INSTITUTE OF COMMUNICATION, INFORMATION AND PERCEPTION TECHNOLOGIES

Scuola Superiore Sant'Anna

Analysis of strain-induced Pockels effect in Silicon

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COMSOL CONFERENCE
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Outline

- Electro-optic effect in crystal
- Strain-induced Pockels effect
- The proposed model and the overlap functions
- Conclusion

Nonlinear optics

Nonlinear effect can be mathematically described by the polarization vector

$$D_i(\boldsymbol{r},t) = \varepsilon_0 E_i(\boldsymbol{r},t) + P_i(\boldsymbol{r},t)$$

The general formula of the polarization vector is quite complicated*

$$\begin{aligned} P_i(\boldsymbol{r},t) = {} & \varepsilon_0 \int \chi^{(1)}_{ij}(\boldsymbol{r}-\boldsymbol{r'},t-t')E_j(\boldsymbol{r},t')d\boldsymbol{r'}dt \\ & + \varepsilon_0 \int \chi^{(2)}_{ijk}(\boldsymbol{r}-\boldsymbol{r_1},t-t_1,\boldsymbol{r}-\boldsymbol{r_2},t-t_2)E_j(\boldsymbol{r_1},t_1)E_k(\boldsymbol{r_2},t_2)d\boldsymbol{r_1}dt_1d\boldsymbol{r_2}dt_2 \\ & + \varepsilon_0 \int \chi^{(3)}_{ijkl}(\boldsymbol{r}-\boldsymbol{r_1},t-t_1,\boldsymbol{r}-\boldsymbol{r_2},t-t_2,\boldsymbol{r}-\boldsymbol{r_3},t-t_3)E_j(\boldsymbol{r_1},t_1)E_k(\boldsymbol{r_2},t_2)E_l(\boldsymbol{r_3},t_3)d\boldsymbol{r_1}dt_1d\boldsymbol{r_2}dt_2d\boldsymbol{r_3}dt_3 + \quad \ldots \end{aligned}$$

For a local medium, in the frequency domain we have

$$\begin{aligned} P_i(\boldsymbol{r},\omega) = {} & \varepsilon_0\, \chi^{(1)}_{ij}(\boldsymbol{r},\omega)E_j(\boldsymbol{r},\omega) \\ & + \varepsilon_0\, \chi^{(2)}_{ijk}(\boldsymbol{r},\omega_1,\boldsymbol{r},\omega_2)E_j(\boldsymbol{r},\omega_1)E_k(\boldsymbol{r},\omega_2) \\ & + \varepsilon_0\, \chi^{(3)}_{ijkl}(\boldsymbol{r},\omega_1,\boldsymbol{r},\omega_2,\boldsymbol{r},\omega_3)E_j(\boldsymbol{r},\omega_1)E_k(\boldsymbol{r},\omega_2)E_l(\boldsymbol{r},\omega_3) \\ & + \quad \ldots \end{aligned}$$

Linear term (refractive index)

Quadratic term (Pockels effect, SHG)

Cubic term (Kerr effect, FWM)

***Einstein notation** is used. The sum over repeated indices is always understood.

Electro-optic effect in crystal

The quadratic nonlinear susceptibility is responsible of the Pockels effect

$$P_i^{(2)}(\omega) = \varepsilon_0 \chi_{ijk}^{(2)} E_j(0) E_k(\omega)$$

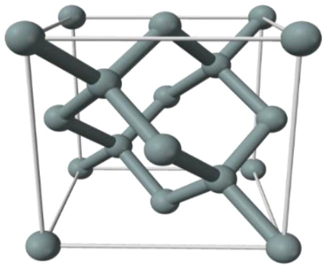
$\chi^{(2)}$ is a 3-index tensor → **27** components

Symmetry → **18** independent components

Our goals

1. $\chi^{(2)}$ as a function of the strain gradient tensor
2. Theory and optimization of nonlinear effects in silicon waveguides

Centrosymmetric crystal and deformations



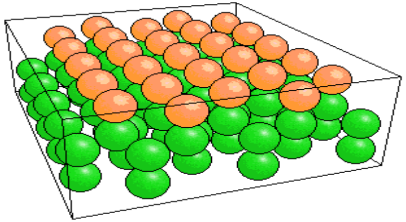
Silicon fundamental cell

- Inversion point

for every point (x, y, z) in the unit cell
there is an indistinguishable point (-x, -y, -z).

