Simulation of Piezoelectric Vibration Energy Harvester Based on Thickness-Tapered Cantilever

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Abstract: The paper presents simulations of uniform thickness and thickness-tapered types of piezoelectric vibration energy harvesters using COMSOL Multiphysics. The simulated vibration energy harvesters have bimorph cantilever structure with end mass. Tapering the thickness of the piezoelectric layer in bimorph towards the free end is found to improve the stress distribution in the beam and produce greater power.

Keywords: vibration energy harvester.

1. Introduction

As research in low-power electronic device design and fabrication technology advanced, it opened the possibility of self-powered autonomous devices and motivated many researchers towards harvesting electrical energy from various ambient sources, including solar power, thermal gradients, and vibrations. Harvested power from a vibration energy harvester (VEH) is an important parameter in the design, and finite element (FE) analysis can be used to design and estimate harvested power [1] [2] [3]. In the conventional fixed-free cantilever beam, the stress is maximum at the fixed end and gradually decreases towards the free end. Therefore in VEH based on cantilever of uniform thickness, the low-stress portion of the beam remains underutilized.

In first part of the paper, FE analysis in COMSOL Multiphysics is used to simulate cantilever based piezoelectric VEH. In the second part of the paper, thickness profile of the piezoelectric VEH beam is modified to ensure an improved stress distribution on the beam surface and its performance as a VEH is studied. The distribution of stress on the beam surface and generated power are compared for VEHs with uniform and thickness-tapered beams.

2. VEH Based on Cantilever

Schematic diagram of a conventional piezoelectric VEH comprising of a substrate layer sandwiched between the piezoelectric layers of uniform thickness is shown in figure 1. The piezoelectric layers are metalized on top and bottom surfaces to form electrodes, the thickness of the electrodes is negligible compared to overall thickness. The motion of the end mass due to excitation at the fixed base exerts stress on the piezoelectric layer. The stress on the piezoelectric layer generates charge on the electrodes and current passes through the resistive load connected with the VEH.

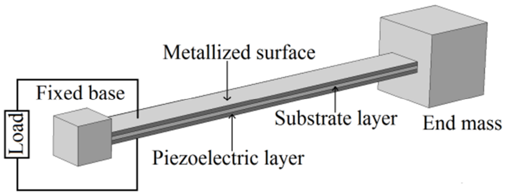

Figure 1. The geometry of a conventional VEH.

Piezoelectric constitutive equations (1) and (2) describe mathematically how stress (***T***), strain (***S***), charge-density displacement (***D***), and electric field (***E***) in the piezoelectric material interact with each another [6]

$$S = s^E T + d E \tag{1}$$

$$D = d T + \varepsilon^T E \tag{2}$$

where, s is compliance of Young's modulus, d is piezoelectric strain coefficient, ε is permittivity.

According to the numerical model developed by Erturk and Inman [4], [5] the steady state voltage response and the transverse displacement of a VEH across a resistive load for harmonic base excitation can be written as

$$v(t) = \frac{j2\omega R_l k F e^{j\omega t}}{(2+j\omega R_l C)(\omega_1^2-\omega^2+j2\zeta\omega_1\omega)+j2\omega R_l k\chi} \tag{3}$$

$$w(t) = \frac{(2+j\omega R_l C) k F e^{j\omega t}}{(2+j\omega R_l C)(\omega_1^2-\omega^2+j2\zeta\omega_1\omega)+j2\omega R_l k\chi} \tag{4}$$

where ω is the excitation frequency, ω_1 is the first natural frequency of the beam, R_l is the connected load resistance, C is the capacitance of the piezoelectric layer, k is the piezoelectric coupling term, F is the amplitude of the modal mechanical forcing function, ζ is the mechanical damping ratio and χ is the backward modal coupling term.

3. VEH Geometry

The uniform thickness piezoelectric VEH geometry consists of two piezoelectric layers and a substrate layer having uniform thicknesses and widths. On the other hand the thickness-tapered VEH has substrate layer of uniform thickness and a linearly varying thickness of the piezoelectric layer along the length. Both the VEH have the same width and length.

The uniform thickness VEH is simulated using the geometrical and physical parameters as described in [5], and is given in Table 1. Figure (2) shows the uniform thickness VEH geometry as described in Table 1.

Table 1: Geometrical and material properties used in the simulation.

