Nonstandard High-Voltage Electric Insulation Models

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Abstract: The design of modern electric insulation devices for medium and high voltage (HV) applications requires computational work that goes far beyond solving just a Laplace equation for the electric potential, and limiting the electric field below critical design values. Both field calculation and determination of breakdown limits are often large challenges for the development of HV insulation systems. In this contribution two prototype examples are discussed. First, it is shown how the field distribution is modeled and simulated if injected space charge dominates the electrical behavior, which occurs particularly in gas insulation with Corona discharge or in in HV direct current (HVDC) solid polymer insulation. Secondly, a recipe is provided for modeling and simulation of dielectric breakdown of gases via electric sparks in the form of streamers. The simulations of both examples require a certain modeling flexibility of the used simulation tool. It is shown that Comsol provides this flexibility.

Keywords: Electric field simulations, space charge injection, electric breakdown, streamer inception and propagation.

1. Introduction

The investigation of whether an electric insulation design for a HV device is acceptable for application consists of two steps. First, the electric field is calculated for required test conditions. Secondly, the field distribution has to be compared with dielectric withstand limits.[1] The standard procedure is thus to solve the relevant part of the Maxwell equations for the electric field[2] $\boldsymbol{E}(\boldsymbol{x})$ with given relative dielectric permittivity ε_r and conductivity σ, and to compare then the calculated E-field with design field values, e.g., the known breakdown field minus a safety margin.

However, there exist cases that are different in nature and thus require more sophisticated models in order to calculate the electric field and/or to determine dielectric withstand conditions. In this contribution we discuss two examples. The first one refers to a case where the material cannot be described simply by a conductivity as a bulk property, because the majority carriers origin from injection [1]. The second one provides a recipe for the simulation of streamer breakdown in gas insulation [2].

These examples are non-standard in the sense that they cannot be treated with simulation tools that are restricted to the above mentioned standard procedure. In fact, the associated models require tools with a high modeling flexibility. Fortunately, Comsol allows a large freedom in defining physical models.

The need for non-standard models is growing because previously irrelevant physical effects become more and more important, mainly due to the continuous increase of voltage levels and decrease of device sizes, as well as by new emerging technologies like novel insulation materials or the trend towards high voltage direct current (HVDC) systems.

2. Unipolar space charge injection

A prerequisite for modeling conduction by a bulk conductivity σ is a sufficiently large intrinsic free charge carrier density. However, this is invalid if the transit time $\tau_{tr} = L/\mu E$ of the charge carriers with mobility μ through the insulation material of size L is smaller than the dielectric relaxation time $\tau_M = \varepsilon_r\varepsilon_0/\sigma$ of the material. This occurs particularly for very low intrinsic carrier density. The electric behavior depends then strongly on the contact properties, and can be far from the ohmic (linear) current-voltage behavior due to strong E-field distortions. In the following, we consider as an illustrative example a material without intrinsic carriers and traps, and a single (unipolar) injected carrier species, which can lead to so-called space charge limited currents (SCLC) [1].

[1] If thermal runaway is a critical failure mode, one has to investigate the coupled electro-thermal problem; this will not be considered here.

[2] Vectors will be denoted throughout in bold letters.

2.1 The model

According to the absence of intrinsic carriers, only injected carriers can contribute to the particle current. The space charge density ρ equals the carrier density times the carrier charge.[3] The equations for electric transport are the Poisson equation for the electric potential ϕ and the continuity equation for ρ [3,4]

$$-\nabla\cdot(\varepsilon_r\varepsilon_0\nabla\phi)=\rho\,, \tag{1}$$

$$\partial_t\rho+\nabla\cdot\boldsymbol{j}=0\ . \tag{2}$$

Here, ∂_t, and ∇ denote time derivative and Nabla symbol, respectively. The current density $\boldsymbol{j}$ consists of drift and diffusion currents[4]

$$j=|\rho|\mu(E)\boldsymbol{E}-D\nabla\rho \quad , \tag{3}$$

where the mobility μ can depend on the modulus of the electric field, $\boldsymbol{E}$ = -$\nabla\phi$, and D is the diffusion constant, which may be related to μ via the Einstein relation in case of local equilibrium. The absolute value $|\rho|$ in (3) is considered because μ is defined positive here, but in fact it contains the carrier charge as a factor.

