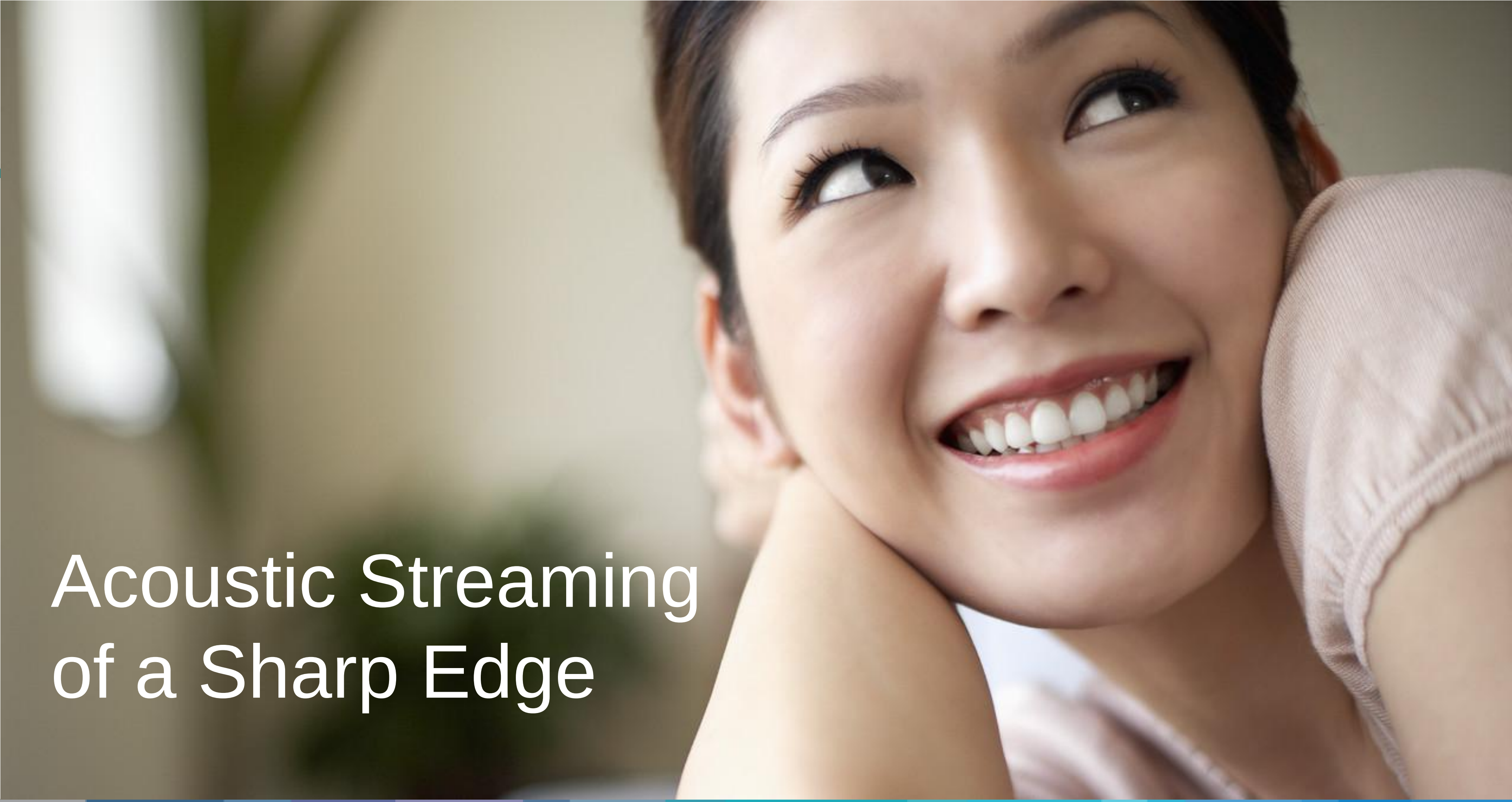

# Acoustic Streaming of a Sharp Edge