Variables	Description	Values
L	Beam length	50.8 mm
b	Beam width	31.8 mm
h_p	Thickness of Piezoelectric layer	0.26 mm
h_s	Thickness of substrate layer	0.14 mm
M	End mass	12 g
Y_s	Young's modulus of substrate	105 GPa
Y_p	Young's modulus of PZT-5A	66 GPa
ρ_s	Mass density of PZT-5A	7800 kg/m^3
ρ_p	Mass density of substrate (Brass)	9000 kg/m^3
$\ddot{y}$	Input acceleration	1 g m/s^2

The piezoelectric material used in the simulations is PZT-5A and the substrate material is brass. The material properties of the PZT-5A are used from the material library of COMSOL.

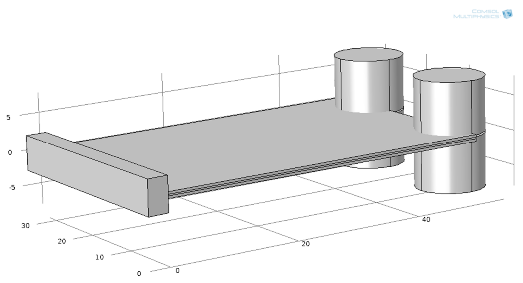

Figure 2. 3-D view of uniform thickness VEH.

The end mass in the thickness-tapered VEH is changed to keep the harvester resonance frequency the same as that of the uniform thickness VEH. The tapered piezoelectric layer has 50% increase in thickness at the fixed end and 50% decrease in thickness at the free end, keeping the volume of the piezoelectric material the same as that of the uniform thickness VEH. The material properties, width and substrate layer thickness are the same as that of the uniform thickness VEH, and the changes in piezoelectric layer thickness and end mass are shown in Table 2.

Table 2. Geometrical parameters for thickness-tapered VEH.

Variables	Description	Values
h_{p0}	Thickness of piezoelectric layer at the fixed end	0.39 mm
h_{p1}	Thickness of piezoelectric layer at the free end	0.13 mm
M	End mass	19.6 g

Figure (3) shows 3-D view of the thickness-tapered VEH.

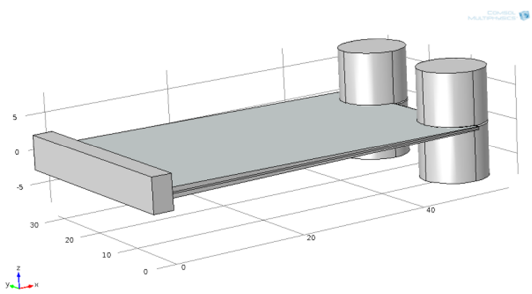

Figure 3. 3-D view of thickness-tapered VEH.

4. Simulation in COMSOL Multiphysics

COMSOL Multiphysics is used to simulate the uniform and thickness-tapered types of piezoelectric VEH. Both the VEHs are modeled in COMSOL using Piezoelectric Devices (*pzd*) and Electrical Circuit (*cir*) physics. The piezoelectric material used in this simulation is PZT-5A, and the substrate is made of brass. The piezoelectric layers are connected in series. The first resonance frequency of vibration is obtained from Eigen Frequency analysis while the displacement of the end mass and generated power is obtained from Frequency Domain analysis. The results obtained from simulation are the displacement of the end mass, harvested power and distribution of stress on the beam surface. The harvested power varies with the load resistance connected and at an optimal load the energy harvester generates maximum power. The VEHs are simulated for different load resistance and the optimal value is obtained for both the harvesters. The input excitation is considered as a sinusoidal excitation of 1 g acceleration.

5. Simulation Results

The VEHs are simulated to obtain displacement of the end mass, generated voltage, generated power and distribution of stress along the length of the beam for different input frequencies. The plots of generated voltage and displacement of the end mass versus frequency for the uniform thickness VEH connected with a resistive load of 1 kΩ are shown in figure (4) and figure (5) respectively. The results are consistent with the experimental results from [5].

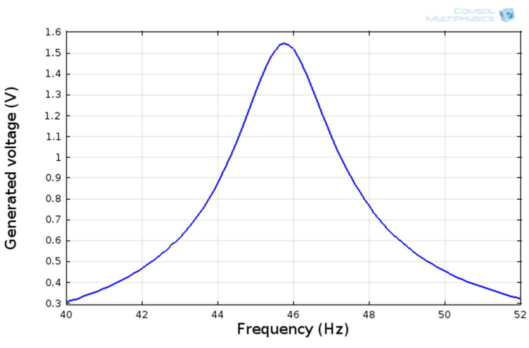

Figure 4. Plot of generated voltage versus frequency for uniform thickness VEH with 1 kΩ load.

