held bat which can be treated as a clamped cantilever beam (D. A. Russell 2006). According to Russell (2015) while the hands quickly dampen the vibrations from the bat, the vibrational frequencies or the mode shape does not change with a tightly gripped bat. Figure 1 shows the first four (4) bending modes of a baseball bat. According to Russell (2006) the first bending mode occurs at a frequency of 170 Hz as shown in Figure 2. This mode is very important in the performance of the bat since it helps in the location of the "sweet spot" where the node is about 5-7 inches from the barrel end (toe) of the bat. Impacts at this point on the bat would not cause any vibrations to be produced and thus none of the initial energy of the ball would be lost to the bat due to deformation and the player would not feel any vibrations at this frequency. Russell (2006) also stated that the second bending mode occurs at a frequency of 600 Hz and is also important in bat performance since it has a node about 5-7 inches from the barrel end of the bat. With this node and the previously mentioned node, a "sweet zone" is established in between the two bending modes as shown in Figure 3. Impacts in this region would cause minimal vibrations felt by the player, and the ball would not lose all of its energy to the bat's vibrations (D. A. Russell 2006).

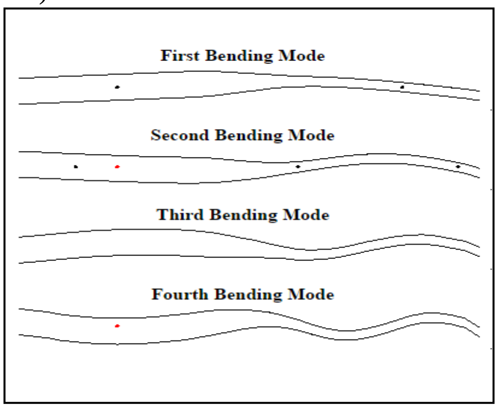


Figure 1 – Showing the Bending Modes of a Baseball Bat (D. A. Russell 2015)

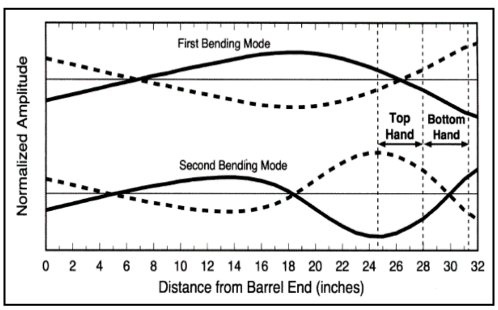


Figure 2 - Mode shapes for the first two bending vibrational modes in a 32-inch youth baseball bat (D. A. Russell 2006).

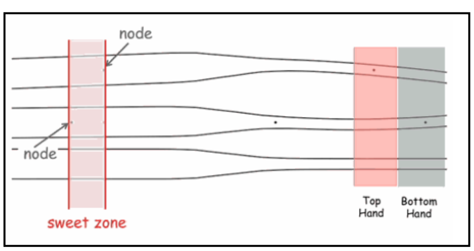


Figure 3 – Showing the sweet zone of a Baseball Bat (D. A. Russell 2006)

The coefficient of restitution (COR) is used to measure the elasticity or inelasticity of two objects when they make contact with each other in the case of a cricket bat as it strikes a cricket ball (Cheng and Ku 2015).

The coefficient of restitution e is define as follows:

$$e = \frac{v_{Bf} - v_{Af}}{v_{Bi} - v_{Ai}}$$

where A and B are two objects with mass M_A and M_B, initial velocity V_{Ai} and V_{Bi} and final velocity V_{Af} and V_{Bf} respectively.

When the COR has a value of one (1), the system is considered to be perfectly elastic and there is no energy loss due to deformation of the colliding objects. However, when the COR has a value of zero (0), the system is considered to be perfectly plastic and there is no velocity left in the two objects after collision.

The "sweet spot" can be defined and located in numerous ways. There are a multitude of definitions of the "sweet spot". These include the following:

1) The region where least vibration sensation (sting) in the batsman's hand is produced, in addition, the locality where maximum batted ball speed is produced and where maximum energy is conveyed to the ball is located.
2) The region of maximum coefficient of restitution also known as the centre of percussion.
3) The node of the fundamental vibration mode.
4) The region between nodes of the first two vibration modes (D. A. Russell 2015).