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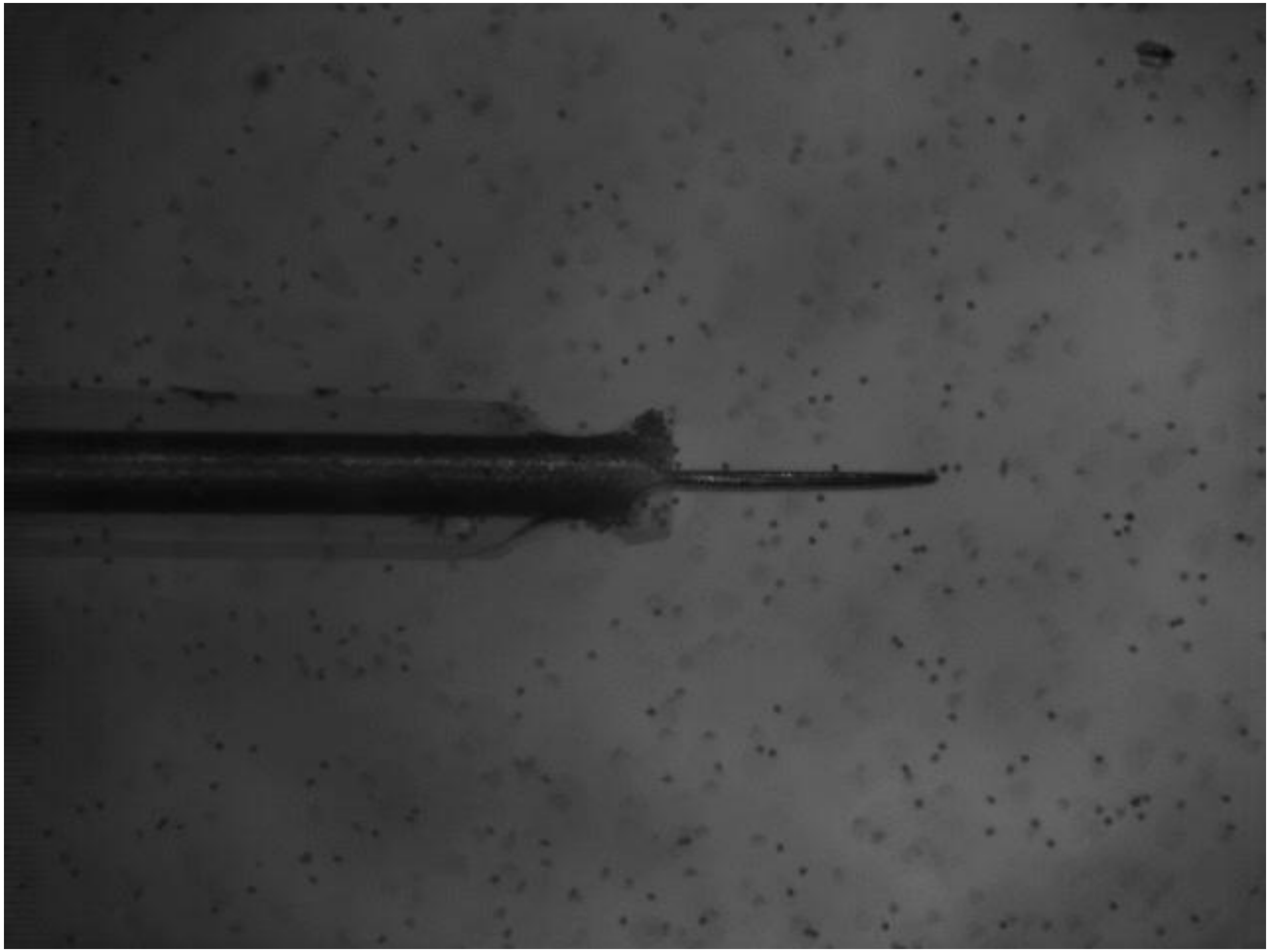
# What is acoustic streaming?

