

# Inverse Problem and a New Device to Estimate Thermal Conductivity of Composite Phase Change Material

KARKRI Mustapha <sup>(1)</sup>, NACHAOUI Abdeljalil <sup>(2)</sup>

(1) Université Paris-Est, CERTES, 61 Avenue du Général de Gaulle, 94010 Créteil Cedex

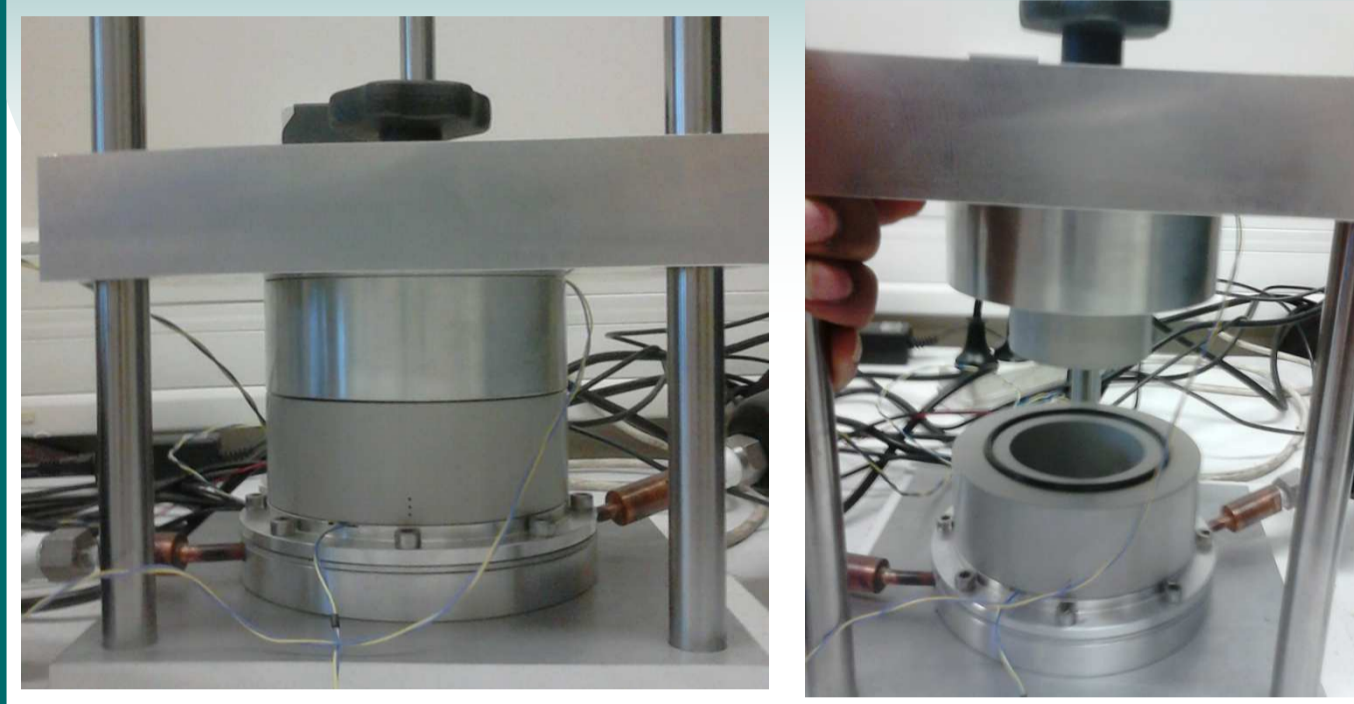
(2) Université de Nantes, Département de Mathématiques 2, rue de la Houssinière BP 92208, 44322 Nantes Cedex 3