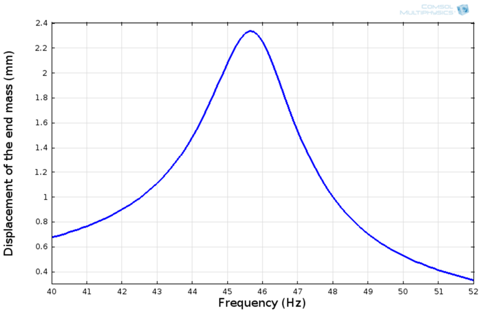

Figure 5. Plot of displacement of the end mass versus frequency for uniform thickness VEH with 1 kΩ load.

5.1 Power comparison

The generated power varies with the value of connected load resistance and is maximum for the optimal value. The variation of generated power with load resistance is plotted for uniform and thickness-tapered VEHs in figure (6) for the input sinusoidal acceleration of 1 g and frequency of 45.75 Hz. The peak power for 30 kΩ load resistance at 45.75 Hz excitation frequency is 21.95 mW for the uniform thickness VEH whereas peak power in the thickness-tapered VEH is 28.83 mW.

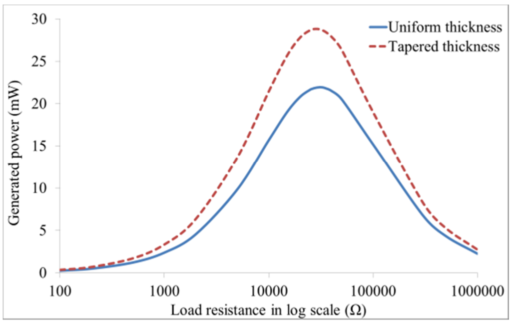

Figure 6. Plots of generated power versus load resistance for uniform and thickness-tapered VEHs.

5.2 Stress comparison

The stress profile in the uniform thickness VEH is shown in figure (7), and it is seen that the stress is highest at the fixed end and gradually decreases towards the free end of the beam. The stress profile for the thickness-tapered VEH given in figure (8) shows well distributed stress in the beam. The average stress on the surface is 4.25 MPa for uniform thickness VEH and 6.06 MPa for thickness-tapered VEH. The maximum stress is 9.65 MPa in uniform thickness VEH and 8.82 MPa in thickness-tapered VEH.

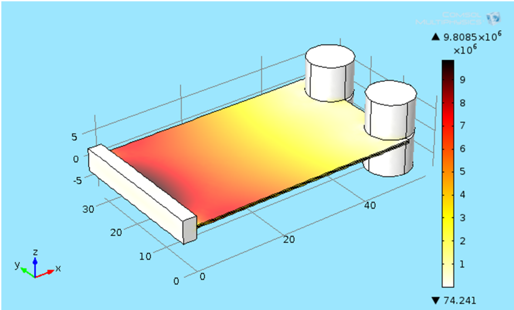

Figure 7. Stress profile in the uniform thickness VEH.

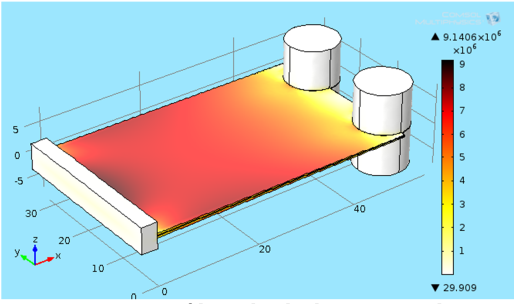

Figure 8. Stress profile in the thickness-tapered VEH.

6. Conclusions

Bimorph piezoelectric VEHs consisting of brass substrate sandwiched between two PZT-5A layers with uniform thickness and thickness-tapered beams have been simulated using COMSOL Multiphysics. The variation of generated power with excitation frequency is studied, and the results in the two cases are compared. The thickness-tapered VEH is found to have reduced peak stress and more uniform stress profile. The average stress on the surface increases by 42% and generated power increases by 31% in the thickness-tapered VEH for the same input excitation.

7. References

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