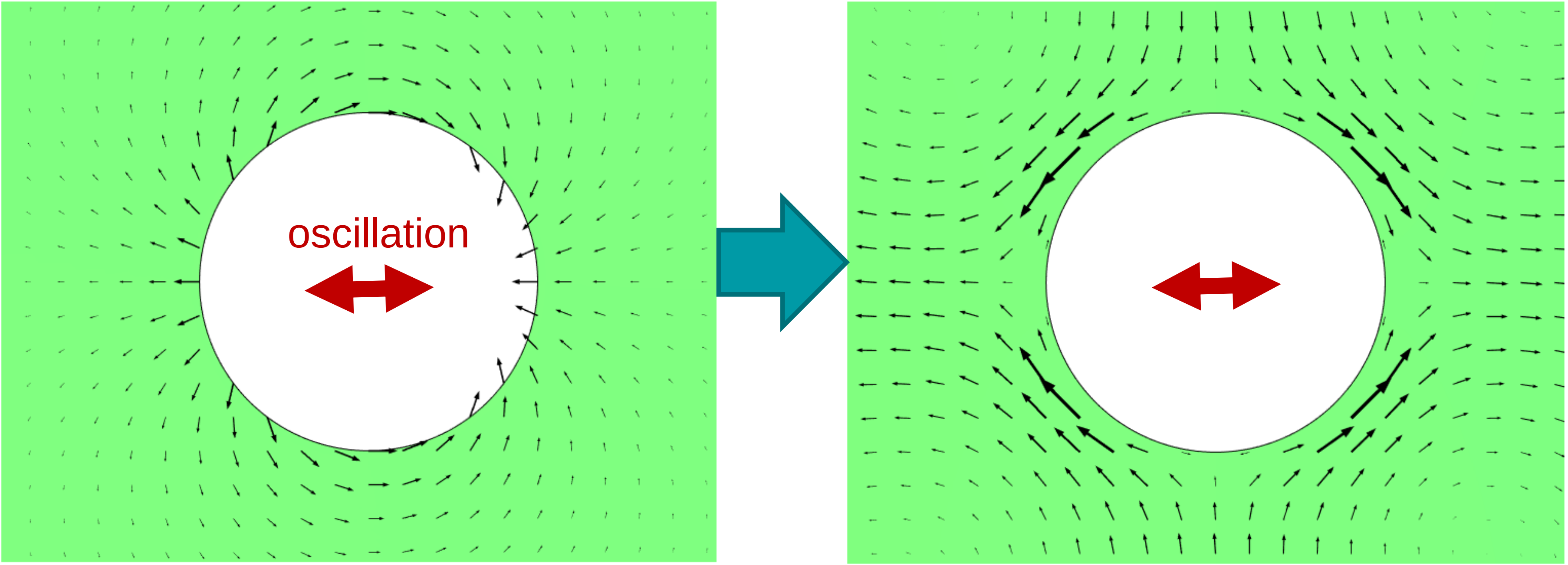
An ultrasonic instrument vibrating in water generates a strong steady stream of fluid



# Classic Example

Acoustic velocity

Weak steady streaming



# Why?

## Fluid dynamics (Navier-Stokes eq-n):

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v}$$

# Why?

## Fluid dynamics (Navier-Stokes eq-n):

$$\frac{\partial \mathbf{v}}{\partial t} + \cancel{(\mathbf{v}\cdot\nabla)\mathbf{v}} = -\frac{1}{\rho}\nabla p + \cancel{\nu\nabla^2\mathbf{v}}$$

## For small amplitude vibrations



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# Why?

