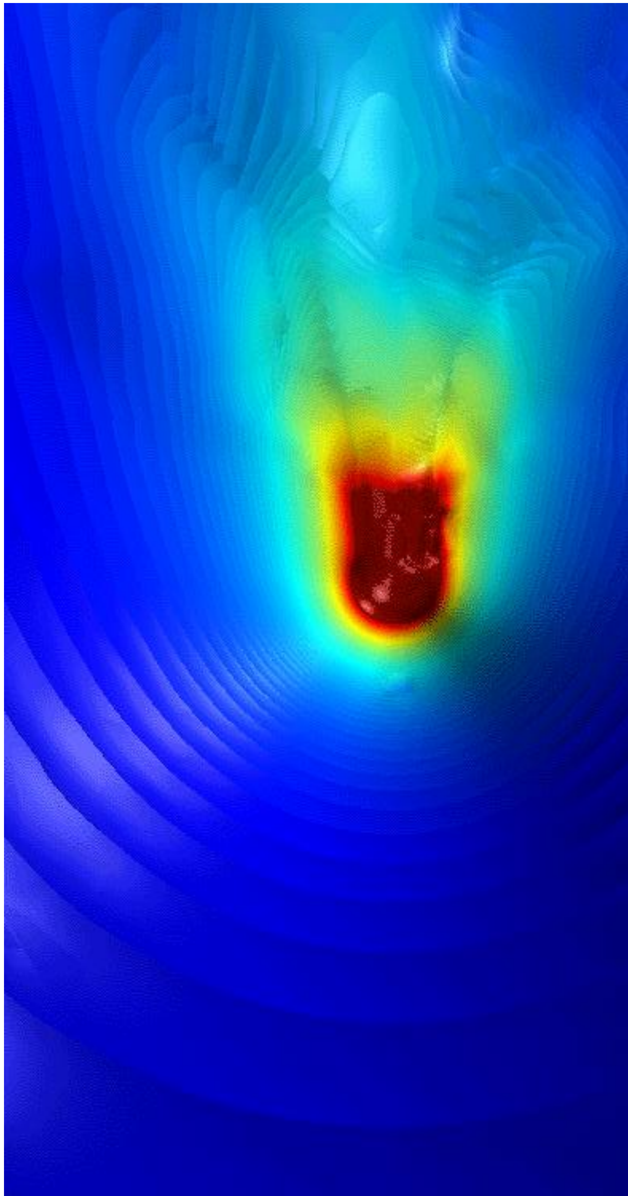
2012 COMSOL Conference in Boston



Optimization of Artificial Diffusion Stabilization Techniques and Corresponding Mesh Density Distribution in Drift Dominated Transport of Diluted Species

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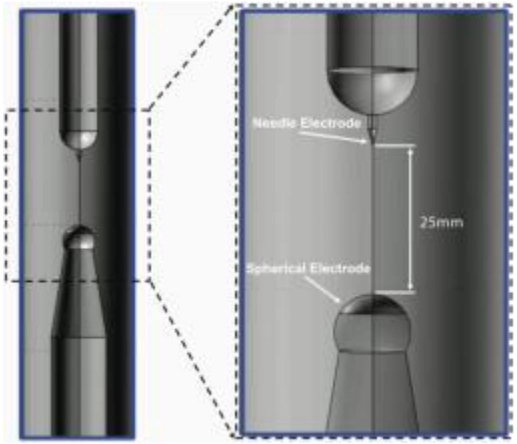
Charge Transport Modeling of Streamers
Governing equations and simulation geometry

$$
\left\{
\begin{aligned}
&\nabla\cdot(\varepsilon\vec{E}) = \rho_p+\rho_n+\rho_e \\
&\frac{\partial\rho_p}{\partial t}+\nabla\cdot(\rho_p\mu_p\vec{E}) = G_M(|\vec{E}|)+\frac{\rho_p\rho_e R_{pe}}{q}+\frac{\rho_p\rho_n R_{pn}}{q} \\
&\frac{\partial\rho_n}{\partial t}-\nabla\cdot(\rho_n\mu_n\vec{E}) = \frac{\rho_e}{\tau_a}-\frac{\rho_p\rho_n R_{pn}}{q} \\
&\frac{\partial\rho_e}{\partial t}-\nabla\cdot(\rho_e\mu_e\vec{E}) = -G_M(|\vec{E}|)-\frac{\rho_p\rho_e R_{pe}}{q}-\frac{\rho_e}{\tau_a} \\
&\frac{\partial T}{\partial t}+v\cdot\nabla T = \frac{1}{\rho_l c_v}(k_T\nabla^2 T+\vec{E}\cdot\vec{J})
\end{aligned}
\right.
$$

Gauss' law

Conservation of charge equations

Heat transfer equation

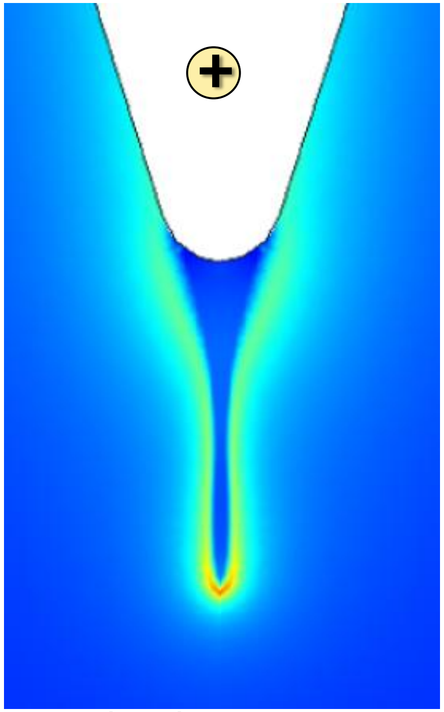


Cylindrical needle-sphere electrode geometry field with transformer oil.

Previous Streamer Model: Containing Non-physical Results

Previous 2D- axisymmetric model is unable to correctly model **negative streamers** and **positive streamers** for applied voltages **higher** than 130 kV (both structure and velocity).