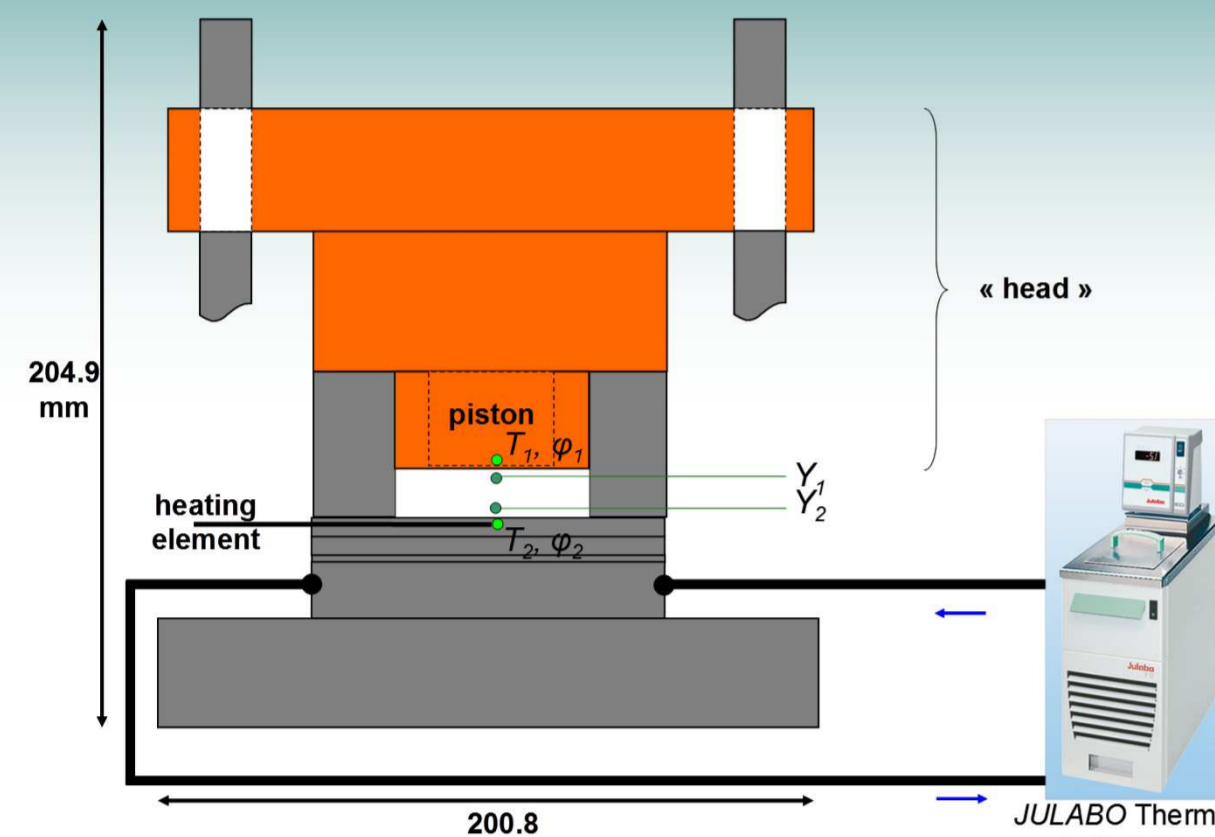
## Context / Objective

Due to the current energy crisis, thermal-energy storage systems become more and more crucial for reducing fossil fuel consumption and CO<sub>2</sub> emissions. The phase-change materials (PCMs) are attractive solutions because they can change their state (usually solid-liquid transitions) at relatively low temperatures while absorbing or releasing high amounts of heat. However, it is important to know precisely their thermal properties in order to perform accurate thermal simulations and improve PCM thermal performance. It is in this context that a new experimental device has been developed to characterize PCM thermal properties (thermal conductivity, sensible and latent heat thermal energy storage) in the solid phase, during the solid-liquid transition and in the liquid phase. *(In this presentation, we show only the solid phase characterization)*

## Description of the thermal characterization method



Overview of the experimental device



Simplified drawing

### Principle of the measurement:

A typical measurement consists in imposing a vertical temperature gradient through the PCM sample driven by a heat source and monitoring during the experiment duration all the boundary conditions (temperatures and heat fluxes) and temperatures within the PCM sample. These experiment data are used to inverse the 2D heat conduction by applying the conjugate gradient method and finally, to determine the PCM thermal properties.

**Inverse heat conduction problem:** a coupling between COMSOL and MATLAB has been performed in order to solve the inverse heat conduction problem by using the conjugate gradient method. The conjugate gradient algorithm is implemented in MATLAB which drives COMSOL by giving it the input parameter models.

