Solving the Inverse Problem of Resonant Ultrasound Spectroscopy on Dumbbell-shaped Compression Samples using COMSOL

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Abstract: The dumbbell geometry is an ideal shape for testing the compressive strength of advanced ceramic materials. This report details the combined use of COMSOL and LiveLink for MATLAB for determining the elastic properties of dumbbell-shaped samples by measuring their acoustic resonance frequencies. Degenerate bending modes which are sensitive to inhomogeneities or machining defects that disrupt axial symmetry have been identified. A sensitivity analysis of the effect of geometry and density variations to the resonance frequencies is included. Additionally, the potential use of a linear model for solving the inverse problem is discussed.

Keywords: dumbbell, resonance, RUS, eigenfrequency, ceramics, characterization

1. Introduction

Compressive strength is an important material property of advanced ceramics. In order to accurately characterize this property through uniaxial compression testing, a dumbbell-shaped geometry [1] was developed to ensure that compressive fracture and failure resided only in the reduced diameter gage section (Figure 1). However, the manufacturing of any sample can lead to machining defects which introduce fracture nucleation sites and apparent lower strength values. The elastic properties of test specimens are also of interest and can lend insight into the compressive strength. Popular non-destructive techniques to characterize these properties involve observing the interaction of acoustic waves with the sample, from which the elastic properties can be determined. A very accurate method for determining elastic properties of small samples is Resonant Ultrasound Spectroscopy (RUS) [2, 3]. This method characterizes the resonance peaks that arise in a swept frequency spectrum and are determined by the sample's geometry, density, and elastic tensor. The sample geometry and density are usually known or can be easily measured and the elastic tensor is then determined through solving the inverse problem by finding the match between a numerical model and the measured resonance spectrum.

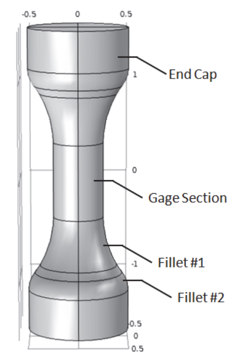

Figure 1: Dumbbell compression test specimen. Scale is in centimeters.

2. Modeling and Parameter Extraction

The eigenfrequency study of solids included in the structural mechanics module of COMSOL, along with the LiveLink for MATLAB provide a very powerful toolset for solving this inverse problem. The method employed in this study was the following: First, the 3D dumbbell geometry (Figure 1) was parameterized and constructed as a linear elastic solid, allowing for variation in the measured sample geometry due to machining variability. All boundaries are left in COMSOL's default state as free. An extra-fine physics-controlled mesh was used. The material parameters (Young's modulus, Poisson's ratio, and density) are instantiated to best-guess values and the forward problem of determining the first 16 eigenfrequencies is solved for using COMSOL's MUMPS solver. Often, there are more experimental resonance peaks observed than modeled eigenfrequencies due to slight deviations between the actual sample geometry and the idealized geometry, or due to constraints or added components in the experimental setup. As a result, the resonance peaks and modeled eigenfrequencies cannot be matched one-for-one. To help account for this, the modeled frequencies are brought into

MATLAB in which the Jonker-Volgenant algorithm [4] for solving the linear assignment problem of matching the modeled eigenfrequencies to the most appropriate measured frequencies is used to determine a fitness score for the current model. To aid in minima detection, the natural logarithm of the assignment score provided by the Jonker-Volgenant algorithm is used to enhance the rate of decent towards the lowest scores (best match). This process is then repeated through a direct search method (MATLAB, *fminsearch*) with new model parameters until the score is minimized and the optimal solution is found.