V =130 kV



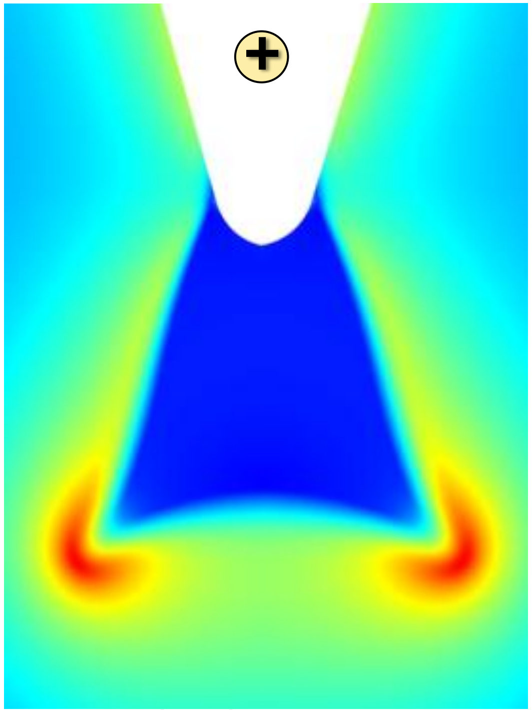
rise times = 1 ns

V(t = 2000 ns) = 200 kV

Constant Δ (IP) Heaviside

No propagation

V=200 kV



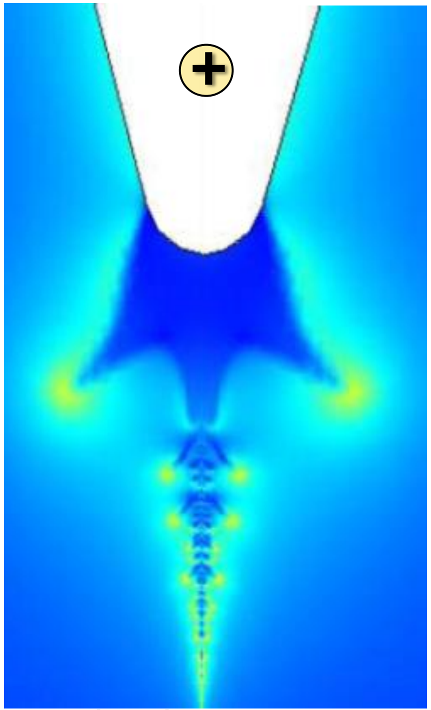
rise times =10ns

V(t = 20 ns) = 190 kV

Constant Δ (IP) Model

Heaviside Function

V=300 kV



rise times =100 ns

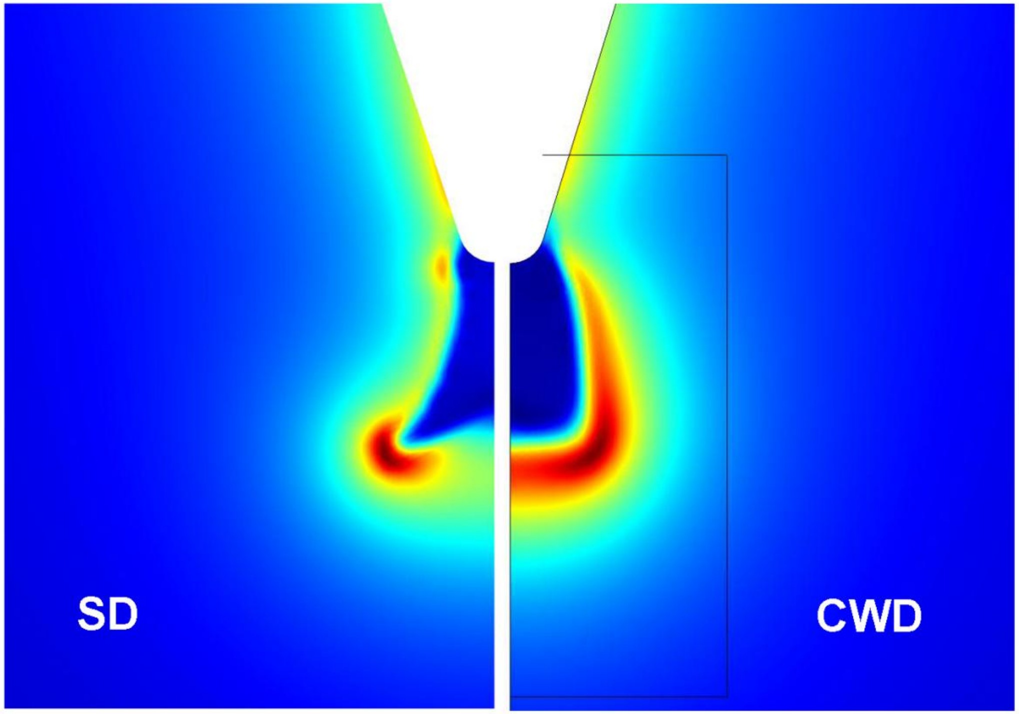
V(t = 100 ns) = 180 kV

Constant Δ (IP) Model

$n_0=1\times10^{26}$, $\Delta=1.58\times10^{-18}$

Artificial Diffusion Stabilization Techniques

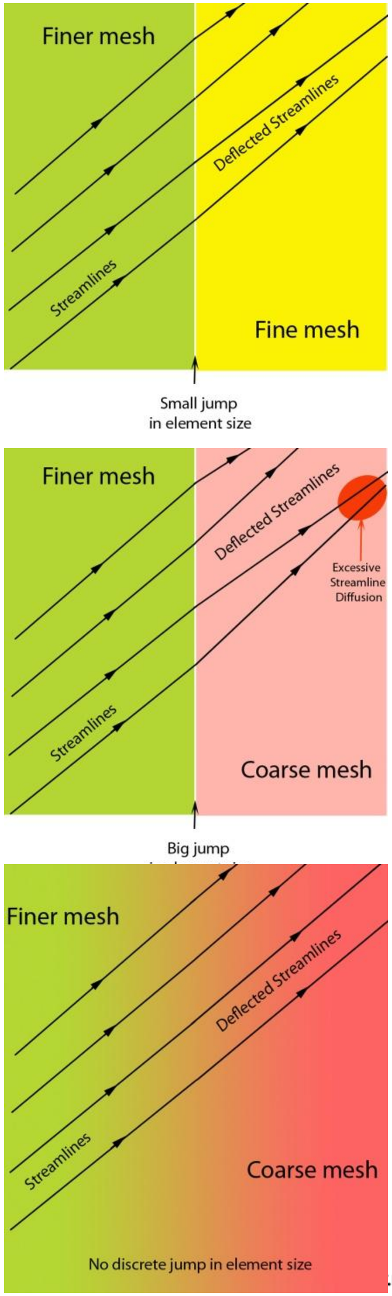
- Crosswind diffusion (CWD) effectively damps oscillations in particle number density and prevents them from becoming negative which is nonphysical. It also increases the streamer diameter and decreases streamer velocity and maximum electric field ahead of the streamer.
- CWD adds some artificial diffusion terms orthogonal to the flow of species to stabilize the numerical solution.
- This difference is more evident for simulations with extremely dense meshes.
- COMSOL 4.2 only employs one type of Streamline Diffusion (SD), Galerkin-Least-Squares (GLS) without any tuning parameters.
- The crosswind diffusion (CWD) method specifies the smallest allowable concentration change across an element. As the concentration gradient appears in the denominator in the equations describing crosswind diffusion, the gradient ensures that unreasonable values do not occur in regions with small to negligible concentration changes.
- It is possible to obtain similar results to 3.5a (using anisotropic streamline diffusion) if the isotropic artificial diffusion is selected in COMSOL 4.2 with tuning parameter ~10.



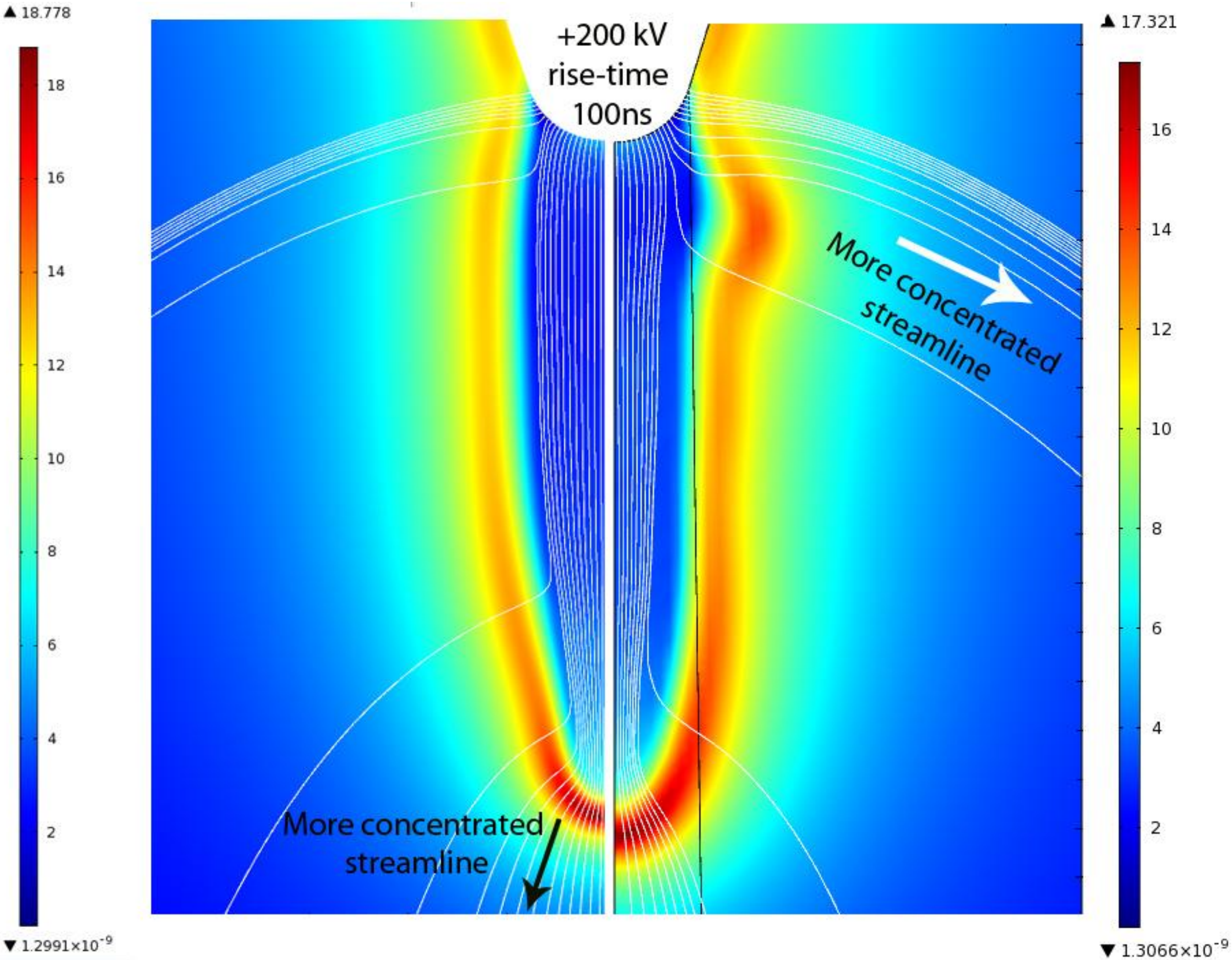
Type	Computational time	Convergence Accuracy of coarse mesh result	Negative number for particle density	Off-axis branch (for fast rising voltages)	Which versions has this type
Isotropic	No convergence	N/A	No	N/A	3.5a - 4.2
SD anisotropic	Fair	Fair	Yes	Yes	3.5a
SDPG	Fair	Good	Yes	Yes	3.5a
SDPGC	Fair	Good	Yes	Yes	3.5a
SUPG	Fair	Fair	Yes	Yes	4.2
CWD	Long	Good	No	Yes	3.5a - 4.2

Different mesh refinement policy around needle electrode

- To overcome these off-axis instabilities we tested several dense mesh distributions in the needle-sphere geometry. It already turned out that only refining the mesh around the needle tip cannot solve the problem as for example shown in Fig. 1 for a box with an excessively dense mesh (maximum mesh element of 0.5 µm).
- We obtained better results with smoother meshes which encouraged us to conduct a series of numerical experiments on jumps in mesh element sizes. Our numerical experiments show that a big jump in element size distribution over space may cause a sort of positive feedback effect and form nonphysical branching especially when the electric field is extremely divergent (which is the case in applied voltages with higher peaks and smaller rise-times). Such big jumps create small numerical perturbations that grow due to a streamline diffusion accumulative effect in our model.