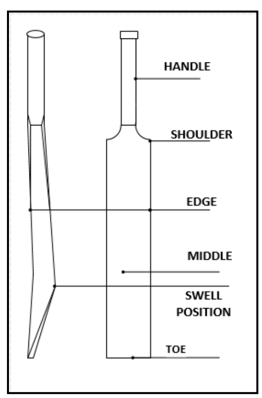


Figure 4 – Showing the Profile of a Cricket Bat

Figure 4 shows the profile of a typical cricket bat. The handle of the bat is the most susceptible to strain when a ball is being played. The edge of a cricket bat is generally 2 – 3 inches thick. According to Jones (2015) a thicker edge increases the durability of the bat and as a result it reduces the likelihood of the bat becoming worn. The "sweet spot" is distributed over a wider area of the bat as the swell position and depth changes as there is more wood behind the blade as seen in Figure 4. This leads to improved rebounding qualities and greater force is transferred to the ball when struck. Jones (2015) also stated that knowledge of the swell position is important when selecting a bat to play these particular shots most effectively. The back profile of the bat aid in the pick-up and the balance of the bat. (Jones 2015).

3. Governing Equations

According to COMSOL (2013) eigenfrequency analysis also referred to as free vibration of a structure, is used to find the damped and undamped eigenfrequencies and eigenmode shapes of a model. When performing an eigenfrequency analysis in structural mechanics context, the fundamental eigenvalue, λ, mode, i, and the eigenfrequency, f, is more commonly used.

$$f = -\frac{\lambda}{2\pi i}$$

It is possible to compute eigenfrequencies for structures which are not fully constrained, this referred to as free-free modes. For a 3-dimensional structure there is a possibility of six (6) rigid body modes which comprises of three (3) translations and three (3) rotations. If rigid body modes are present in the model, only the shape and not the size of the modes have physical significance (COMSOL 2013).

3.1 Linear Elastic Materials

According to COMSOL (2013) the total strain tensor, ϵ, in terms of the displacement gradient, ∇u, and temperature, T, is written as:

$$\epsilon = \frac{1}{2}\left(\nabla u + (\nabla u)^T\right)$$

Or in components as:

$$\epsilon = \frac{1}{2}\left(\frac{\partial u_m}{\partial x_n} + \frac{\partial u_n}{\partial x_m}\right)$$

The Duhamel-Hooke's law relates the stress tensor to the strain tensor and temperature:

$$s = s_0 + C : (\epsilon - \epsilon_0 - \alpha\theta)$$

Where C is the 4th order elasticity tensor, ":" is the double-dot tensor product, s_0 and ε_0 are initial stresses and strains, $\theta = T - T_{ref}$ and α is the thermal expansion tensor (COMSOL 2013).

3.2 Degrees of Freedom and Natural Frequencies of Vibration

According to Stokey (1988) system with mass and elastic parts are characterized by a finite number of degrees of freedom and some degree of elasticity. The number of natural frequencies of vibration of any system corresponds to the number of degrees of freedom. With each natural frequency is an associated shape this is commonly known as the natural mode shape which is assumed by the system during free vibration at the corresponding frequency (Stokey 1988).

For many elastic bodies, the Rayleigh method is most appropriate in finding an approximation to the fundamental natural frequency. All the energy associated with vibration is in the form of elastic strain energy. (Stokey 1988).The Strain energy. V and Kinetic Energy, T of a uniform beam in bending can be found by using the following:

$$V = \frac{EI}{2}\int_0^l \left(\frac{\delta^2 y}{\delta x^2}\right)^2 dx$$

$$T_{general} = \frac{S\gamma}{2g}\int_0^l \left(\frac{\delta y}{\delta t}\right)^2 dx$$

$$T_{maximum} = \frac{S\gamma\omega_n^2}{2g}\int_0^l (Y)^2 dx$$

$$\omega_n = 2\pi f_n$$

Where:
E = modulus of elasticity
I = moment of inertia of beam
l = length of beam
S = area of cross section
γ = weight density
g = modulus of rigidity
y = lateral deflection of beam
(Stokey 1988).