Equations (1)-(3) must be supplemented by boundary conditions for ϕ and ρ, which for ϕ are as usual of Dirichlet type at conductor surfaces and of Neumann type at insulating boundaries. The crucial part of the model describes the injecting contact with a relation between ρ and the normal component j_n of $\boldsymbol{j}$. This boundary condition affects the charging state in the bulk material. For instance, it has been shown that a charge injection instability of the insulating state can occur in inhomogeneous fields under certain conditions [5,6]. In the following, we will use the physically motivated boundary condition [3,4,7]:

$$j_n = j_{sat} - w\rho \tag{4}$$

at the (positive) electrode where charge injection might occur. The normal component of the net current density is equal to the difference of the injected saturation current density, j_{sat}, and a current density $w\rho$ associated with carriers moving backwards to the injecting electrode. This backscattering current is proportional to the local density ρ at the contact and a characteristic velocity w. According to Eq. (4), a finite amount of space charge appears at the contact even at equilibrium (j_n=0). Despite of not being qualitatively wrong, the resulting small diffusion voltage U_b is negligible compared to the externally applied high voltages U. Furthermore, although $j_{sat}(E,T)$ and $w(E,T)$ are related to contact physics, which may be determined from fundamental contact physics [3,7], it is sufficient to consider them as phenomenological E and T dependent model parameters.

Electrodes that act as counter-electrodes are described by carrier-absorbing boundaries, i.e., $\rho=0$. Insulating boundaries without large formation of surface charges are modelled with vanishing normal current density, $j_n=0$, as boundary condition. The occurrence of large amounts of surface charges on insulators can also be modelled [4], which goes, however, beyond the scope of this article.

2.1 Implementation and Simulation Results

The implementation of Eqs. (1)-(3) together with the boundary conditions as discussed is straight-forward in Comsol, for instance by using the Poisson equation (“Electrostatics”) coupled with a drift diffusion equation (“Transport of diluted species”). The important injecting boundary condition (4) can be formulated in the predefined “Flux” boundary condition. It has been shown in Refs. [3,4] that the simulation results are in accordance with analytical and expected results for the transient space charge limited current as well as for the steady-state current-voltage characteristics. For instance, the steady state solution for large injection current ($j_n \ll j_{sat}$), large voltage ($U \gg U_b$), and small diffusion current, in a plate geometry is associated with the space charge limited current (SCLC) characteristics $j=9\mu\,\varepsilon_r\varepsilon_0U^2/8L^3$, where L is the sample thickness. Figure 1 compares two simulated and analytical [3] I-U characteristics for a plane plate-plate arrangement with L=0.1 m, ε_r = 1, j_{sat},= $j_{sat,0}$ $\Theta(E\text{-}E_{inc})$, where Θ is the

[3]Without restriction of generality, we consider positive carrier polarity in order to prevent unnecessary sign manipulations in the calculations.

[4]In the presence of a flow velocity v of fluid insulation media, a convective current vρ can be added to (3).

Heaviside step function, E_{inc} is the inception voltage, and $j_{sat,0}$ is a constant saturation current density. Hence, one can model with this approach insulators with charge injection occurring at a finite inception field. This is, for instance, characteristic for Corona discharge at metal electrodes in gases. In contrast to SCLC, the *I-U* starts then at a finite voltage level, associated with the inception voltage, and shows a crossover to the SCLC characteristics, followed by a further crossover to the contact limited current where current saturation $j = j_{sat,0}$ occurs at large voltages (Fig. 1).