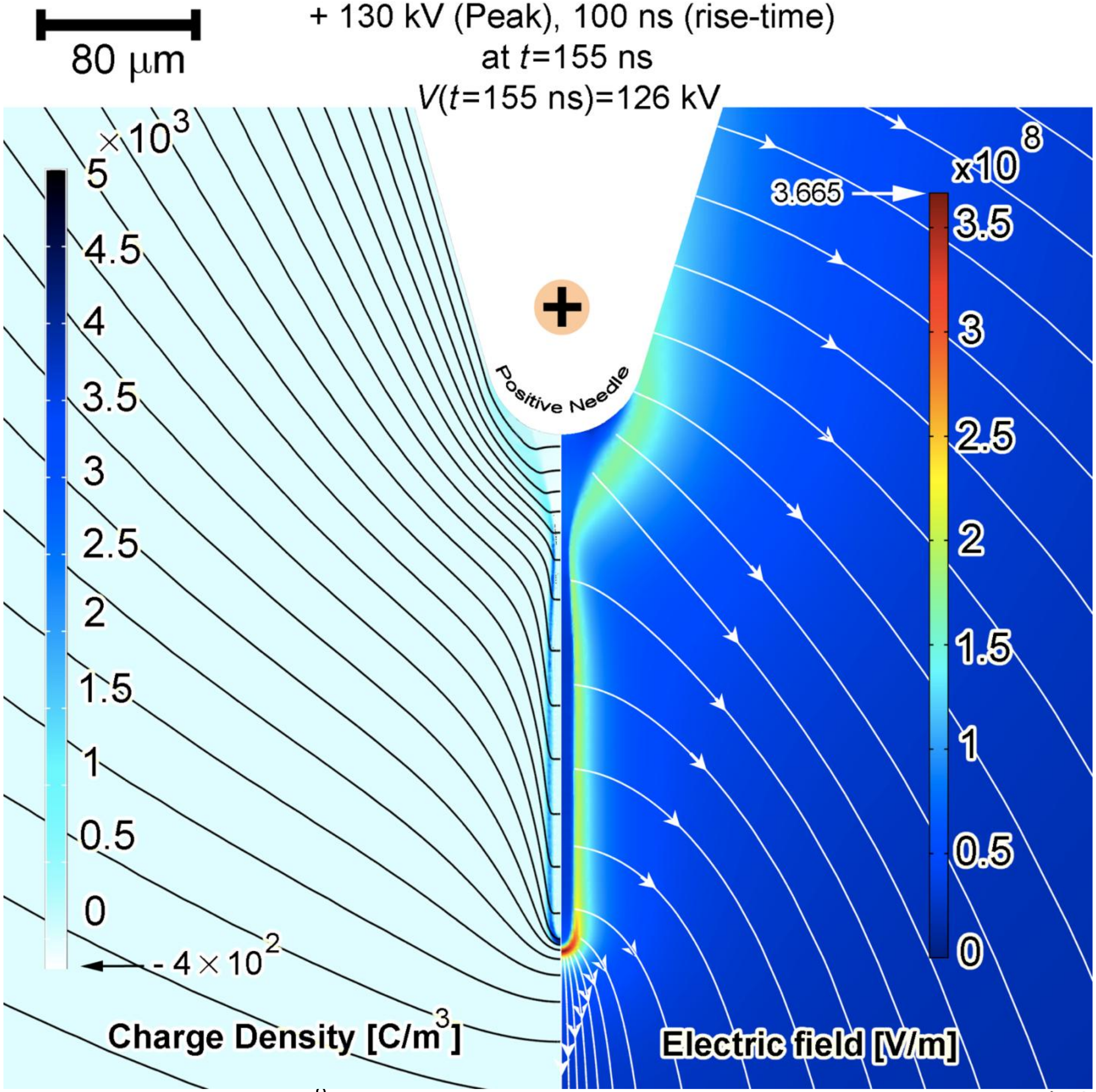
Different mesh refinement policy around needle electrode



Non-dimensionalized base electric field (E_b=1.2e7) for two different mesh element size distributions for a positive applied voltage with 200 kV peak and 100 ns rise-time at time 85 ns. The two simulations are separately computed with the left side plot having a smooth fine mesh (derived by COMSOL adapted mesh) while the right side plot has a fine mesh within 40 µm (maximum element size of 2 µm) and for the outer area beyond this box it has been freely meshed (course). Streamline and crosswind diffusions are applied in both sides.

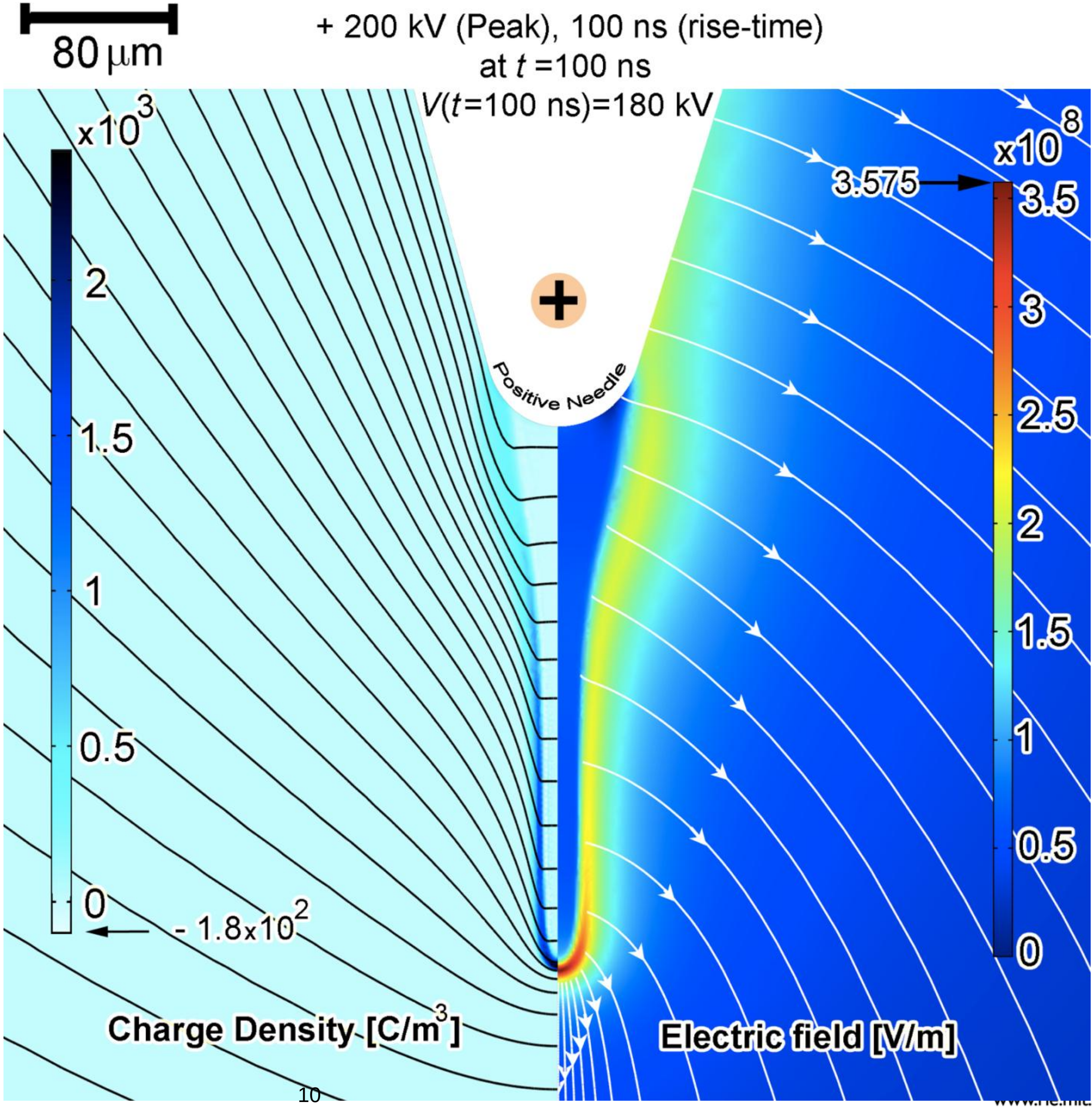
2D results: effect of polarity, magnitude and rise-time:

- Electric field magnitude and field lines (right side)
- Net charge density and equipotential lines (left side) for a positively applied lightning impulse voltage with 130 kV peak and 200 ns rise-time at *t*=155 ns. No streamer is observed for a 130 kV negatively applied impulse voltage.