The natural frequency, f_n of a uniform cantilever beam with mass on the end can be found by using the following:

$$f_n = \sqrt{\frac{k}{M + 0.23m}}$$

$$k = \frac{3EI}{m^3}$$

Where:
M = mass
m = length of the beam
k = stiffness
I = Moment of inertia
(Stokey 1988).

4. Method and Use of COMSOL Multiphysics® Software

4.1 Selecting the Physics

A 3-dimensional conceptual model of the cricket bat used was constructed using the Structural Mechanics of the COMSOL Multiphysics software. The importance of the structural mechanics modules is for analyzing deformations, stresses and strains of solid structures. Solid Mechanics was chosen since it is based on solving equations and computed results for displacements, stresses and strains of the cricket bat. Eigenfrequency was then chosen for the study since it is used to compute Eigenmodes and Eigenfrequencies of a linearized model. In solid mechanics, the Eigenfrequency corresponds to the natural frequencies of vibrations and the Eigenmodes corresponds to the normalized deformed shapes at the Eigenfrequencies.

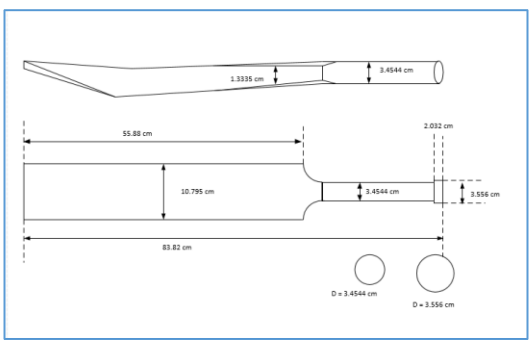
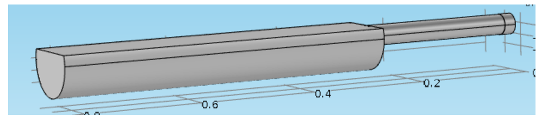


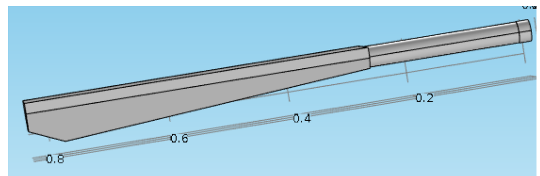
Figure 5- Model of the cricket bat with dimensions constructed using COMSOL Multiphysics software

4.2 Defining Geometry

Firstly the handle of the cricket bat was constructed based on the parameters outlined above. This was done by constructing a cylinder of radius 0.01727 m and length 0.254 m. Then the upper end of the handle was constructed by placing a cylinder of radius 0.01776 m and length 0.02032 m. The base of the bat was also represented as a cylinder of radius 0.05715 m and length 0.5588. Figure 6 (a) to (d) shows the progression of the stages in the construction of the bat using COMSOL.



(a)



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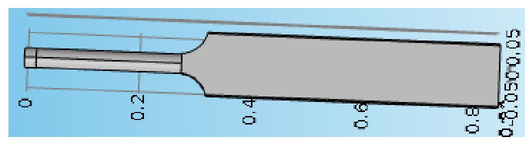

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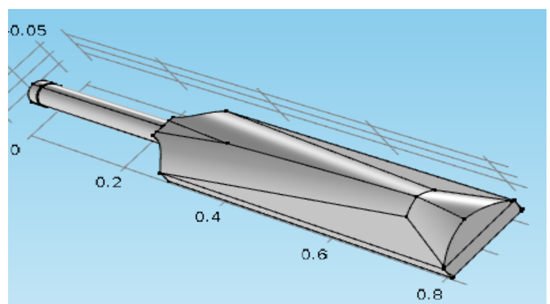

(d)

Figure 6- Construction stages for the Cricket Bat model

After the bat was constructed, materials were then added to the bat to replicate an actual cricket bat. From research conducted from Northgate Ltd (2016) cricket bats are made of a fibrous wood known as willow wood. Northgate Ltd (2016) also stated that carbon fiber is commonly used in the handles of cricket bats since it acts as an absorber from the shock when the bat is impacted. Titanium was also used in the handles of the bat as an added support and reinforcement, thus added to more power in striking the ball. However, for this experiment, only willow was used throughout the entire bat.