For general electrode geometries, charge injection occurs at high field regions. This may lead to travelling charge clouds that deposit charge on insulator surfaces, and eventually to field distributions which can strongly differ from the Laplace fields [4]. It is needless to say that in this case a field calculation based on a standard σ–ε model yields wrong results, which cannot be used for designing HV insulation devices.

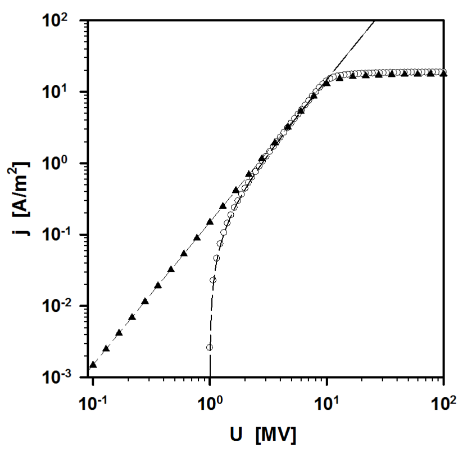

Figure 1. Current density vs voltage (parameter values given in the text). Simulation: triangles (SCLC) and circles (E_{inc}=10kV/mm). Underlying curves: analytical results for the limit $j_{sat,0} \to \infty$ (see [4]).

An illustrative example for a non-plane geometry is the tip of a rounded rod electrode in front of a plane counter electrode. This type of rod electrodes are used, for example, as plug contacts in HV gas circuit breakers. The high field at the plug head may lead to Corona discharges with the consequence of space charge injection into the gas. The space charge motion in the simple test geometry, obtained from a Comsol simulation is show in Fig. 2 at different times $0.01\tau_{tr}$, $0.1\tau_{tr}$, $0.5\tau_{tr}$, and $1.2\tau_{tr}$,with transit time $\tau_{tr}=L^2/\mu U_0$=5ms, where L=0.1m is the distance from plug tip to the counter electrode, μ =10^{-5} m^2/Vs is the carrier mobility, and U_0 = 200kV is the amplitude of the voltage impulse with rise time of $0.07\tau_{tr}$ and subsequent decay time of ca. $0.7\tau_{tr}$. At $0.5\tau_{tr}$ the voltage dropped down to almost half of the peak value, and the residual space charge is significant only near the plug edge where the field is highest. After $1.2\tau_{tr}$ almost all of the injected charge has escaped through the plate contact. In Figure 3, which shows the simulated field together with the Laplace field at the front of the plug, one can clearly see the effect of the space charge on the electric field: after its appearance, the plug is partially shielded leading to a reduced field, which intermediately saturates at the E_{inc}-value. It would stay there if the voltage would not decay further. Only when the capacitive field drops below E_{inc}, the space charge will vanish.

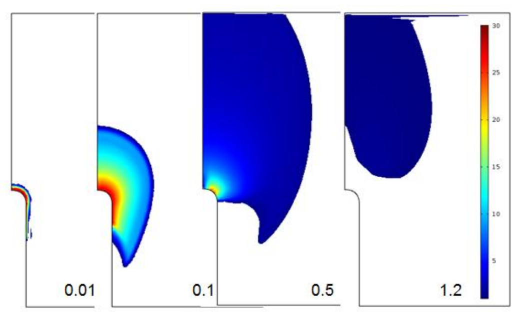

Figure 2. Simulation of the space-charge density-cloud (surface plot, in units of $\varepsilon_0 U_0/L^2$) injected from an electrode plug during impulse voltage at different times (indicated numbers in units of τ_{tr}).