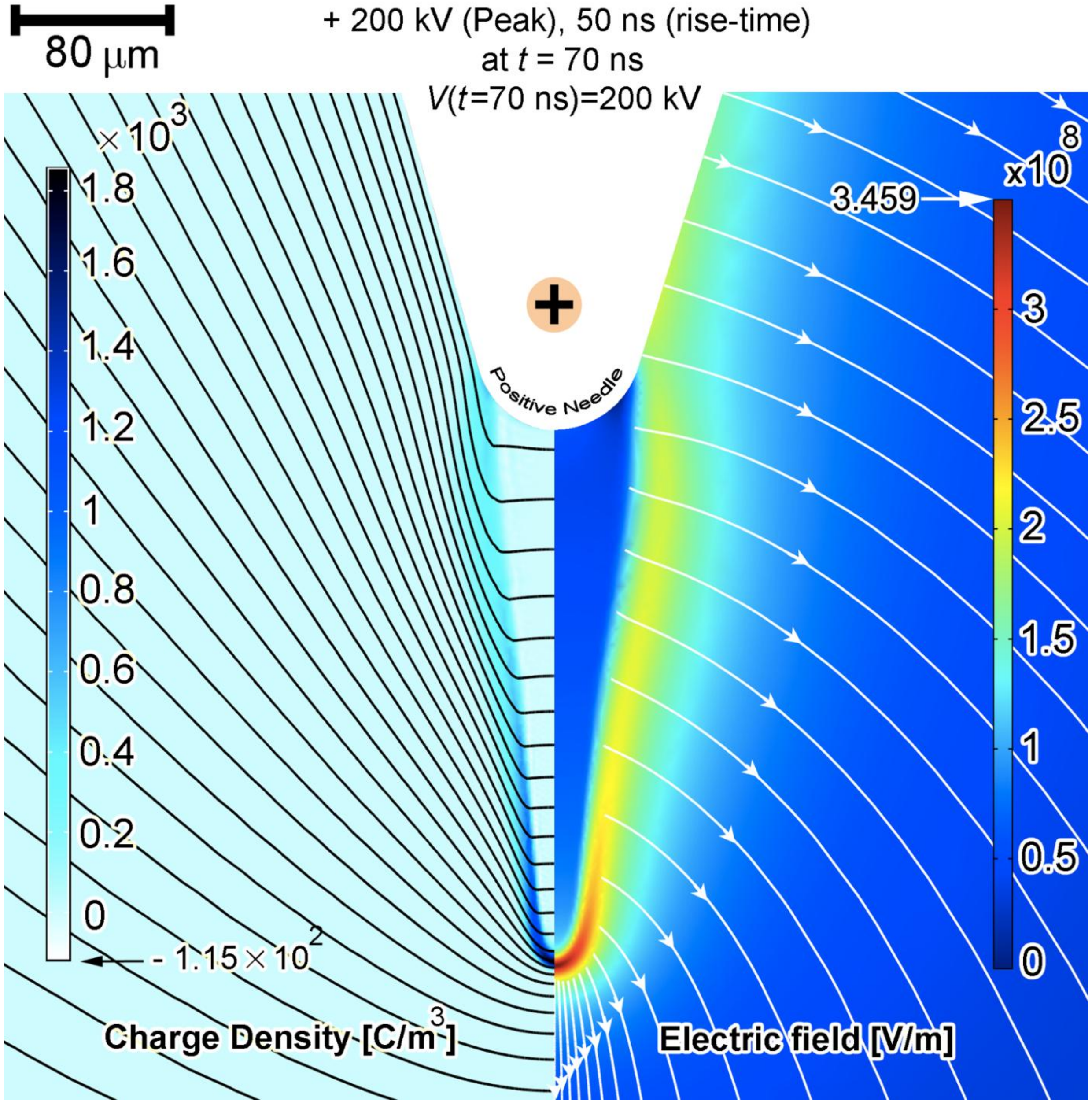
2D results: effect of polarity, magnitude and rise-time:

- Electric field magnitude and field lines (right side)
- Net charge density and equipotential lines (left side) for a positively applied lightning impulse voltage with 200 kV peak and 100 ns rise-time at *t*=100 ns.
- As can be seen in the following slides the diameter of the streamer and head curvature of the streamer head increases by decreasing the applied voltage rise-time appreciably.



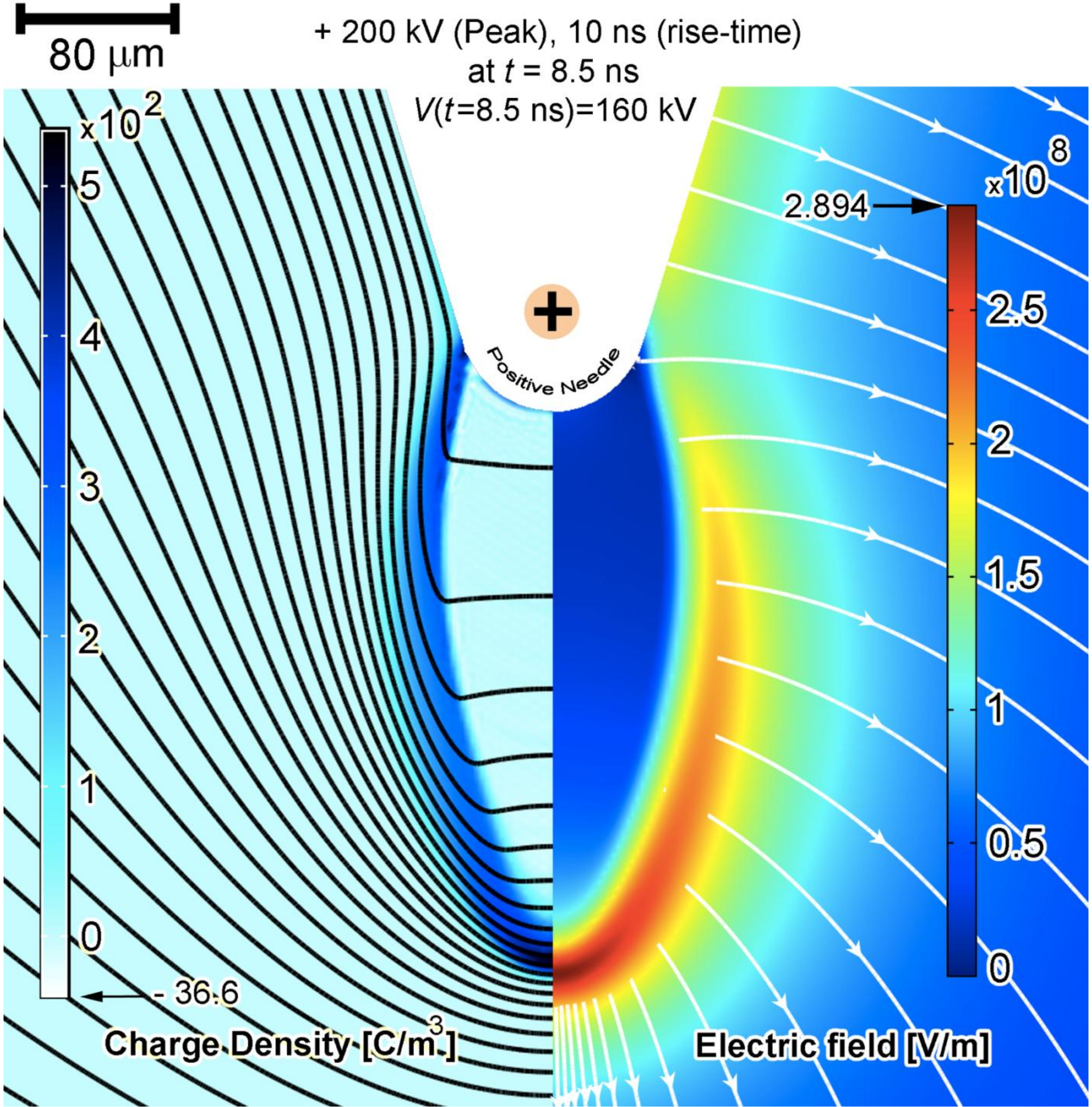
2D results: effect of polarity, magnitude and rise-time:

- Electric field magnitude and field lines (right side)
- Net charge density and equipotential lines (left side) for a positively applied lightning impulse voltage with +200 kV peak and 50 ns rise-time at *t*=70 ns.



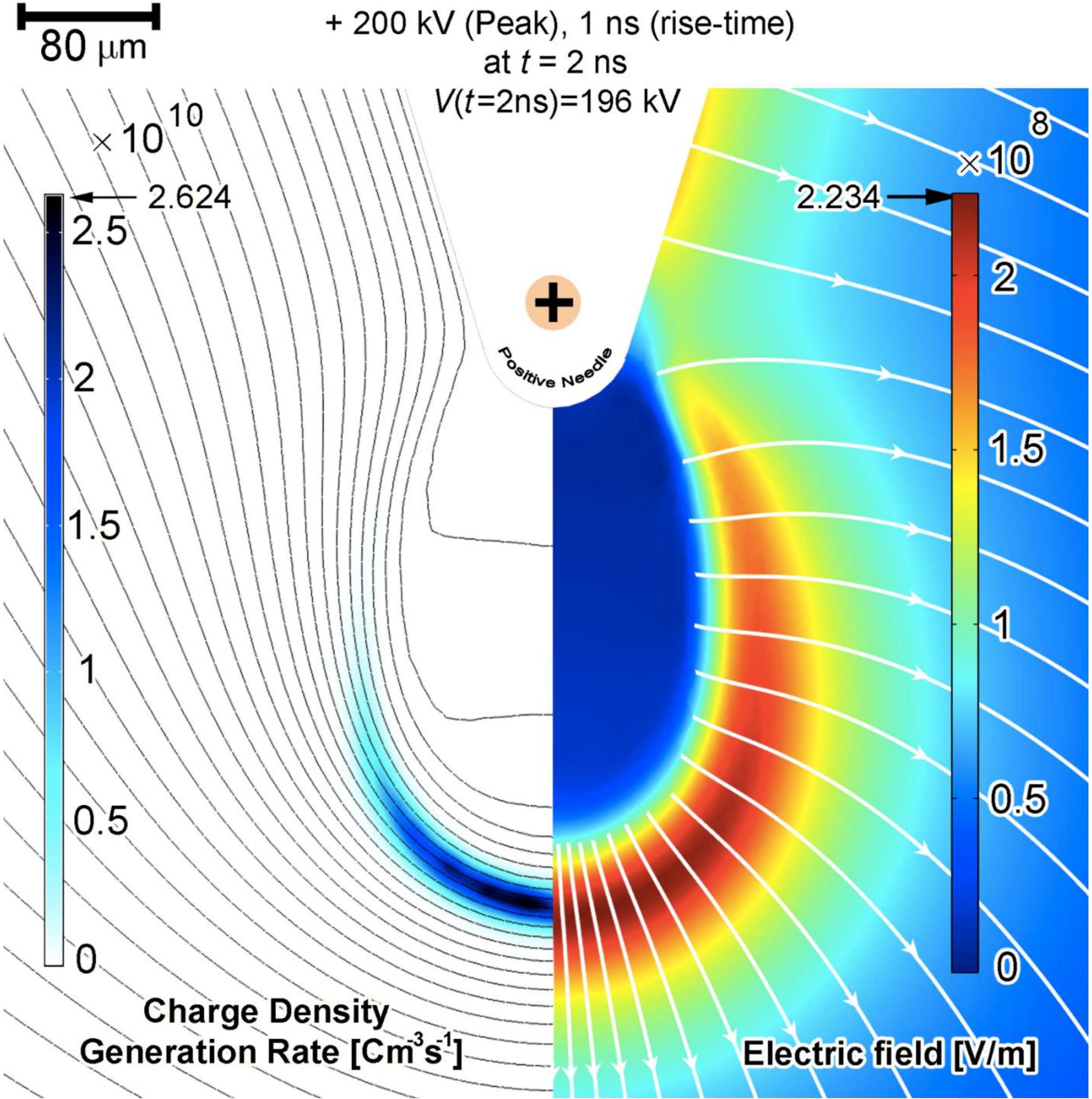
2D results: effect of polarity, magnitude and rise-time:

- Electric field magnitude and field lines (right side)
- Net charge density and equipotential lines (left side) for a positively applied lightning impulse voltage with +200 kV peak and 10 ns rise-time at $t = 8.5$ ns.