4.3 Defining Materials

To apply the material properties of the bat structure which was previously constructed. The function to add material was selected and wood was searched. The only type of wood found was kingwood and was applied it to the entire structure. Some parameters were missing such as Poisson's ration and Young's modulus. These were added based on research for willow wood since the material used for the construction of a cricket bat is willow.

4.4 Setting up the physics

The bat was set as being free object everywhere except at the handle where it is fixed in space.
In the solid mechanics physics, Linear Elastic Material was applied to the entire bat. This generated the Eigenfrequencies for the entire structure using the following equation from COMSOL:

$$-\rho\omega^2 u = \nabla \cdot s + Fv$$
$$s = S_0 + C : (\epsilon - \epsilon_0 - \epsilon_{inel})$$
$$\epsilon = \frac{1}{2}\left(\nabla u + (\nabla u)^T\right)$$

Initial Values was applied to the entire bat and the Displacement field and Structural Velocity field was all set to zero. This was to denote the equilibrium position where the bat did not undergo any deformation. The handle was then set as a Fixed Constraint. This would set the displacement for the Eigenfrequency to zero (u = 0).

4.5 Meshing

The free tetrahedral mesh was chosen for this analysis. An extra fine mesh was chosen to achieve a more accurate result. Extremely fine wasn't chosen since it took longer for the model to mesh and the results to compute. An edge mesh was also done on the model, this was to ensure that all the edges of the bat was properly meshed.

4.6 Simulation

The Eigenfrequency mode was chosen in the study mode, with a preset value of six (6) Eigenfrequencies. This meant that the program would look for the first six (6) Eigenfrequency of the system during the simulation. The results were computed allowing the program to solve differential equations for each point on the mesh.

5. Results

The following figures shows the first six mode shapes of the model of the cricket bat created using the COMSOL simulation. Figure (7 - 12) shows the first six Eigenmodes and Eigenfrequencies of the cricket bat model in solid mechanics in COMSOL simulation. These Eigenmodes correspond to the normalized deformations of the cricket bat at the Eigenfrequencies. The Eigenfrequencies correspond to the natural frequencies at which these vibrations occurs. Below are the eigenmodes and corresponding eigenfrequencies.

Eigenmode	Eigenfrequency /Hz
1	1.1075
2	1.5165
3	9.6195
4	10.344
5	17.217
6	27.884

Figure 7 – 9 and 11 – 12 shows the deformation of the bat produced a vertical motion about the handle. The bat experienced the greatest amount of displacement and vibrations at the toe of the bat. Figure 9, 11 and 12 shows a lower mid region where the bat acted as a pivot and there was no displacement and vibrations. However only figure 12 showed as an upper mid region where the bat acted as a pivot and there was no displacement and vibrations in this region. Figure 10 shows the deformation of the bat produced a twist motion about the handle. The bat experienced the greatest amount of displacement and vibrations at this frequency at the edges of the bat. It was noticed at the middle of the bat along the entire length there was no displacement and vibrations.
On the left of these figures there is a colour bar indicating displacement of the bat from its natural position. A colour of red indicated a larger amount of movement of the cricket bat, whereas a colour of blue indicated that position of the bat is not experiencing any vibration and is left in it rest position at the specified frequency.