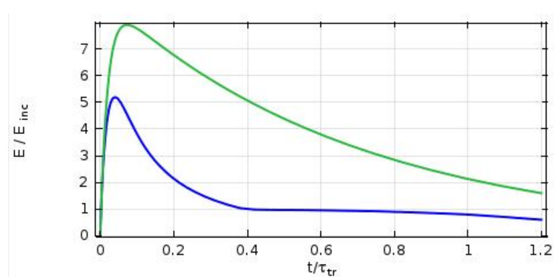

Figure 3. Electric field vs time in front of the rod. Blue/green: with/without charge injection. The field is reduced by the shielding space charge and is limited to E_{inc} after a certain time.

3. Streamer Electric Breakdown

As mentioned in the introduction, knowledge on the electric field distribution alone is not sufficient for a judgment on acceptance of an insulation design, but additional information on electric withstand is needed. There are different kinds of electric breakdown mechanisms in insulation. In the following, we focus on streamer breakdown. Streamer inception (SI) and propagation (SP) are the first two steps of a spark discharge. A streamer is a non-equilibrium cold plasma channel, which is able to start at electric fields around 2.6 kV/mm in normal air and to propagate if the field succeeds about 0.5 kV/mm. Breakdown can occur if a streamer propagates to the counter electrode, which leads to a connection between the stressed electrodes by a plasma channel. Streamer breakdown is a typical failure mechanism of insulation gas in nonuniform electric fields for electrode distances smaller than about 1 meter [2,8-10]. It is therefore a particularly critical failure mode in medium voltage devices. For larger distances, streamers will transform to so-called leaders by a heating-up of the cold plasma channel. A leader simulation is very complex, because the leader propagation must include a self-consistent determination of the electric potential. However, streamers can approximately be assumed to move in the fixed background potential. We show now that streamer breakdown can be modeled in a simple phenomenological manner with a finite element tool like Comsol.

3.1 The model

We assume that the solution of the Laplace equation for the electric potential $U(\boldsymbol{x})$ in the spatial region of interest is given for appropriate boundary conditions. Let the potential be positive at the electrode where SI is expected, and ground on the counter electrode (positive streamers are more critical than negative streamers [2]). Hence, the electric field lines of $\boldsymbol{E}(\boldsymbol{x}) = -\nabla U$, point away from the critical electrode. The SI criterion is associated with a critical electron avalanche size and is formulated as an integral condition to the effective ionization coefficient $\alpha(E)$ along a field-line path γ where α is positive and which ends at the critical electrode [2],

$$\int_{\gamma} ds\, \alpha(E) \geq C_{crit} \tag{5}$$

with field strength $E(\boldsymbol{x}) = |\boldsymbol{E}|$. α and C_{crit} are assumed to be given. For a field distribution $\boldsymbol{E}(\boldsymbol{x})$ in an arbitrary geometry, it is not a priori obvious which are the critical field lines satisfying Eq. (5); they are not necessarily related to electrode locations with maximum field [9]. The required search for critical field lines, and the extraction of information thereof for realistic geometries, as it is needed for (5), is usually not a feature provided by typical commercial simulation tools. But we will show that there is a simple way to determine the critical SI region.

For this purpose, we introduce a new scalar field variable $\phi(\boldsymbol{x})$, which satisfies the 1st order partial differential equation (PDE)

$$-\mathbf{v} \cdot \nabla\phi = \alpha(E)\Theta(\alpha) \tag{6}$$

where

$$\mathbf{v}(\boldsymbol{x}) = \boldsymbol{E}/E \tag{7}$$

is the normalized electric vector field, which points along the field lines. Equation (6) states that the derivative of ϕ along the backward direction of the field lines (i.e., towards the critical electrode) equals α. Hence the solution of Eq. (6) is the integral of α along field lines and therefore equal to the streamer integral (5), provided $\phi = 0$ in regions where $\alpha \leq 0$. The latter condition is ensured by using a homogeneous Dirichlet boundary condition, $\phi = 0$, at the counter electrode(s), where the flow lines of $\mathbf{v}$ end. The Θ-function in Eq. (6) ensures integration only for $\alpha \geq 0$. The SI region where streamers will emerge is given by $\{x| \phi(\boldsymbol{x}) \geq C_{crit}\}$. Because the inception region is a volume region, the present procedure allows also the determination of electrodeless SI at insulator surfaces [8,9].