### Conjugate gradient algorithm:

1. Choose an initial guess  $k^{(0)}$  (MATLAB).
2. Compute the **direct problem** (COMSOL) and the functional  $J(k^{(i)})$  (MATLAB)

$$J(k) = \sum_{i=1}^M \|T(x_i, y_i, t) - Y_i(x_i, y_i, t)\|^2$$

3. Compute the **adjoint problem** and the **gradient** (COMSOL)

$$\frac{\partial J}{\partial k} = - \int_0^{t_f} \int_{\Omega} \left( \frac{\partial T}{\partial x} \frac{\partial \Psi}{\partial x} + \frac{\partial T}{\partial y} \frac{\partial \Psi}{\partial y} \right) dS dt$$

4. Define the conjugation coefficient (MATLAB)

$$\gamma^{(i)} = \frac{\int_0^{t_f} \int_{\Omega} \nabla J^{(i)} [\nabla J^{(i)} - \nabla J^{(i-1)}] dS dt}{\|\nabla J^{(i-1)}\|^2}$$

5. Define the direction of descent (MATLAB)

$$w^{(i)} = -\nabla J^{(i)} + \gamma^{(i)} w^{(i-1)}$$

6. Define the **step size** by solving the **sensitivity problem** (COMSOL)

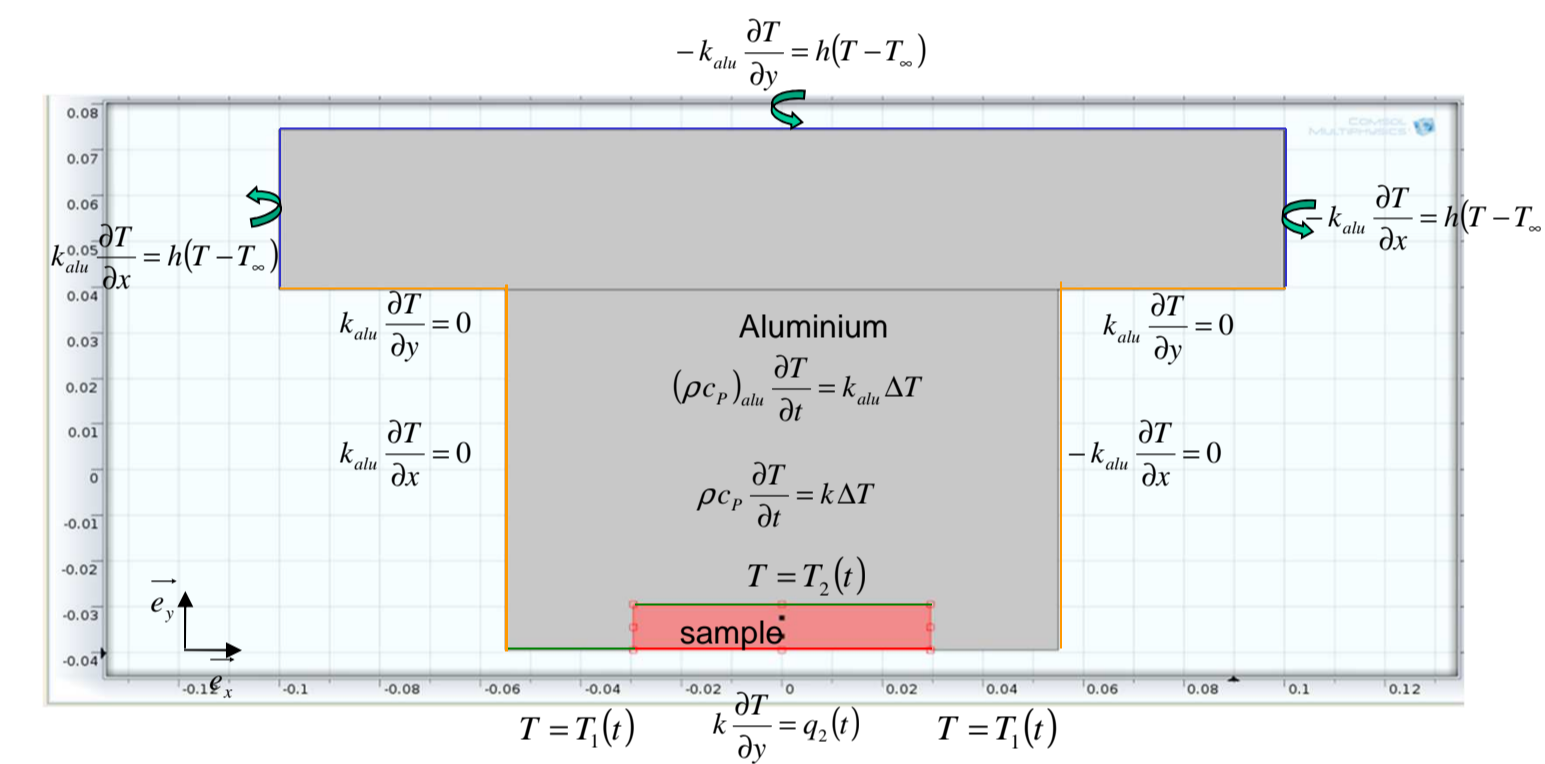
$$\rho = \frac{\int_0^{t_f} \sum_{i=1}^M [T(x_i, y_i, t) - Y_i(x_i, y_i, t)] \Delta T_k(x_i, y_i, t) dt}{\int_0^{t_f} \sum_{i=1}^M [\Delta T_k(x_i, y_i, t)]^2 dt}$$