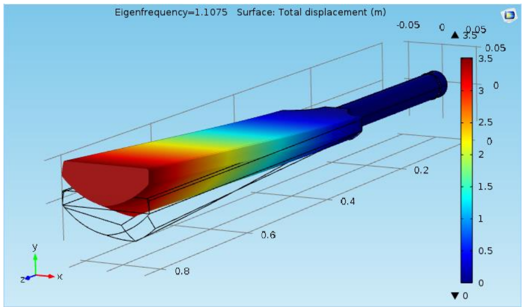

Figure 7 - Showing Mode Shape 1 at Eigenfrequency 1.1075 Hz

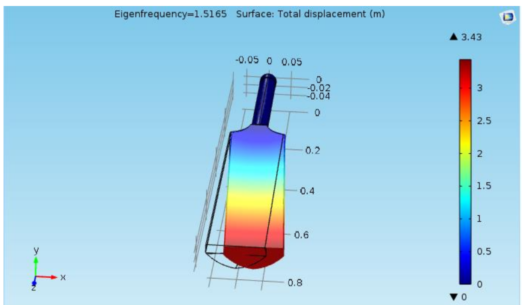

Figure 8 - Showing Mode Shape 2 at Eigenfrequency 1.5165 Hz

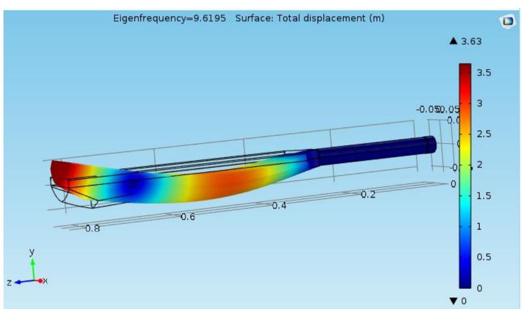

Figure 9 - Showing Mode Shape 3 at Eigenfrequency 9.6195 Hz

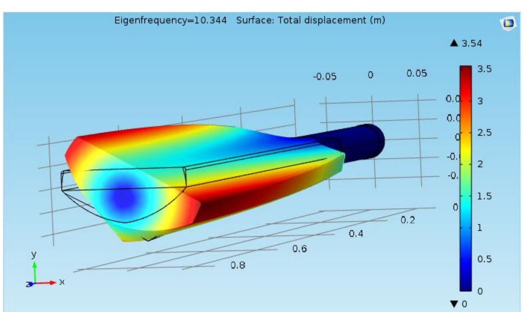

Figure 10 - Showing Mode Shape 4 at Eigenfrequency 10.344 Hz

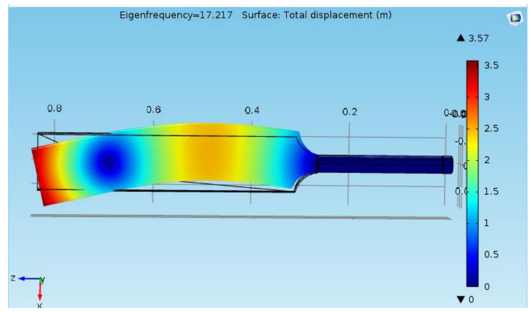

Figure 11 - Showing Mode Shape 5 at Eigenfrequency 17.217 Hz

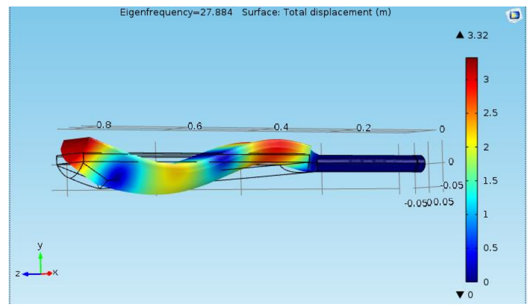

Figure 12 - Showing Mode Shape 6 at Eigenfrequency 27.884 Hz

6. Conclusion

A COMSOL model of the cricket bat was also constructed in the structural mechanics module which aided in determining the "sweet spot" of the cricket bat. The COMSOL model utilized Eigenfrequency analysis to determine the mode shapes and the Eigenfrequencies at which they occur. Additionally for the COMSOL simulation, the "sweet spot" can be seen located along the center of the bat, and is more concentrated in the lower mid region of the blade of the bat. At eigenfrequency 9.619 Hz this was found to be the first bending mode and a frequency of 27.884 Hz this was found to be the second bending mode of the cricket bat. It was observed that another region where no deformation was observed in the higher mid region of the blade of the bat at the second bending mode.

It can be concluded that the "sweet spot" is located in the region of 10 cm to 15 cm from the base and additionally, having another sweet zone at 20 cm from the base of the handle.

It must be noted that due to the simulation time, the finest possible mesh element was not used for the simulations in this study. This would have led to some degree of inaccuracy in the results of the simulations. Also assumptions made were that the simulated cricket bat were of the exact dimensions of the actual cricket bat used in the sport and the material used in COMSOL were similar to that used to construct the cricket bat. Additionally, age of the cricket bat was not taken into consideration for this simulation.

7. References

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8. Acknowledgement

The authors would like to acknowledge the contribution of Mr. Randall Ali for his concept and guidance of this study.