A streamer propagation (SP) model has to predict where and how far the emerging streamer(s) will go. A simple SP model makes use of the observation that a streamer length increase requires a certain voltage drop $U_s(s)$ along the streamer path, which is in the relevant regime empirically well-described by [2,8,9]

$$U_s(s) = U_{s,0} + E_s s \tag{8}$$

where s is the path length, $U_{s,0}$ can be interpreted as the streamer head voltage, and E_s is an approximately constant field value, which can be associated with the SP field or the so-called "streamer stability" field. Under the (maybe somewhat oversimplifying [9]) assumption that streamers follow field lines, the streamer path can be found by solving the ordinary differential equation (ODE) for the location $\boldsymbol{x}(t)$ of the streamer head

$$d\mathbf{x}/dt = h(\Delta U,t)\,\mathbf{v}(\boldsymbol{x}) \tag{9}$$

with initial condition $\mathbf{x}(0)$ in the SI region, and where $\Delta U(\boldsymbol{x}) = U_0 - U(\boldsymbol{x})$ is the (external) voltage drop along the streamer with head location at $\boldsymbol{x}$. The prefactor h is either 1 or 0, depending on whether the SP criteria is satisfied or not, and it ensures that the streamer stops if the local potential drop is insufficient for further propagation, i.e., if $\Delta U < U_s(s)$ outside the inception region. Note that the streamer path is parameterized here by an artificial time t which is equal to the streamer path length s because $|\mathbf{v}| = 1$ ($\mathbf{v}$ is not the true streamer velocity but only its normalized direction vector; the true speed, which is typically of the order of some mm/ns is not needed for determining the streamer length for most practical cases). In the following we will consider, for simplicity, $U_{s,0} = 0$, such that $h = \Theta(\Delta U(\boldsymbol{x}) - E_s t)$. A discussion of more general cases is straightforward.

3.1 Implementation and Simulation Results

In order to simulate the 1st PDE (6) in a simple way with Comsol, one is tempted to write it in the form of the 2nd order PDE $d\Delta\phi - \mathbf{v}\cdot\nabla\phi = \alpha\Theta$, and take then a small d value. However, the limit $d \to 0$ is not equivalent to $d=0$, because of the mathematical structural difference between PDEs of different orders. But for practical purposes the solution of Eq. (6) can be approximated with sufficient accuracy, if d is small enough *and* provided the boundary conditions for ϕ are appropriately chosen. In particular, the disturbance of the solution by the boundary condition at the critical contact where SI occurs should be negligibly small. According to (6), once should chose at the critical electrode the boundary condition $\boldsymbol{n}\cdot\nabla\phi = -\alpha\,\Theta$, where $\boldsymbol{n}$ denotes the surface normal vector. Furthermore, at counter electrodes where the field lines end, one should have $\phi = 0$, and homogeneous Neumann boundary conditions at electrically insulating boundaries.

The implementation of the model in Comsol is then straightforward. For the equation for ϕ one can use, for instance, "heat transfer in fluids". Streamer propagation, which is modeled with ODEs as given by (9), can be simulated with particle tracing; it is even sufficient to use the application "particle trajectories" for massless particles in the post-processing mode. On has to define starting points $\boldsymbol{x}(t=0) = \boldsymbol{x}_0$ that are active only if the SI criterion $\phi(\boldsymbol{x}) \geq C_{crit}$ is satisfied, which can be realized by defining h appropriately, e.g., such that it vanishes at the starting point $\boldsymbol{x}_0$ of the trajectory if $\phi(\boldsymbol{x}) < C_{crit}$.