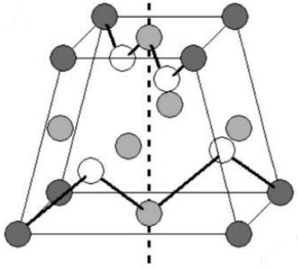
- No native $\chi^{(2)}$

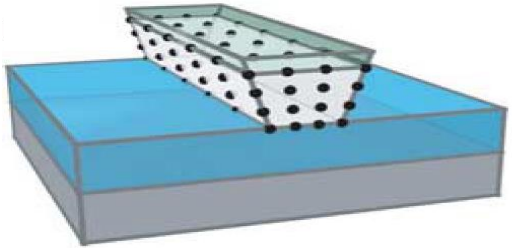
Surface interface



or

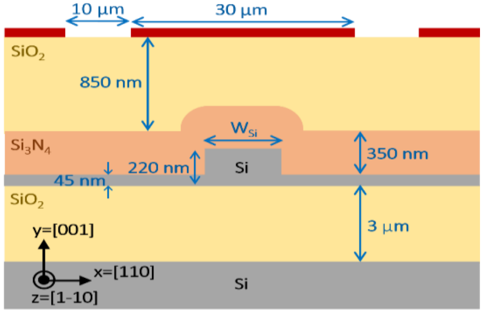
Non uniform strain

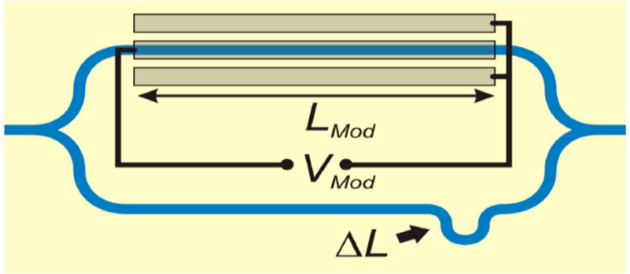


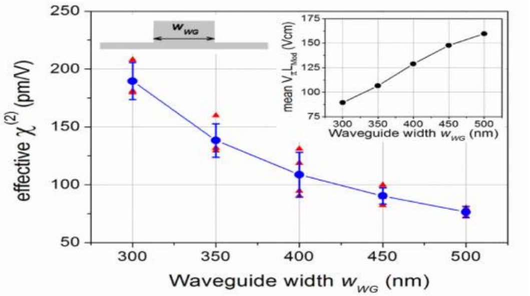


- Centrosymmetry is broken
- $\chi^{(2)}$ appears

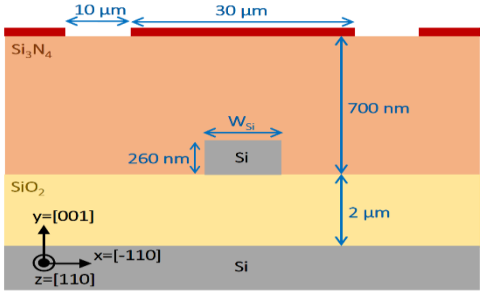
Strained silicon: state of the art

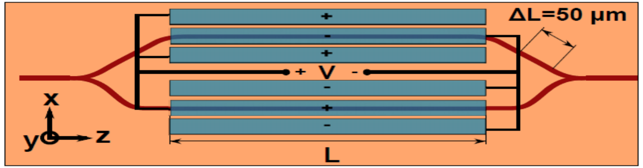


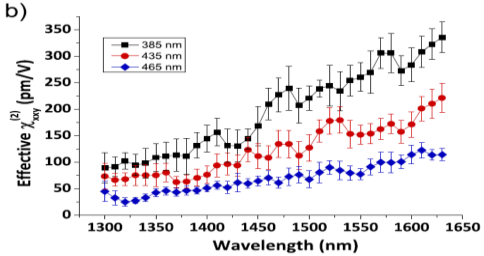




B. Chmielak et al., “Pockels effect based fully integrated, strained silicon electro-optic modulator,” Opt. Exp. **19**, pp. 17212–17219, 2011.
B. Chmielak et al., “Investigation of local strain distribution and linear electro-optic effect in strained silicon waveguides,” Opt. Exp. **21**, pp. 25324–25332, 2013.







P. Damas et al.,“Wavelength dependence of Pockels effect in strained silicon waveguides,” Opt. Express **22**, pp. 22095–22100, 2014.