3. Results and Discussion

3.1 Alumina Sample

To evaluate this method a dumbbell sample was machined from a 94% alumina (CoorsTek) block with a density of 3.56 g/cc, Young's modulus of 280 GPa, and Poisson's ratio of 0.23. The sample was placed between the transducers of a Magnaflux Quasar RUSpec system (Figure 2) and swept in frequency between 1kHz and 3MHz. The goal of the direct search algorithm is to locate the minimum mismatch between the measured RUS spectrum and the model, indicating that the correct elastic properties have been discovered. However, optimization algorithms that do not re-initialize to random starting parameters tend to get trapped in local minima. Also, if the minimum is not well defined or ambiguous then the elastic property prediction may be less accurate. To visualize the search space, a response surface was generated by parametrically varying the Young's Modulus

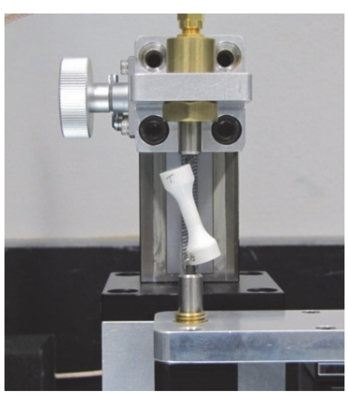

Figure 2: Alumina sample between transducers of the Magnaflux Quasar RUSpec system.

and Poisson's ratio and plotting the score of the mismatch between experimental and simulated resonance frequencies (Figure 3).

The response surface shows a single minimum that is more distinct in the Young's modulus than the Poisson's ratio. This suggests that the search algorithm should easily converge on an accurate Young's modulus but determination of the Poisson's ratio may depend on an accurate description of the measured model parameters (geometry and density).

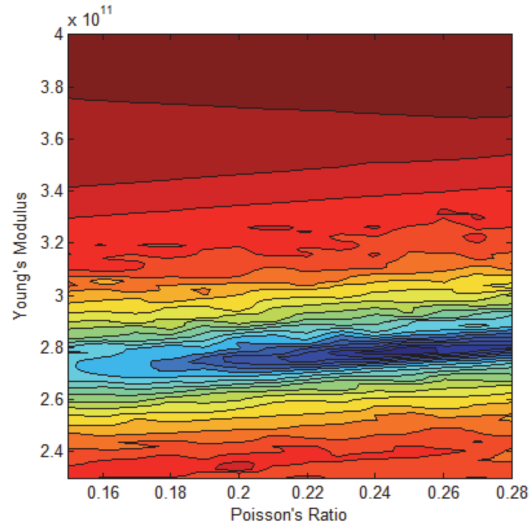

Figure 3: Response surface of the mismatch score between experimental and simulated resonance frequencies.

3.2 Sensitivity Study

A study of the sensitivity of the elastic property estimation to the model inputs was performed. This study, shown in (Figure 4) indicates in which direction, if any, the minima of the response surface (shown in Figure 3) shifts due to a change in the measured model properties such as the density, main diameter, gage diameter, and length of the dumbbell. For example, if there was an erroneous density measurement performed that overestimated the density, then the extracted Young's modulus would also erroneously increase, however the Poisson's ratio would remain the same. If the end cap diameter was measured high, then both the extracted Young's modulus and the Poisson's ratio would also read a false high value. The same positive trend is true for the length measurement. A high reading of the gage