7. Compute the new iterate (MATLAB)

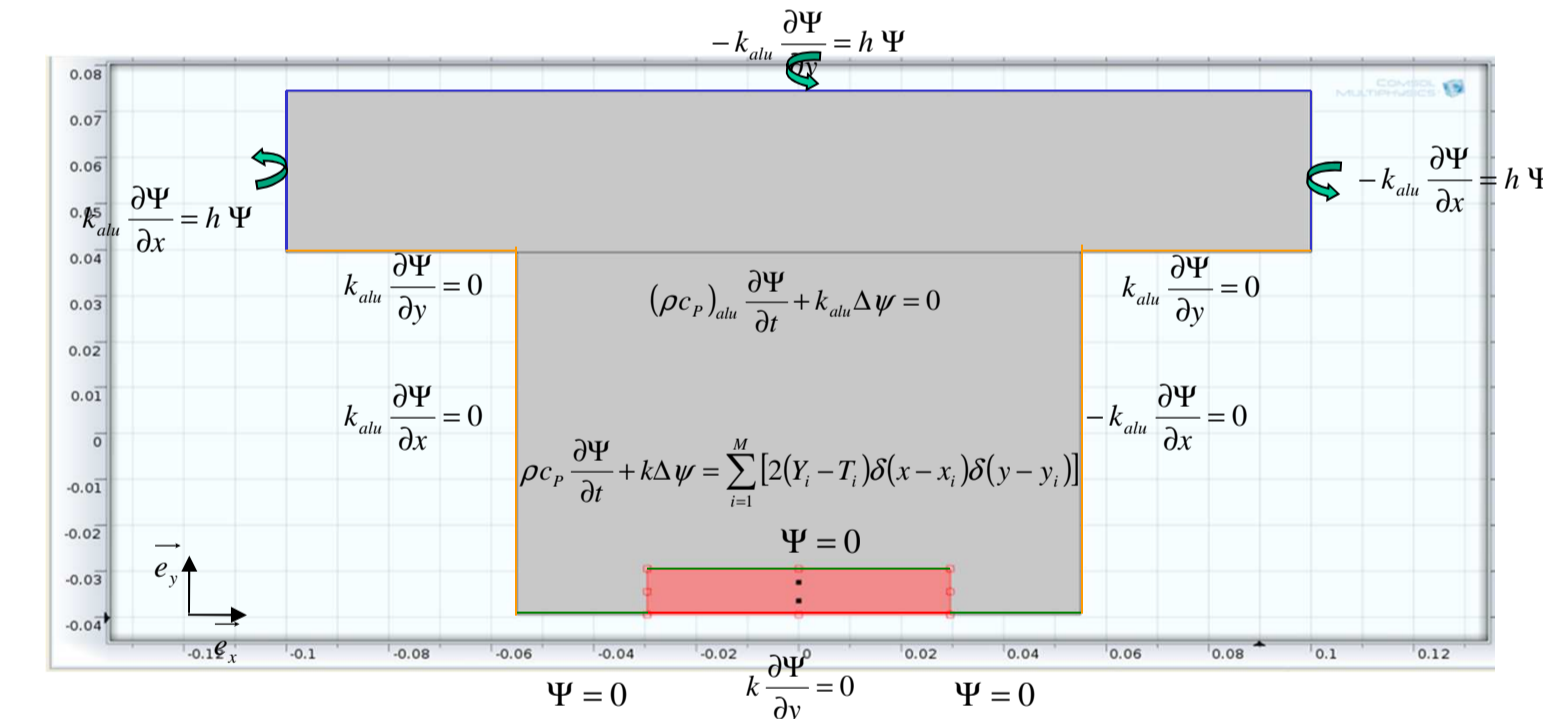
$$k^{(i+1)} = k^{(i)} + \rho^{(i)} w^{(i)}$$