## Fluid dynamics (Navier-Stokes eq-n):

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v}$$

## Acoustics

$$\nabla^2 p = \frac{1}{c^2}\frac{\partial p}{\partial t}$$

# 1st analytical theory: Lord Rayleigh, 1896

## Fluid dynamics (Navier-Stokes eq-n):

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{v}$$

$\cos^2(\omega t)$
Has steady component

Creates Boundary layer




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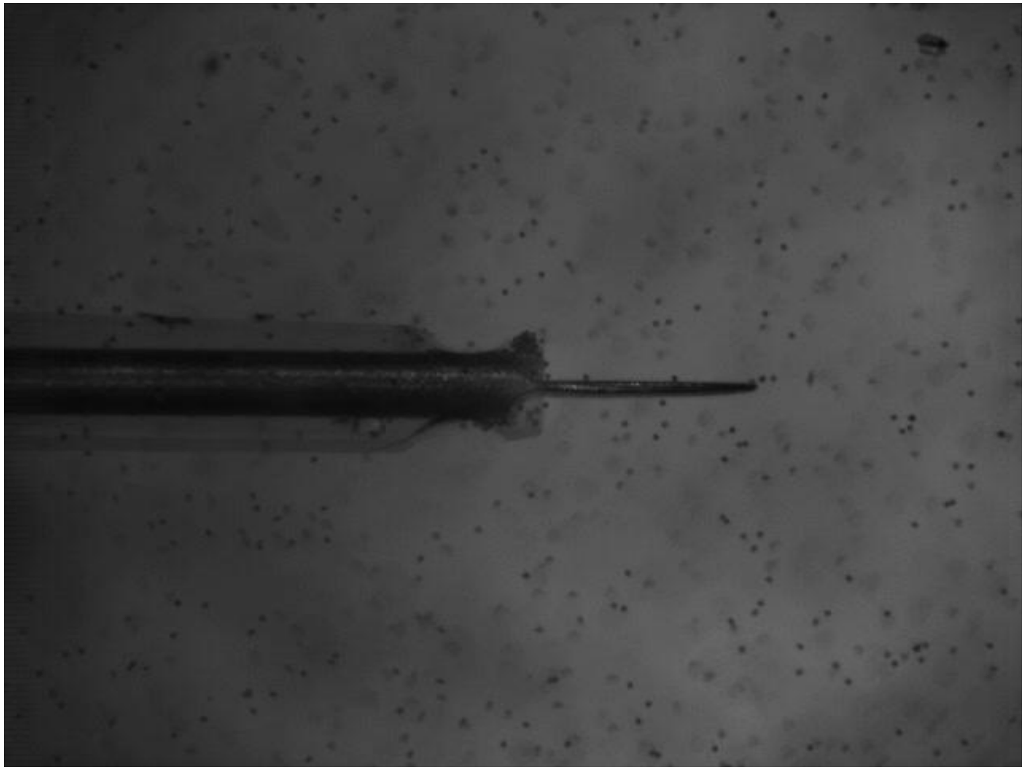
# It does not work for our tips:



- The streams are 100 times stronger
- The problem are sharp edges
- Can we compute the streaming using FEA analysis?
- We have two options:

  1) Do full time dependent CFD analysis – very expensive (must resolve time and length scales); does not give any explanation.

  2) Implement perturbation theory using COMSOL PDE interface



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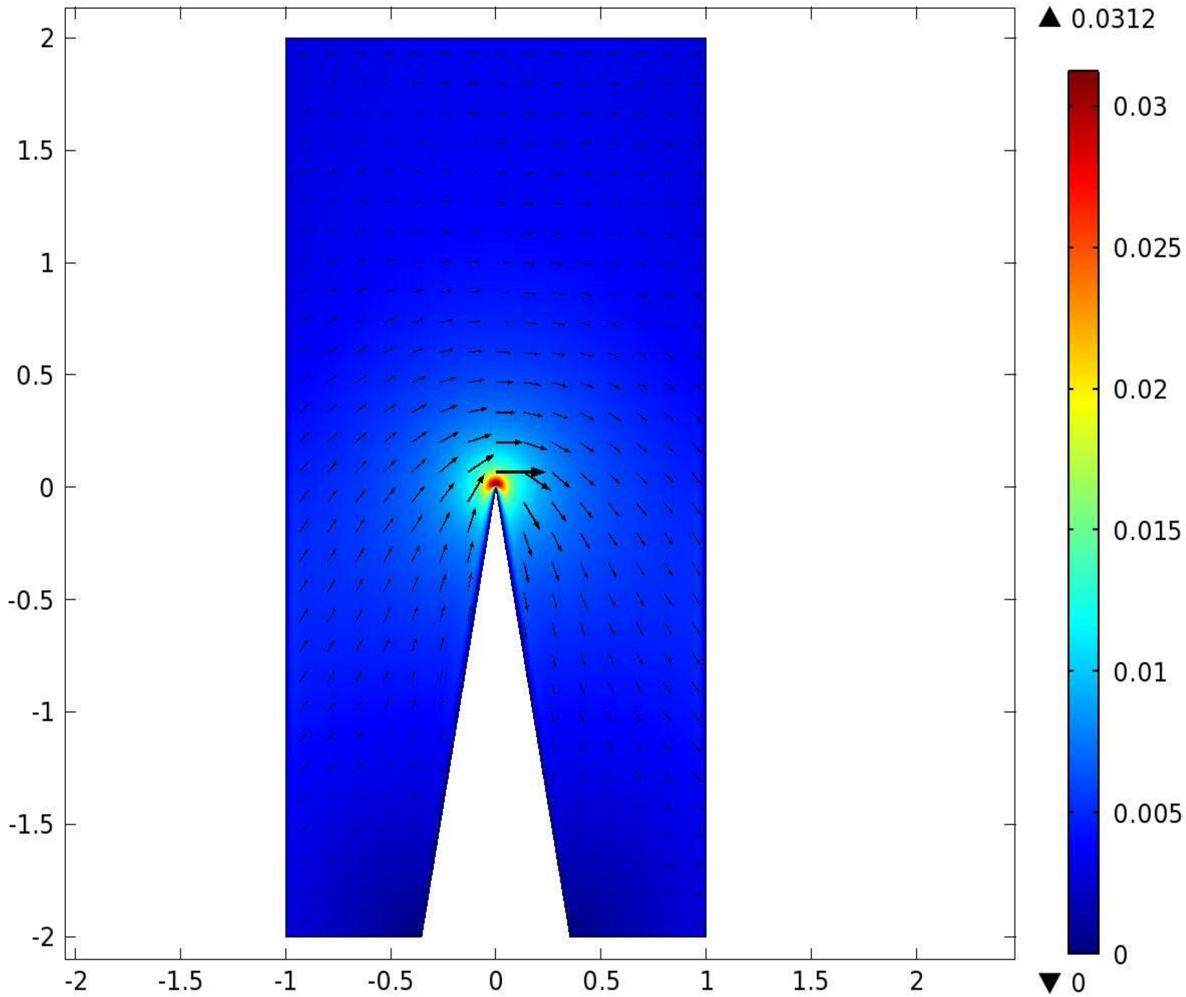
# COMSOL Solution step 1:




Solve equations for the oscillatory motion of the fluid.

$$i\omega \mathbf{v}_\omega = -\frac{1}{\rho}\nabla p_\omega + \nu \nabla^2 \mathbf{v}_\omega$$

The solution of time-independent equations implemented using weak PDE interface.



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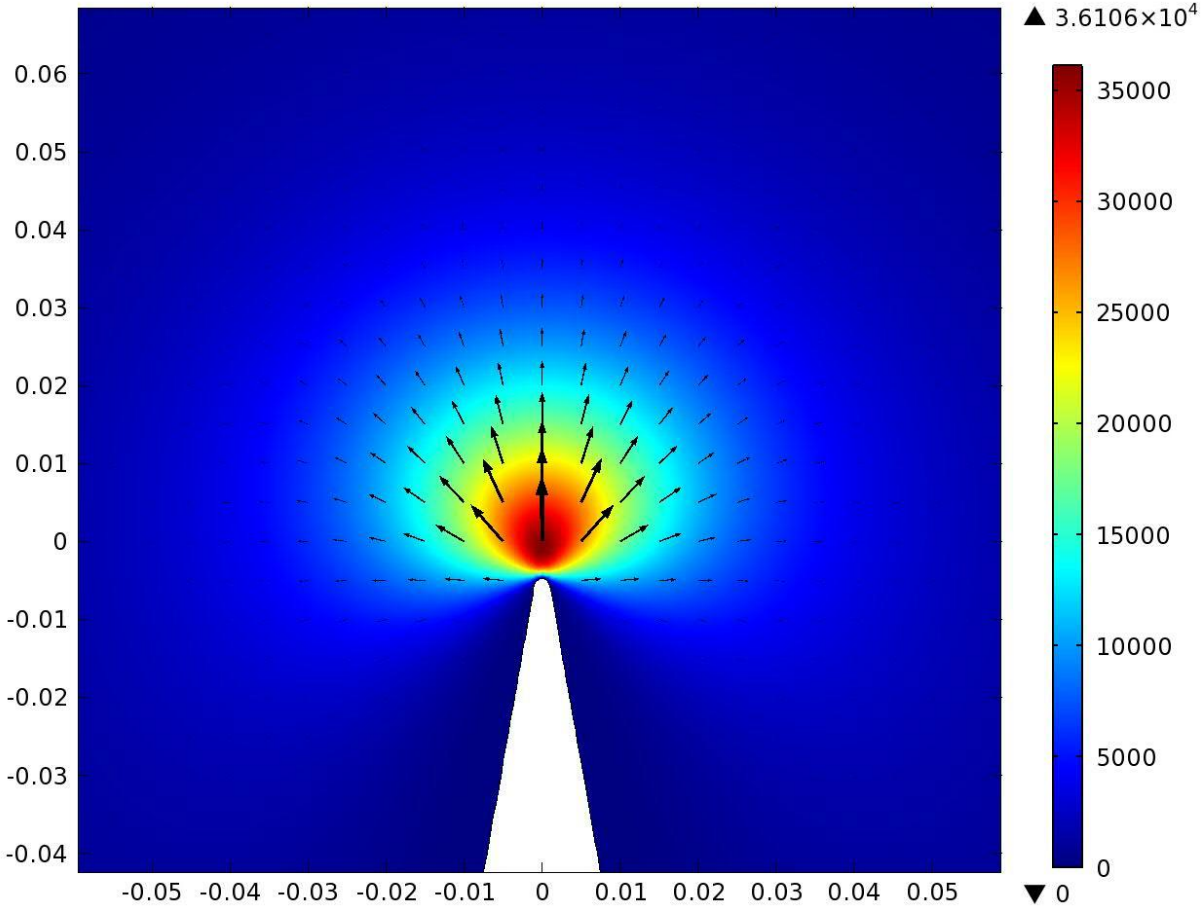
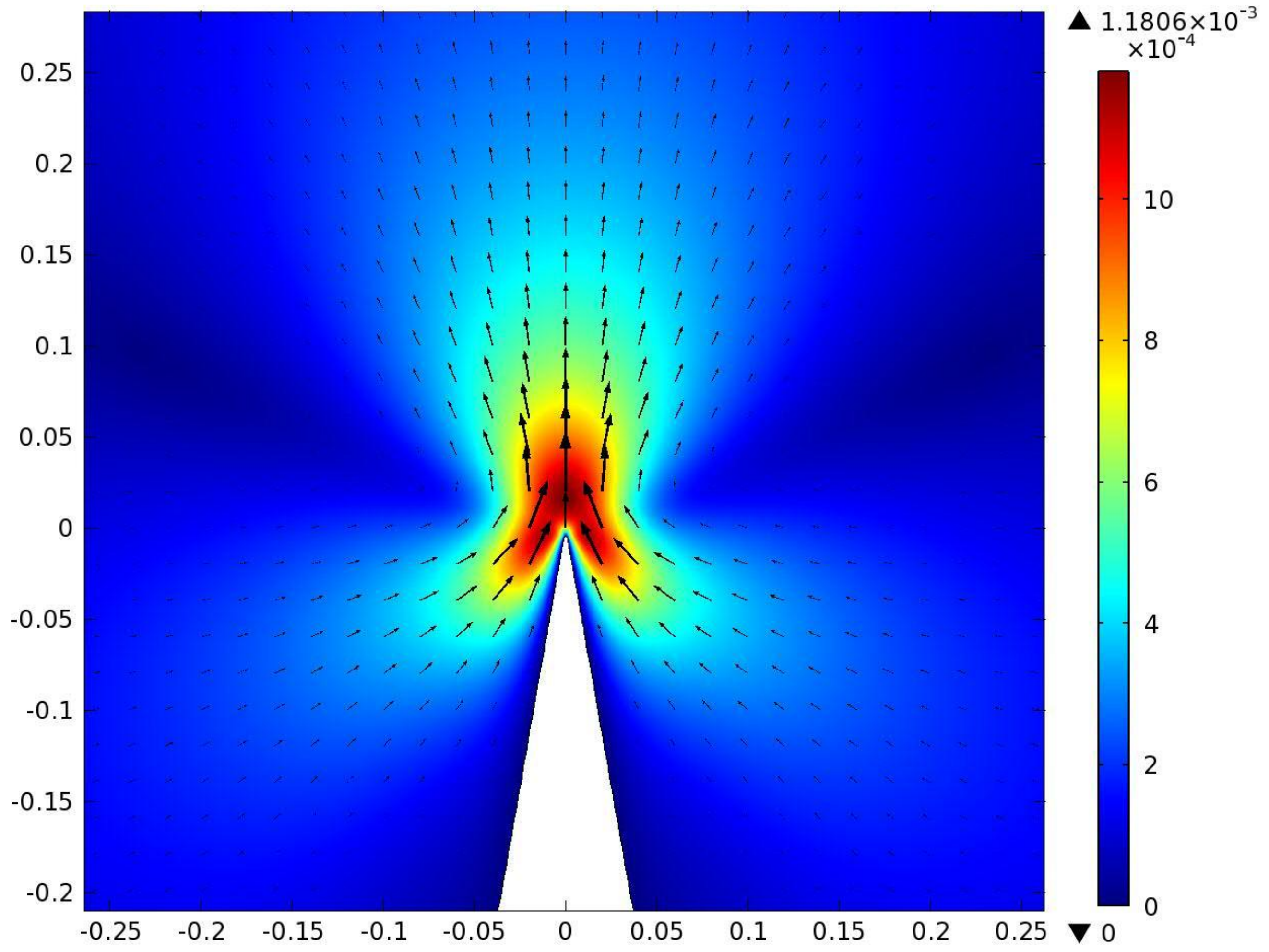
# COMSOL Solution step 2:




Calculate the time independent force driving steady stream of fluid:

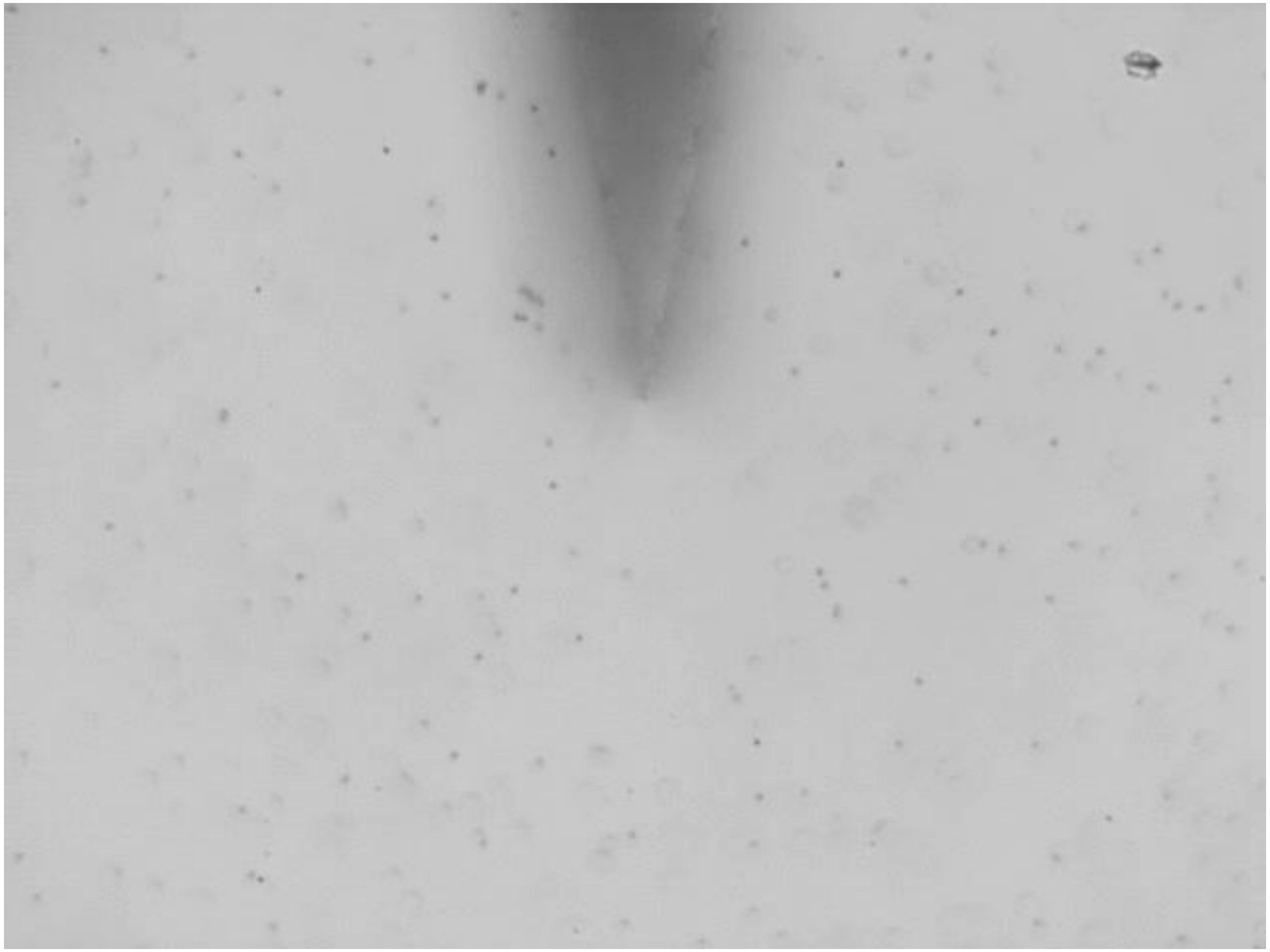
$$F = -\frac{1}{2}\rho \,\mathrm{Re}\left[\left(\mathbf{v}_{\omega} \cdot \nabla\right)\mathbf{v}_{\omega}^{*}\right]$$