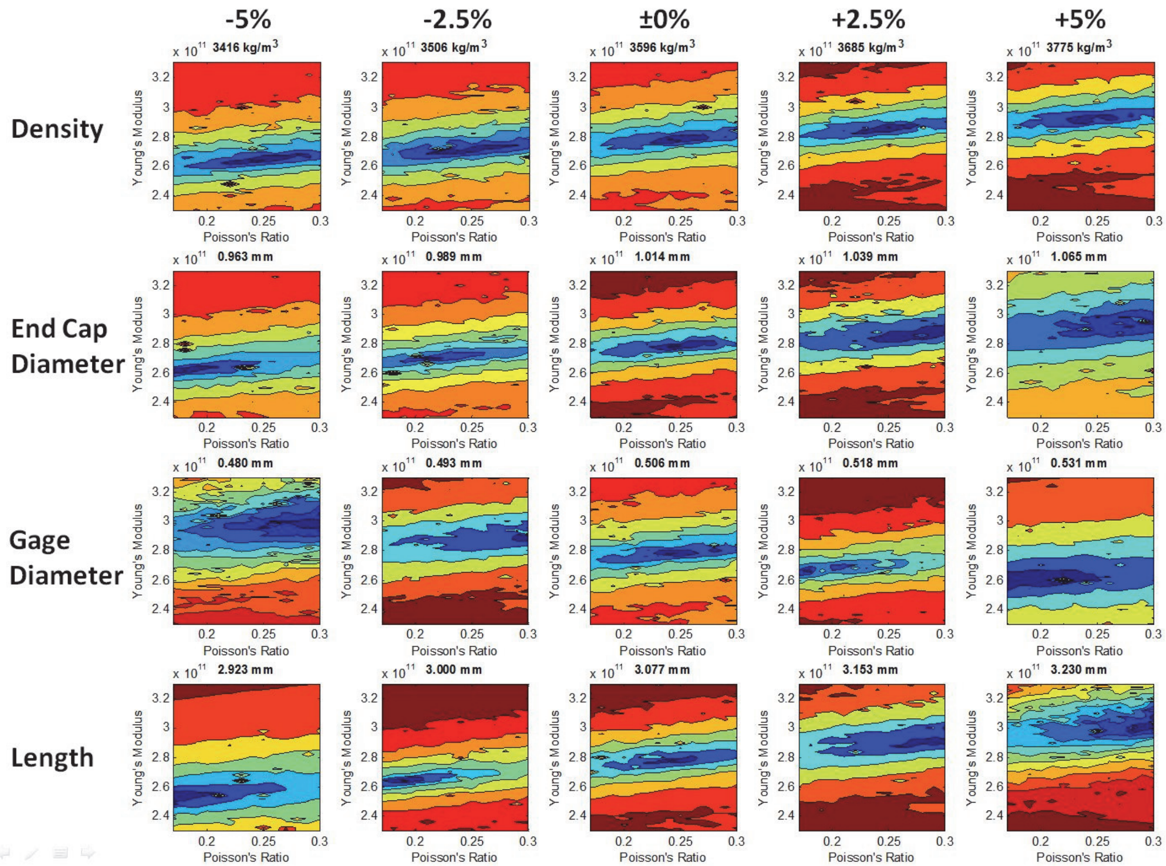

Figure 4: Sensitivity study of the change in expected Young's Modulus and Poisson's ratio given a change in the model parameters.

diameter, however, will lead to a higher Poisson's ratio but a lower Young's modulus value. In general, for this particular alumina sample a 2.5% error in a density or dimension measurement leads to a 2% to 5% Young's modulus error, while the same error can generate a 15%-20% error in the extracted Poisson's ratio.

3.3 Vibrating Mode Modeling

Shown in (Figure 5) is the measured resonance spectrum cropped between 1 kHz and 300 kHz for the alumina dumbbell overlaid with the simulated vibrating modes. Within this frequency range the COMSOL simulation allowed for the identification of 4 degenerate bending modes, which are indicated by large asterisks in the figure. At these locations there are two overlapping modes which may appear as one single peak. If any perturbation such as material inhomogeneity or machining defects disrupts the axisymmetry the peaks will shift in frequency and diverge. As such, the peaks in these locations provide information as to the quality of the test sample and could be monitored for quality control. The model simulations in (Figure 5) also provide insight into what part of the sample is experiencing the most stress. The purpose of the dumbbell sample for compression testing is to test strength of the material within the reduced diameter gage section. Therefore, it may be possible to interpret the modeled modes as to which frequencies are specifically resonating primarily in the gage section. Then particular attention can be directed towards those frequencies when comparing samples to each other.