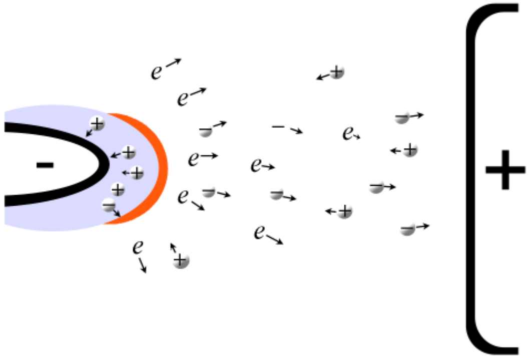


2D results: effect of polarity, magnitude and rise-time:

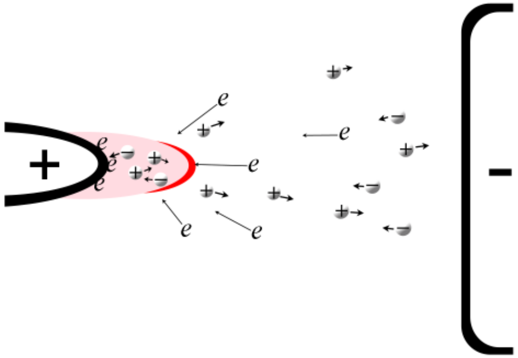
- Electric field magnitude and field lines (right side)
- Net charge density and equipotential lines (left side) for a positively applied lightning impulse voltage with +200 kV peak and 1 ns rise-time at *t* = 2 ns.
- Change in applied voltage rise-time does not affect negative streamers since electron relaxation time, 200 ns is TOO SHORT as compared with applied voltage rise-times.

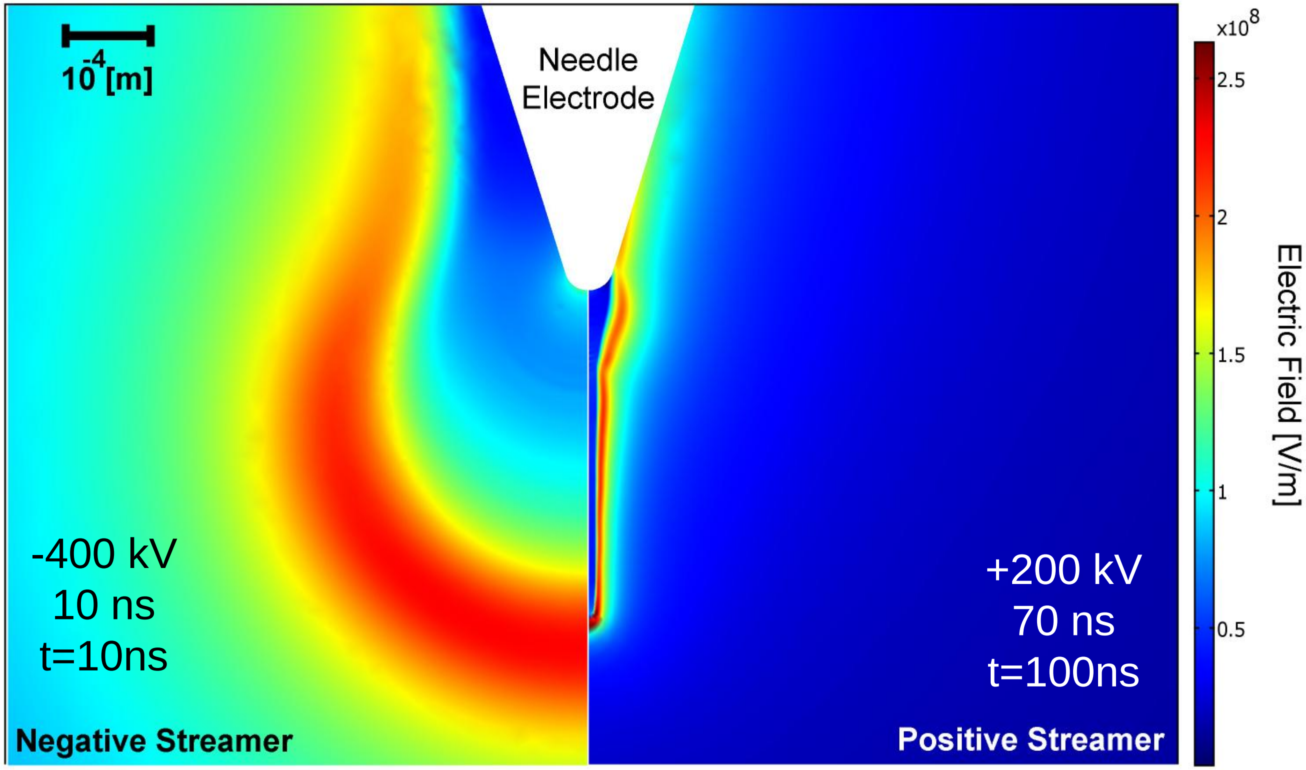


2D results: effect of polarity, magnitude and rise-time:



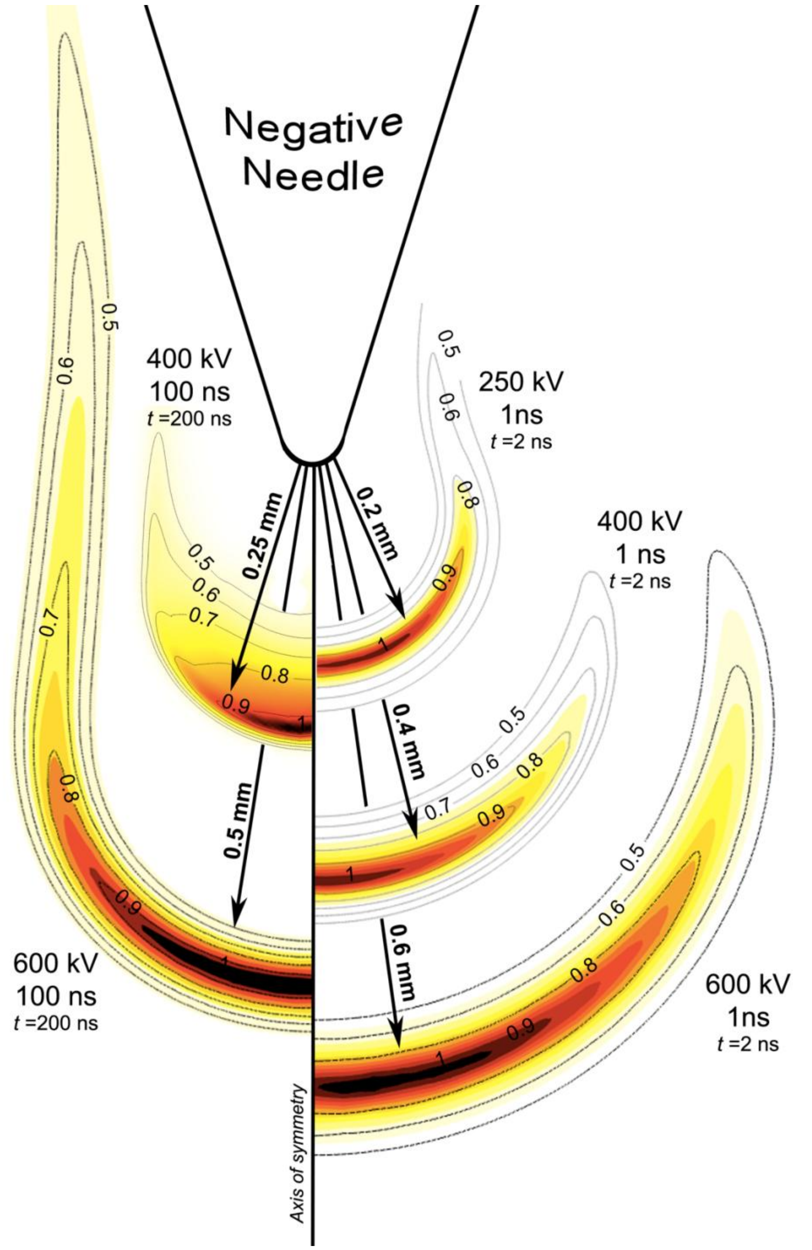
Formation and propagation of (right) positive and (left) negative streamers. In both streamers three areas exist: ionized zone just next to the needle electrode which acts like a weak plasma (low electric field), streamer tip in front of the ionized zone (maximum electric field) and not ionized bulk oil where electric field is relatively low.