8. If  $J(k^{(i+1)}) \leq \epsilon$ , stop. Otherwise, go to step 2 (MATLAB).

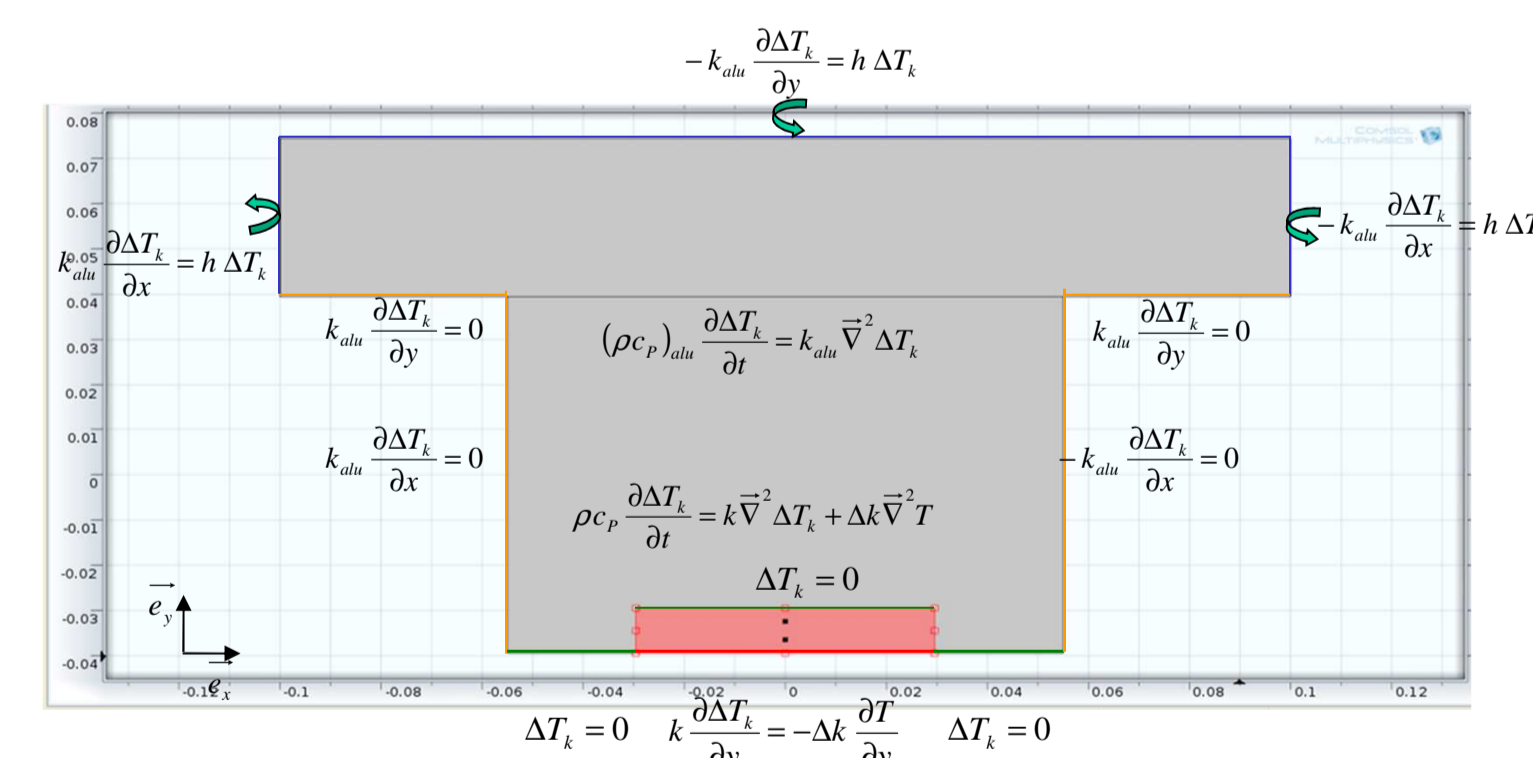
### Direct problem:



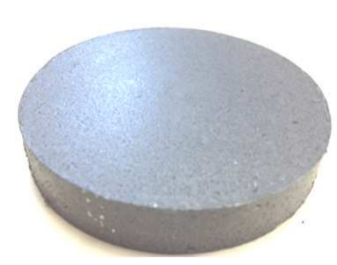
### Adjoint problem:



### Sensitivity problem:

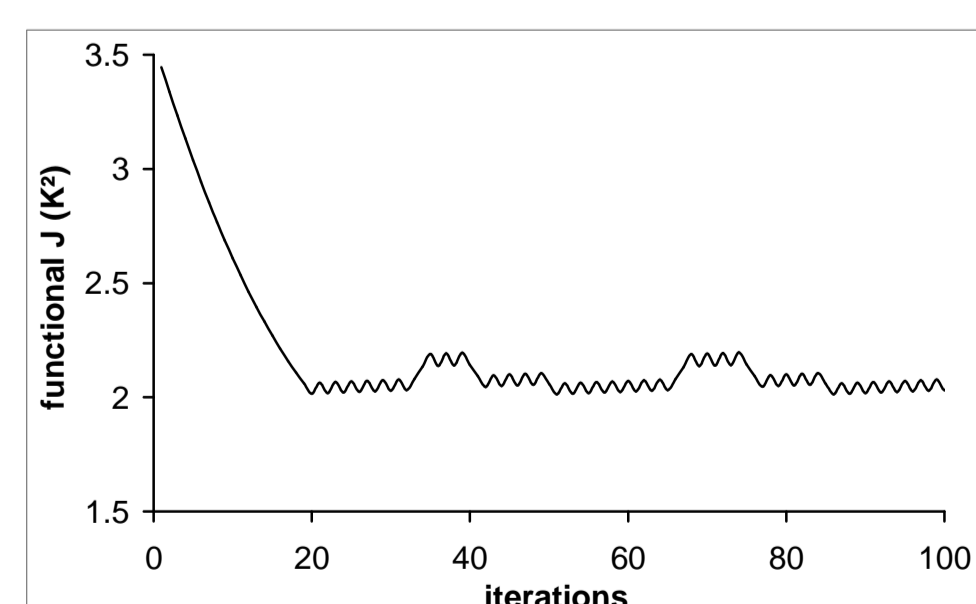


**Results:** for this measurement, we identify only the thermal conductivity  $k$  of a PCM sample.

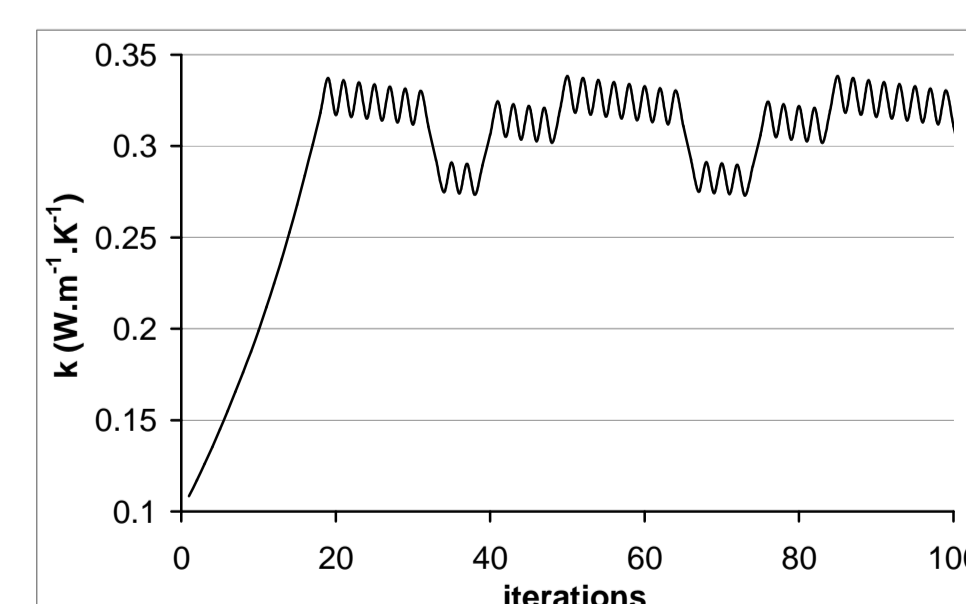


### Composite PCM sample:

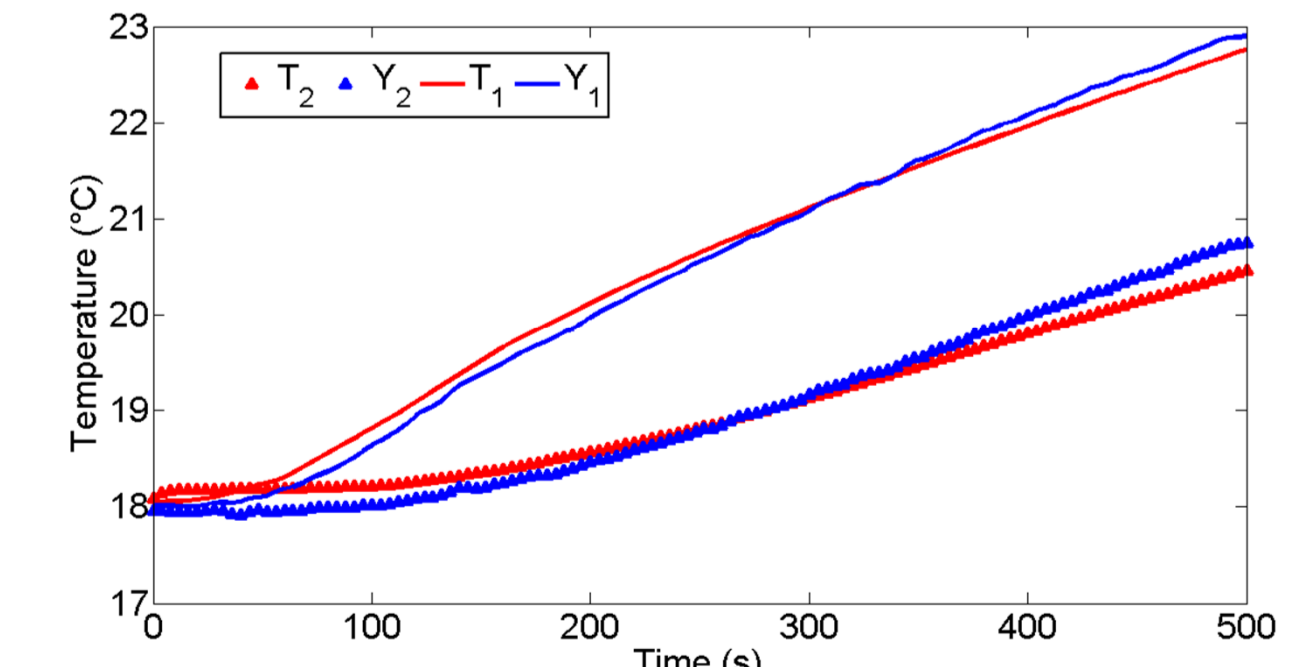
5% wt of graphite waste+paraffin  
(3 cm radius, 1 cm length)



Minimization of the functional  $J$  by the conjugate gradient method



Determination of  $k$



Comparison model vs experiment

The inverse heat conduction procedure gives us the PCM thermal conductivity  $k = 0.318 \text{ W.m}^{-1}.\text{K}^{-1}$

In this work [1-2], a measurement of the same PCM sample performed with the DICO gives  $k = 0.289 \pm 0.012 \text{ W.m}^{-1}.\text{K}^{-1}$

We can conclude that our experimental device and the inverse heat conduction provides consistent and correct value of thermal conductivity.

## Literature cited

- [1] M. Lachheb, M. Karkri, F. Albouchi, S.B. Nasrallah, M. Fois, P. Sobolciak, Thermal properties measurement and heat storage analysis of paraffin/graphite composite phase change material, *Composites Part B: Engineering*, Vol. **66**, 518-525, 2014.  
[2] M. Karkri, M. Lachheb, F. Albouchi, S. B. Nasrallah, and I. Krupa, Thermal properties of smart microencapsulated paraffin/plaster composites for the thermal regulation of buildings, *Energy and Buildings*, vol. **88**, pp. 183-192, 2015. Excerpt from the Proceedings of the 2015 COMSOL Conference in Grenoble