$\chi^{(2)}$ as a function of the strain gradient tensor

Approximation of $\chi^{(2)}$ with its Taylor series with respect to the strain tensor and strain gradient tensor

Strain tensor ε_{lm}

Strain gradient tensor $\zeta_{lmn} = \frac{\varepsilon_{lm}}{x_n}$

$$\chi^{(2)}_{ijk}(\boldsymbol{\varepsilon}, \boldsymbol{\zeta}) = \chi^{(2)}_{ijk}(\boldsymbol{\varepsilon} = 0, \boldsymbol{\zeta} = 0) + \sum_{l,m} \frac{\partial \chi^{(2)}_{ijk}}{\partial \varepsilon_{lm}} \varepsilon_{lm} + \sum_{l,m,n} \frac{\partial \chi^{(2)}_{ijk}}{\partial \zeta_{lmn}} \zeta_{lmn} + o\left(\max_{l,m,n}\{|\varepsilon_{lm}|, |\zeta_{lmn}|\}\right)$$

Vanishes as a 5 index tensor and as all the Tensors with odd number of indices

$$T_{ijk\alpha\beta\gamma} = \left.\frac{\partial \chi^{(2)}_{ijk}}{\partial \zeta_{\alpha\beta\gamma}}\right|_{\varepsilon=0}$$

6-index tensor → Survives!

3^6=729 components!

Symmetries

Symmetry of the strain tensor

$$\varepsilon_{\alpha\beta} = \varepsilon_{\beta\alpha} \quad \Longrightarrow \quad \zeta_{\alpha\beta\gamma} = \zeta_{\beta\alpha\gamma}$$

Symmetry of the Pockels effect

$$\chi^{(2)}_{ijk}(\omega, 0) = \chi^{(2)}_{ikj}(0, \omega) \ ,$$

$$\chi^{(2)}_{ijk}(\omega, 0) = \chi^{(2)}_{ijk}(-\omega, 0) = \chi^{(2)}_{jik}(\omega, 0) \ ,$$ Lossless condition

48 symmetry operations compatible with cubic lattice of silicon

$$\begin{pmatrix} \pm1 & 0 & 0 \\ 0 & \pm1 & 0 \\ 0 & 0 & \pm1 \end{pmatrix}, \quad \begin{pmatrix} \pm1 & 0 & 0 \\ 0 & 0 & \pm1 \\ 0 & \pm1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & \pm1 & 0 \\ \pm1 & 0 & 0 \\ 0 & 0 & \pm1 \end{pmatrix},$$

$$\begin{pmatrix} 0 & \pm1 & 0 \\ 0 & 0 & \pm1 \\ \pm1 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & \pm1 \\ \pm1 & 0 & 0 \\ 0 & \pm1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & \pm1 \\ 0 & \pm1 & 0 \\ \pm1 & 0 & 0 \end{pmatrix}.$$

Due to symmetries,
only 15 components of $\chi^{(2)}$ are independent!

What can we measure? ...the effective susceptibility

The effective index can be experimentally measured

$$\Delta n^{\text{eff}} = \frac{\epsilon_0 c}{N} \int_A E_i^* \chi_{ijk}^{(2)}(\omega, 0) E_j E_k^{dc} \, \mathrm{d}A \,, \qquad N = \frac{1}{2} \int_{A_\infty} (\boldsymbol{E} \times \boldsymbol{H}^* + \boldsymbol{E}^* \times \boldsymbol{H}) \cdot \boldsymbol{i}_z \, \mathrm{d}A \,.$$

The effective susceptibility vector can be defined as $\chi_k^{\text{eff}} E_k^{dc} = n^{\text{eff}} \Delta n^{\text{eff}}$,

$$\chi_k^{\text{eff}}(\omega, 0) = T_{ijk\alpha\beta\gamma}(\omega, 0) \cdot n^{\text{eff}} \cdot \frac{\epsilon_0 c}{N} \int_A E_i^*(\boldsymbol{x}) \zeta_{\alpha\beta\gamma}(\boldsymbol{x}) E_j(\boldsymbol{x}) \, \mathrm{d}A \,.$$