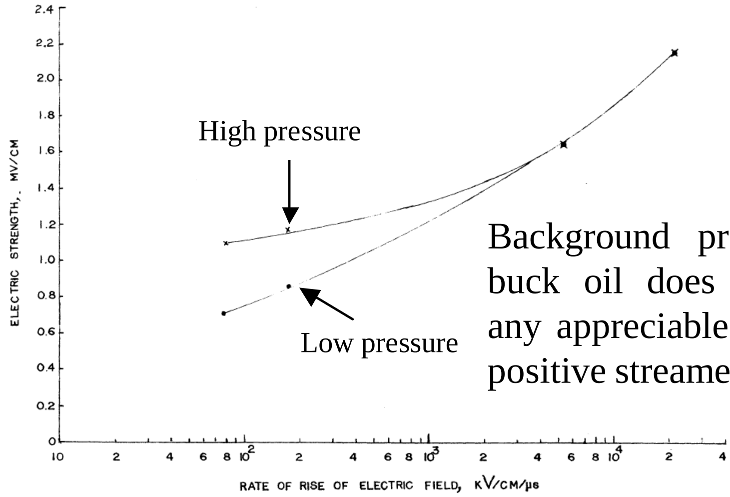


2D results: effect of polarity, magnitude and rise-time:

Volume charge densities and electric field distributions for different negatively applied voltage peak amplitudes and rise-times. Space charge density generation rate, G_I are shown as filled contours from $0.5|G_{max}|$ (the brightest color) to $|G_{max}|$ (the darkest color). Electric field contours are shown as black solid lines from $0.5|E_{max}|$ to $|E_{max}|$. The value of each contour is labeled on the curve as a fraction of $|E_{max}|$. The approximate radius of an ionized bubble can be compared between different applied voltage peaks and rise-times: -250 kV with 1ns rise-time (upper right): $|E_{max}|=1.01\times10^{8}$ V/m and $|G_{max}|=0.7\times10^{11}$ $\mathrm{Cm^{-3}s^{-1}}$; -400 kV with 1ns rise-time (middle right): $|E_{max}|=1.42\times10^{8}$ V/m and $|G_{max}|=1.2\times10^{11}$ $\mathrm{Cm^{-3}s^{-1}}$; -600 kV with 1ns rise-time (bottom right): $|E_{max}|=1.75\times10^{8}$ V/m and $|G_{max}|=6.21\times10^{11}$ $\mathrm{Cm^{-3}s^{-1}}$; -400 kV peak with 100 ns rise-time (upper left): $|E_{max}|=0.95\times10^{8}$ V/m and $|G_{max}|=0.84\times10^{11}$ $\mathrm{Cm^{-3}s^{-1}}$; and -600 kV peak with 100 ns rise-time (bottom left): $|E_{max}|=1.15\times10^{8}$ V/m and $|G_{max}|=1.21\times10^{11}$ $\mathrm{Cm^{-3}s^{-1}}$.

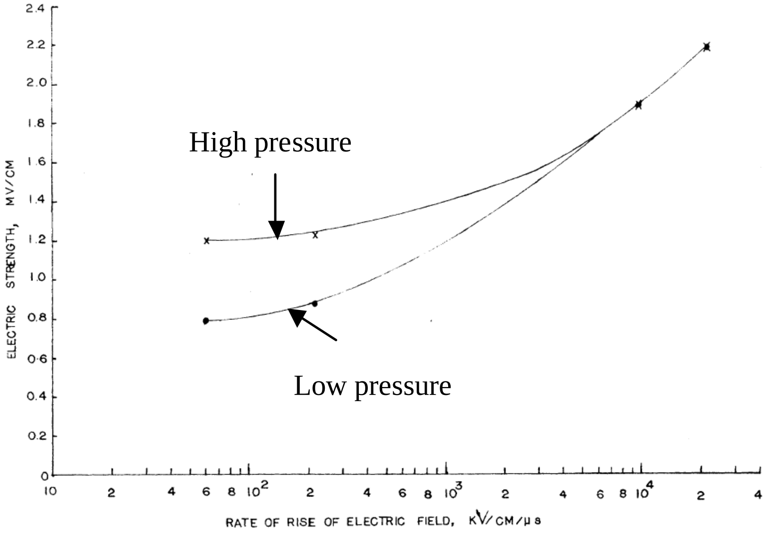


2D results: effect of Background pressure:



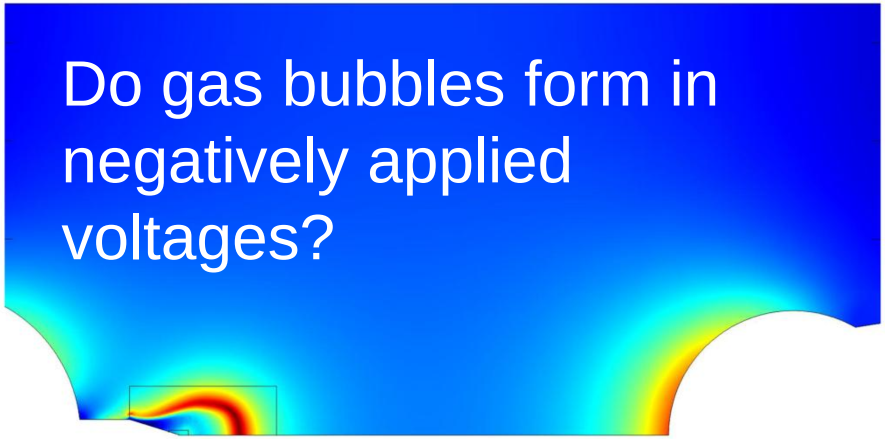
Background pressure of buck oil does not have any appreciable effect on positive streamers

(a)

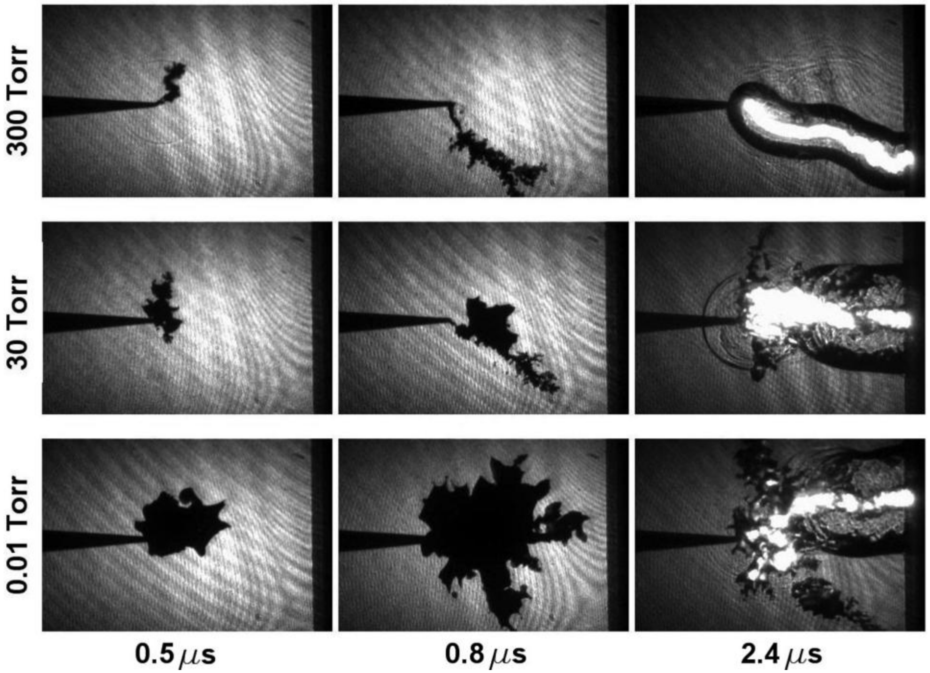


(b)

Dielectric strength of (a): n-hexane and (b): transformer oil for two different pressures. Gap length is about 1 mm [14].



Do gas bubbles form in negatively applied voltages?