3.4 Model Parameter/Frequency Correlation

A correlation map between the model parameters and the simulated resonance frequencies provides another way of visualizing the interrelationship between them. At each simulation point in the sensitivity study the simulated eigenfrequencies were recorded along

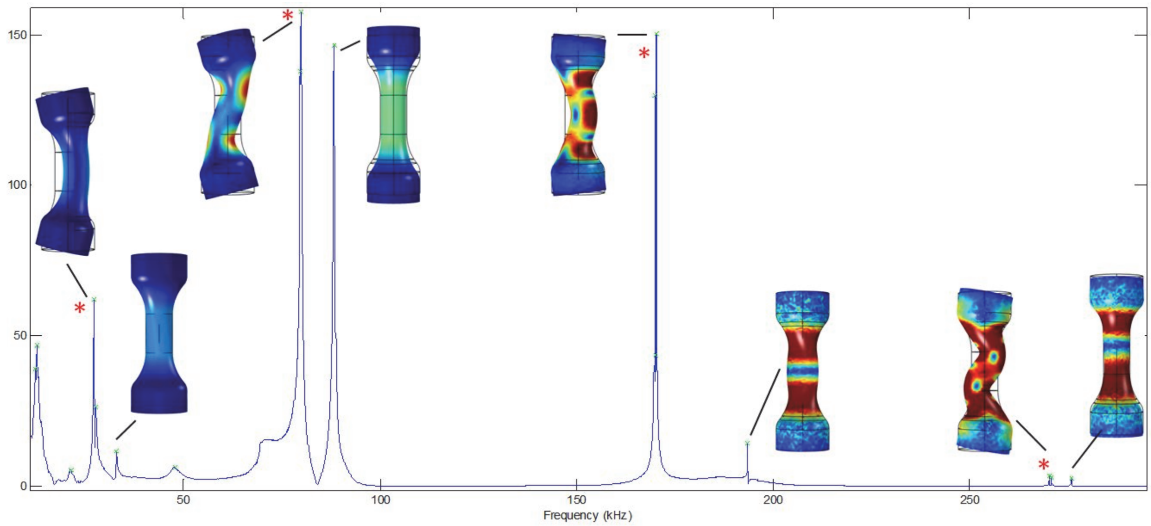

Figure 5: Measured resonance spectrum for an alumina dumbbell sample overlaid with the modeled vibrating modes. Large asterisks identify the degenerate bending frequencies that diverge if the axial symmetry is disrupted by inhomogeneities or machining flaws. Color indicates von Mises stress.

with the model parameters that generated them. Shown in (Figure 6) is a correlation map generated from the data used to create the sensitivity study for the model parameters and the first 9 eigenfrequencies. Clearly visible is a strong positive correlation between all the eigenfrequencies indicating that they tend to move in unison. The map also identifies the first three pairs of degenerate bending modes (Frequencies 1 & 2, 4 & 5, 7 & 8) as having a correlation of 1. Frequencies 1 through 6 appear to be grouped by a stronger correlation, as do frequencies 4 through 8. Of all the model parameters the Young's Modulus appears to have the strongest positively correlated influence on the resonant frequencies. The gage diameter has the second strongest positive correlation with the frequencies. The length has a slight negative correlation at the first degenerate bending mode of frequencies 1 and 2. The other model parameters such as the density, end cap diameter, and Poisson's Ratio appear to have little correlation with the resonance frequencies for the range of values attempted in the study.

3.5 Linear Model Regression

The observed strong correlation between the Young's Modulus and the resonance frequencies suggest that it may be possible to develop a linear model to explain the inverse relationship. A linear model of the inverse problem may be able to provide for a faster alternative to solving the inverse problem than using a direct search to find the best match between modeled eigenfrequencies and the measured resonances. To evaluate the possibility of using a linear model, the large dataset generated in the sensitivity study was regressed using a QR decomposition algorithm provided in MATLAB's statistics toolbox by considering quadratic interaction between the variables (eigenfrequencies and measured model parameters). A test set of 30 samples was generated by choosing model values from a uniform distribution within the ±5% range used to create the sensitivity study's dataset. A comparison between the predicted Young's Modulus and Poisson's Ratio given resonance frequencies is shown in (Figure 7). The results show a reasonable prediction of the Young's Modulus, with a less accurate prediction of the Poisson's Ratio as expected by the observed low correlation with the eigenfrequencies. It appears that a good inverse model has been established over the range of model parameters used in the sensitivity study. A different dataset would need to be generated for specimens having significantly different elastic properties or dimensions. This can be obtained through a design of experiment approach to indentifying testing locations.