# COMSOL Solution step 3:




Perform CFD analysis of the steady-state flow with body force from step 2

# Comparison with experiment





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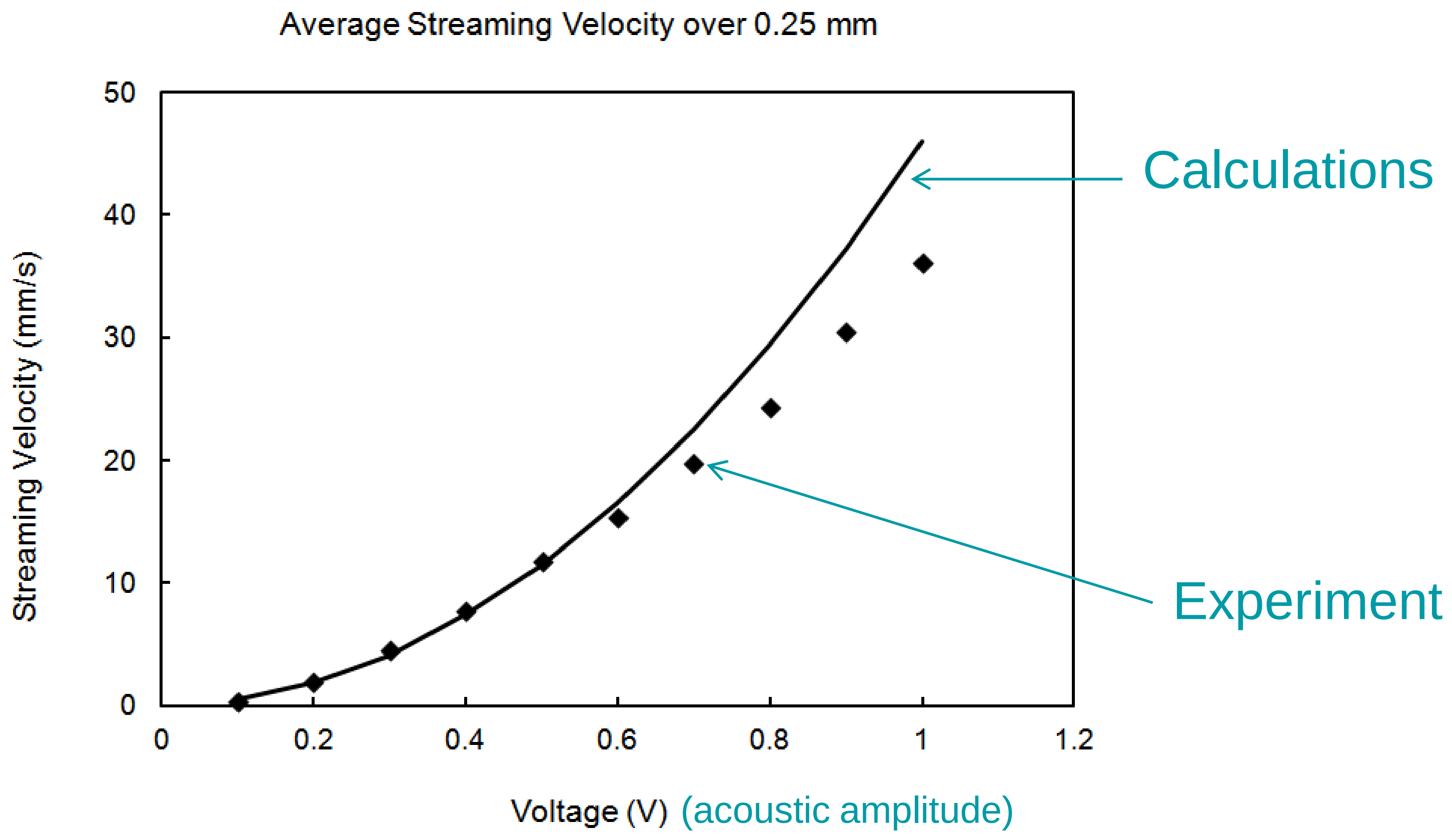
# Comparison with experiment



# Conclusions

- Sharp edges vibrating in a fluid give rise to a new type of acoustic streaming
- Understanding of this phenomenon required development of computational methodology
- COMSOL has enough flexibility to solve this problem using perturbation theory
- The solution gives perfect comparison with experiment and other computational methods
- Perturbation theory provides understanding of the nature of streaming: Streaming is caused by a centrifugal force of the fluid vibrating around a sharp edge



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