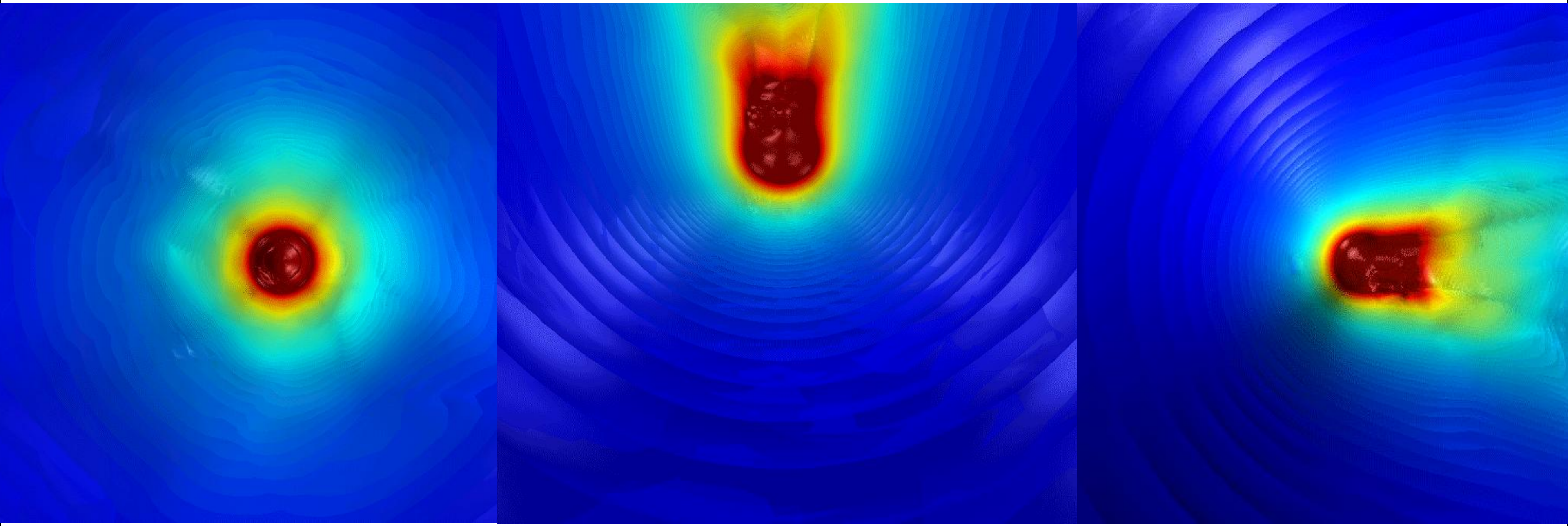
Cevallos, et al., “Imaging of negative polairity DC breakdown streamer expansion in transformer oil due to variation in background pressure”, IEEE TPS, 2005

Streamer Branching *Sanity Check*

3D Simulation Results of Positive streamers

Three dimensional electric field magnitude distribution around the positive needle. Applied impulse voltage has a peak of **200 kV** and rise-time of **100 ns**.

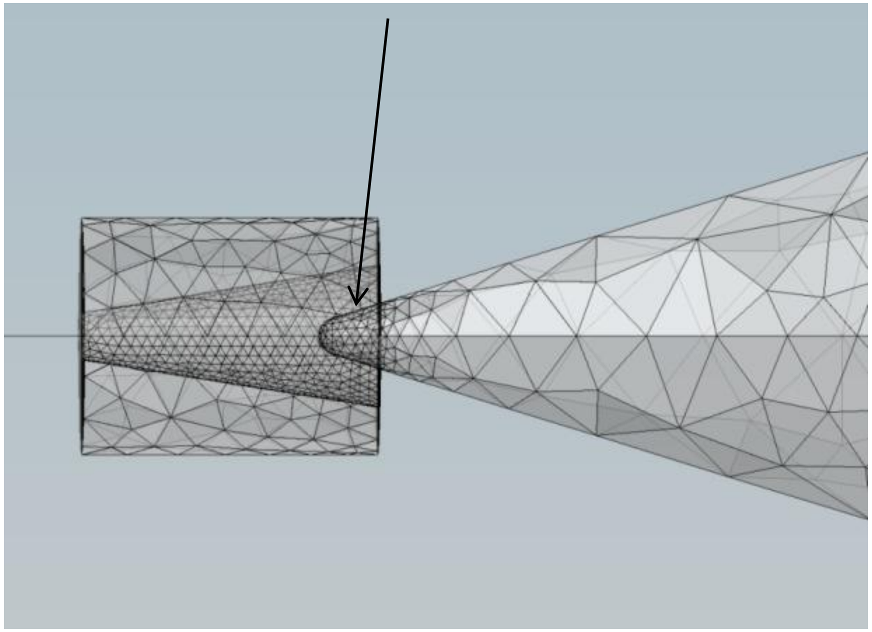
Microscopic perturbation has been added to the spatial distribution of the oil permittivity.



COMSOL 4.2a, 3D Simulation Results of Positive streamers

Distribution of tetrahedral mesh elements around the needle tip. Inside the cone the maximum element size is 2.5×10^{-5} m. Total number of elements is about 50,000. The streamline and crosswind diffusion stabilization technique return convergence for 3D simulations.

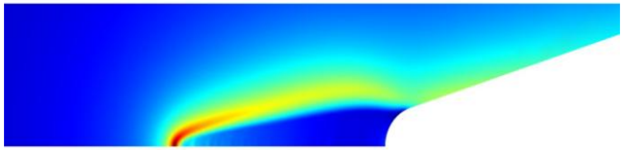
Needle tip radius of curvature is 40 µm.



Three dimensional mesh element distribution around the positive needle. Applied impulse voltage has a peak of 200 kV and rise-time of 100 ns.

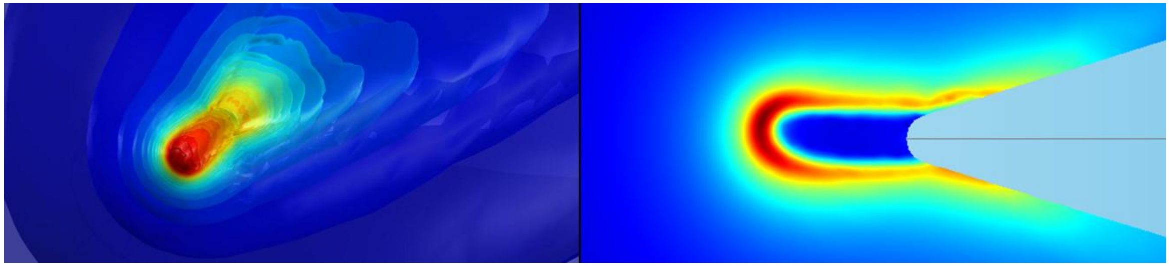
3D Simulation results vs. 2D simulation results: Electric Field

2D axisymmetric



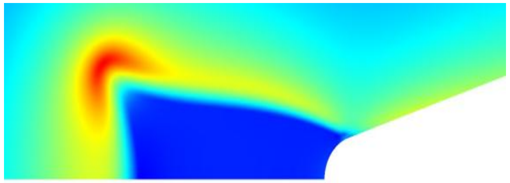
200 kV of peak- 100 ns rise time
at t=100 ns, instantaneous voltage V(t =100 ns) = 180 kV

3D



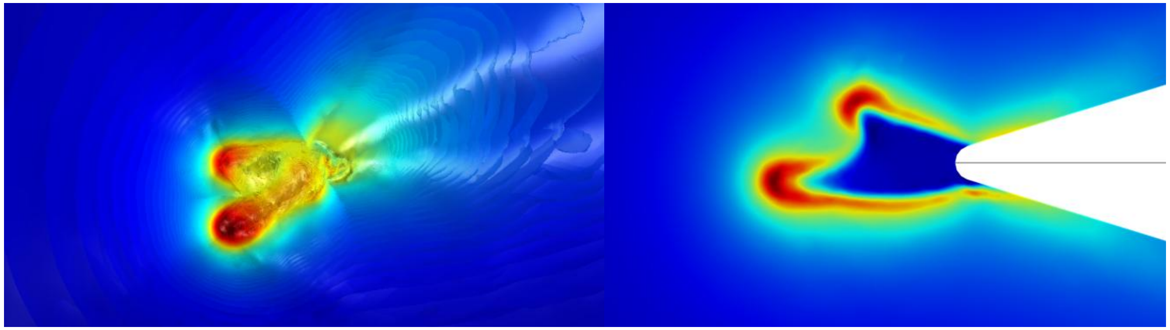
200 kV of peak- 100 ns rise time
at t=100 ns, instantaneous voltage V(t=100 ns) = 180 kV

2D axisymmetric



Nonphysical result of 200 kV of peak- 5 ns rise time
at t=5 ns, instantaneous voltage V(t =5 ns) = 180kV

3D

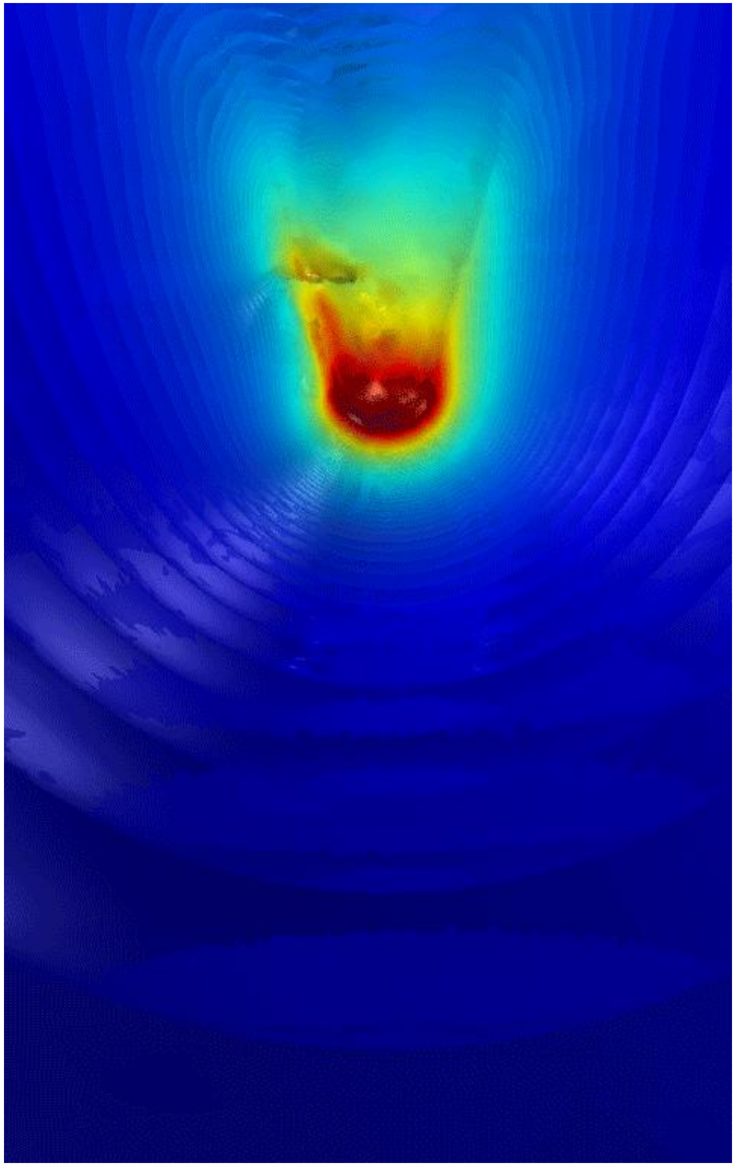


200 kV of peak- 5 ns rise time
at t= 5 ns, instantaneous voltage V(t=5 ns) = 180 kV

Conclusion: Decreasing applied voltage rise time will increase the chance of branching.