4. Conclusions

COMSOL combined with LiveLink for MATLAB has been used to study the resonance spectrum of dumbbell-shaped ceramic compression samples. The Jonker-Volgenant algorithm was used to match measured resonance frequencies to modeled eigenfrequencies and provide a scoring metric. An alumina dumbbell sample was used to determine the shape of the response surface of the match score to a range of possible elastic properties. A smooth surface containing one distinct minimum was found suggesting that a direct search method could easily be used to solve the inverse problem. Frequencies containing degenerate bending modes were identified and can be used for quality control of the sample homogeneity and machining quality. A correlation map showing the relationship between model parameters and resonance frequencies suggested that a linear model may be able to explain the inverse relationship. A model was regressed having reasonable accuracy in the prediction of the elastic properties given a measured resonance spectrum. If time cannot be afforded to obtaining the highest possible accuracy with a direct search method, then a linear model presents a good and fast alternative. While this paper details a study of just one class of material having a dumbbell geometry, it is expected that this method could be readily applied to other material classes with varying degrees of elastic properties (polymers, metals, composites, etc.) as well as other complex geometries.

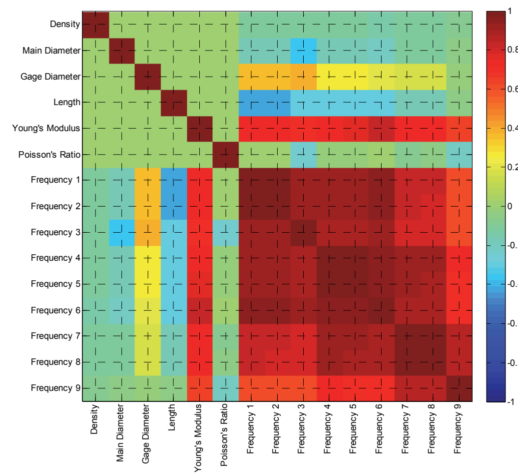

Figure 6: Correlation Map of model parameters obtained from the dataset generated for the sensitivity study.

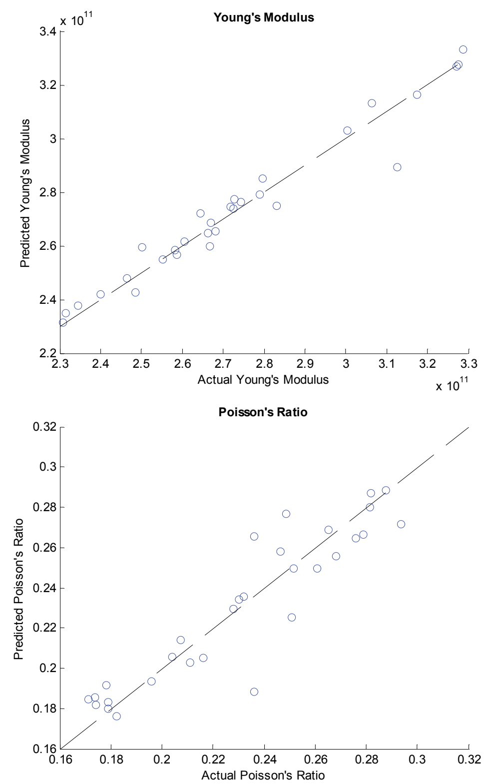

Figure 7: Young's Modulus (top) and Poisson's Ratio (bottom) predicted by a linear model verses the actual value. Dotted line indicates a perfect model prediction.

5. References

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