measured **measured** **computed**

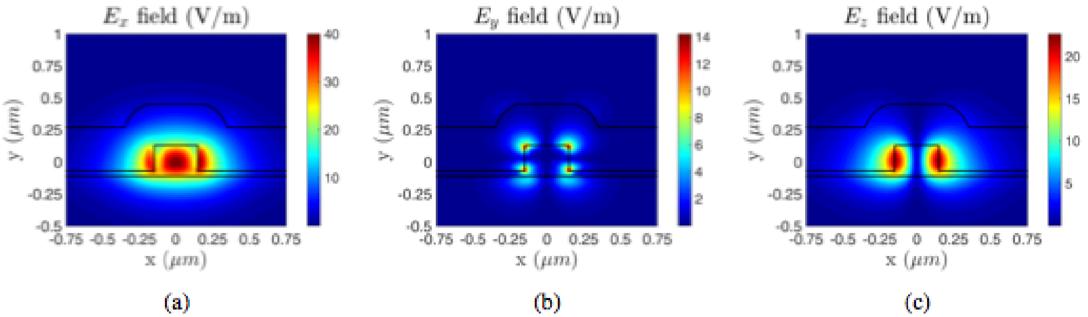
If only $E_y \neq 0$, we can write in a more compact way

$$\chi_y^{\text{eff}}(\omega) = c_i o_i(\omega) \ ,$$

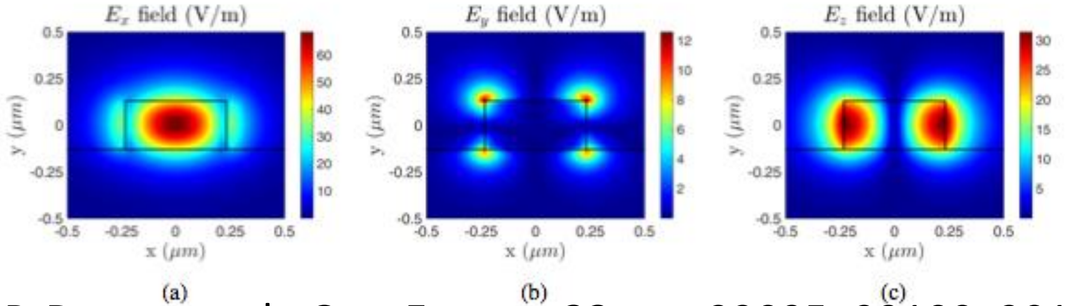
c_i can be derived from the experimental results.

$$n^2 = \epsilon / \epsilon_0 \ , \qquad \epsilon = \epsilon_0 \left[1 + \chi^{(1)} + 2\chi^{(2)} E^{dc} \right] \ .$$

COMSOL modal analysis



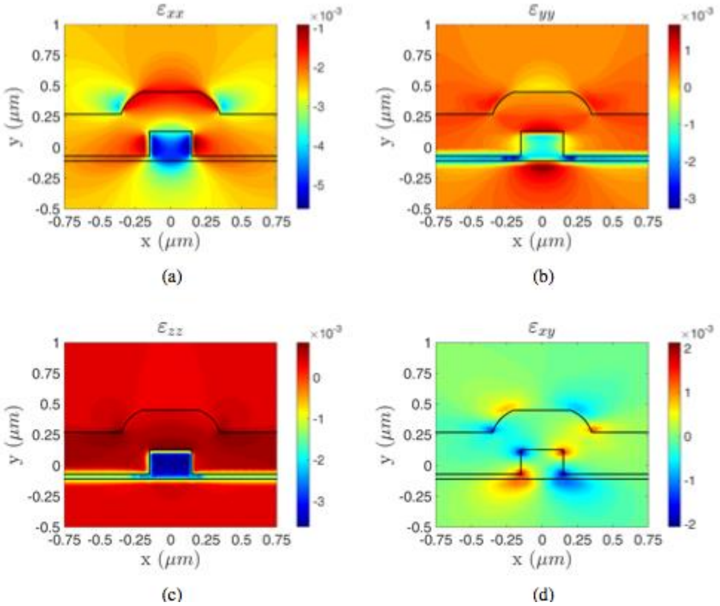
B. Chmielak et al., Opt. Express **19**, pp. 17212–17219, 2011.