As an illustrative example, we consider a similar tip-plate geometry as in the previous section, for air under normal conditions. The effective ionization coefficient is given by $\alpha = p[k(E/p-\Lambda)^2 - A]$ with k = 1.6 mm bar/kV^2, Λ = 2.2 kV/(mm bar), A = 0.3 1/(mm bar), p = 1 bar, tip-plate distance 19 cm, tip radius 1 cm, and E_s = 0.5 kV/mm [10]. Results for different voltage values U_0 are shown in Fig. 4. The SI voltage, i.e., when the first streamer appears, is ca. 67kV. The SI region is made visible by the dark area in front of the tip. Obviously, the streamers start with a finite initial length, which increases further with increasing voltage (Fig. 5). The withstand voltage, i.e, when the first streamer crosses the gap, is U_w = 95 kV.

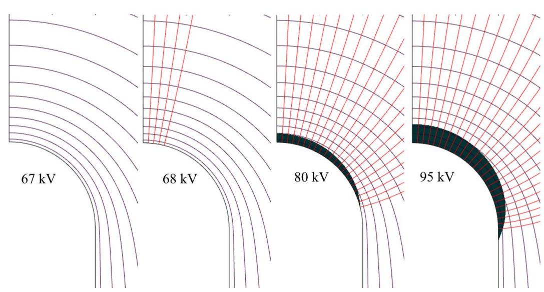

Figure 4. Inception region (black) at the plug head for different voltages (SI at 67-68 kV). Equipotential lines (black), streamer lines (red, cf. Fig. 5) start in the inception region.

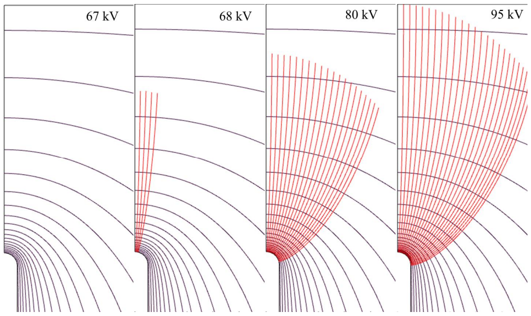

Figure 5. Streamer lines (red) for different voltage values (cf. Fig. 4). At 95 kV the longest streamers bridges the electrode gap.

4. Summary and Conclusions

The development of HV electric insulation devices requires information on the electric field and on the electric withstand, which is usually obtained from simulations and known breakdown field values. The standard procedure is to solve the Maxwell equations for the E-field with known electric permittivities and conductivities, and to compare the calculated field values with empirical design field values. However, the continuous push of technology limits (e.g. increasing field strengths) and the emergence of new technologies (e.g., HVDC) requires more accurate models, which are of course also more specific and sophisticated. As consequence, simulation tools require an appropriate flexibility in building-up models. Two important examples have been discussed, namely insulators governed by space charge injection, and streamer breakdown in gases. We have shown how these non-standard cases can be modeled and simulated with Comsol.

As an outlook we emphasize the general aspects for the two examples. First, the central point of the charge injection model refers to the relevance of the boundary condition, which not only affects the quantitative but also the qualitative behavior of the system. We believe, that in future simulations, the physics on lower-dimensional sub-manifolds like boundaries, interfaces, surfaces, contacts, triple points etc. becomes increasingly important in the context of simulations of HV insulation devices. What concerns the streamer breakdown simulation, we secondly believe that future simulation tools have to contain failure models, like dielectric breakdown in electric systems. The most simple example that exists already in every nonlinear electro-thermal simulation tool is thermal runaway. The steamer breakdown model presented above is another example, which is not generically provided by FEM tools. Even more effort will be required to implement leader physics. Moreover, the two issues – interface physics and breakdown – are often related: for instance, breakdown starts and/or propagates in many cases at interfaces. As an example we mention electrodeless streamer inception and streamer propagation on dielectric surfaces [9]. The development of models for these cases defines the next steps of our research.

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