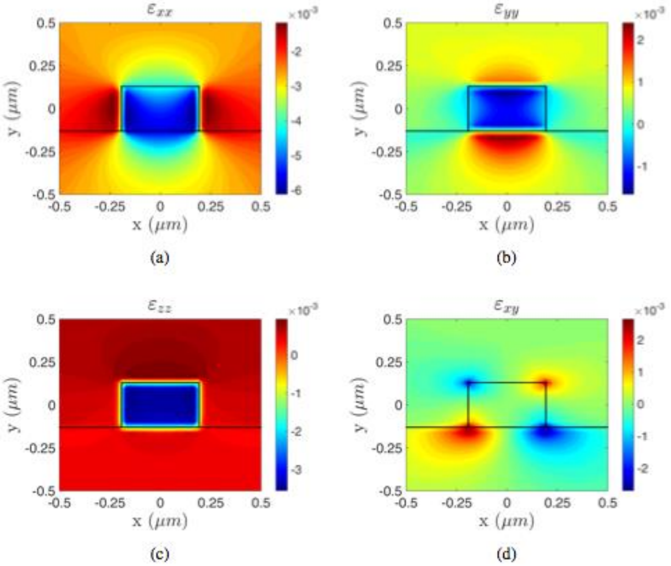
P. Damas et al., Opt. Express **22**, pp. 22095–22100, 2014.

The waveguides show a single mode behavior but E_z and E_y components are not negligible compared to E_x component and thus the mode is clearly not purely transverse electric. The high value of E_z is due to the high index step of the waveguide.

COMSOL mechanical deformation



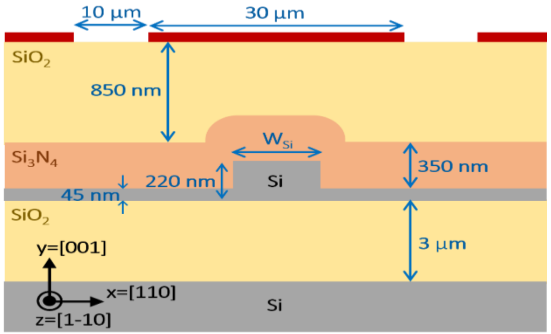
B. Chmielak et al., Opt. Express **19**, 2011.

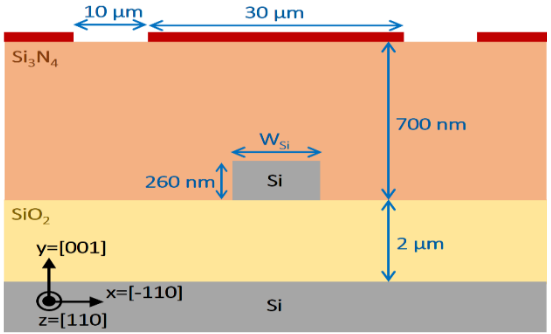


P. Damas et al., Opt. Express **22**,2014.

Silicon anisotropy described by orthotropic model and initial stress of 1 GPa

Overlap function $\chi_y^{\mathrm{eff}} = c_i\, o_i$





B. Chmielak et al., Opt. Express **19**, pp. 17212–17219, 2011.
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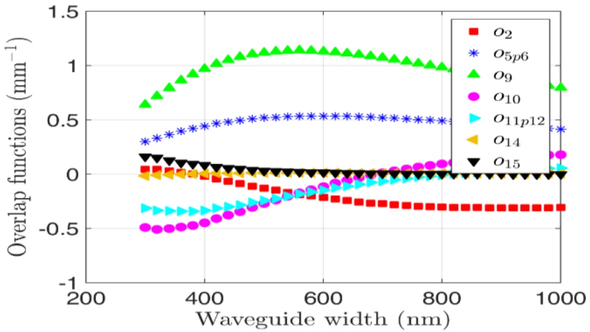
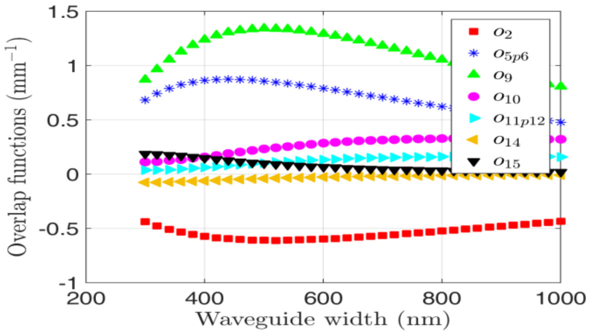
P. Damas et al., Opt. Express **22**, pp. 22095–22100, 2014.





Conclusion

- ✓ A simple model is proposed for the *Pockels effect*
- ✓ *Design and optimization* of strained silicon based device
- ✓ Other second order effect in strained silicon (e.g., *Second Harmonic Generation*) can be modelled
- ✓ Further investigations and experimental results are needed to evaluate all 15 coefficients
- ✓ Paper accepted on Optics Express and on arxive (http://arxiv.org/abs/1507.06589)