Streamer Branching

3D Simulation Results of Positive streamers

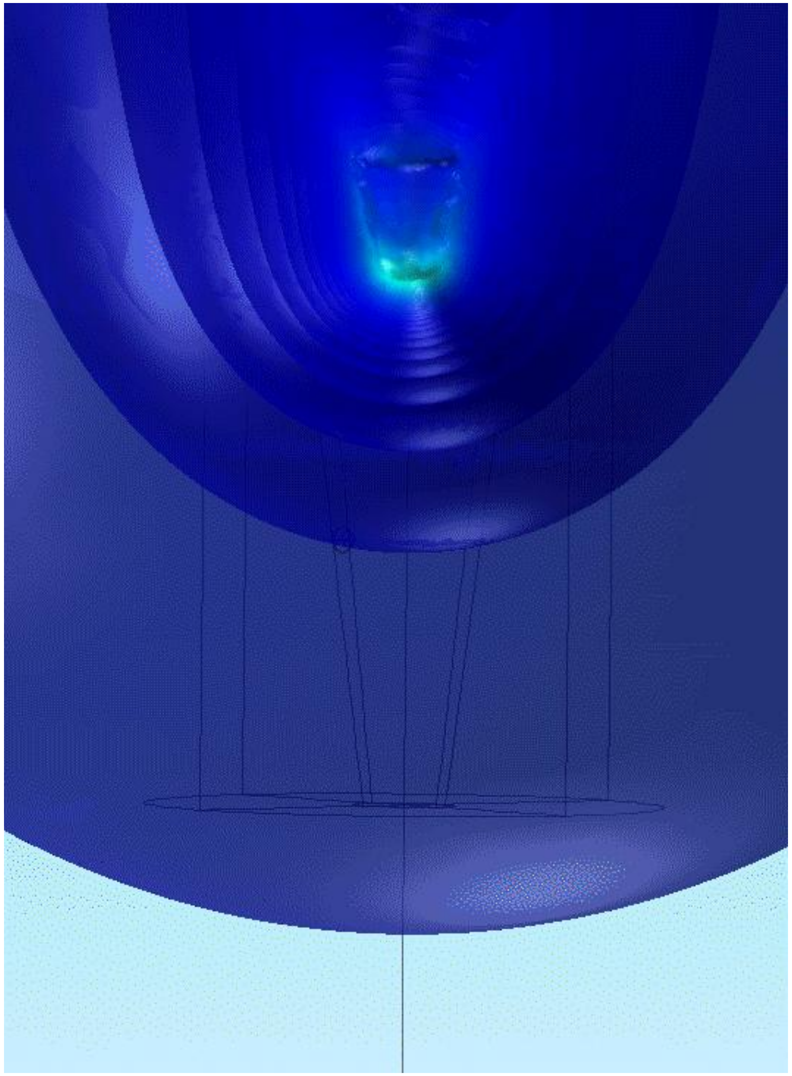


Three dimensional electric field magnitude distribution around the positive needle. Applied impulse voltage has a peak of **130 kV** and rise-time of **100 ns**.

Microscopic perturbation has been added to the spatial distribution of the oil permittivity.

Streamer Branching

3D Simulation Results of Positive streamers



Three dimensional electric field magnitude distribution around the positive needle. Applied impulse voltage has a peak of **200 kV** and rise-time of **100 ns**.

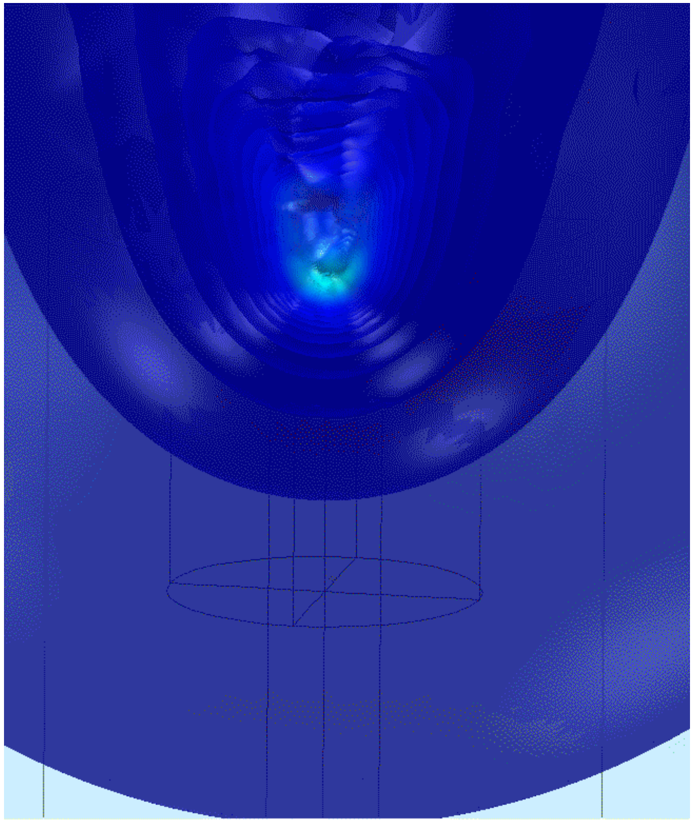
Microscopic perturbation has been added to the spatial distribution of the oil permittivity.
d

Streamer Branching

3D Simulation Results of Positive streamers

Three dimensional electric field magnitude distribution around the positive needle. Applied impulse voltage has a peak of **300 kV** and rise-time of **100 ns**.

Microscopic perturbation has been added to the spatial distribution of the oil permittivity.
?

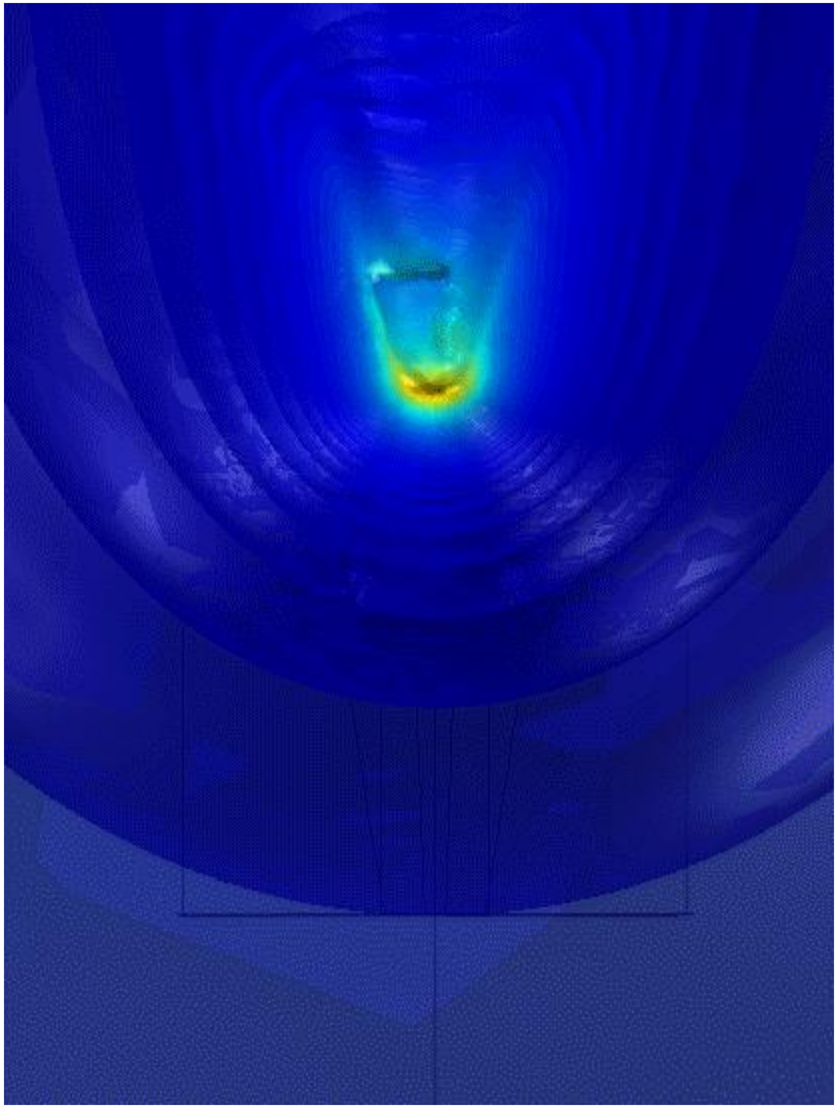


Streamer Branching

3D Simulation Results of Positive streamers

Three dimensional electric field magnitude distribution around the positive needle. Applied impulse voltage has a peak of **350 kV** and rise-time of **10 ns**.

Microscopic perturbation has been added to the spatial distribution of the oil permittivity.
h

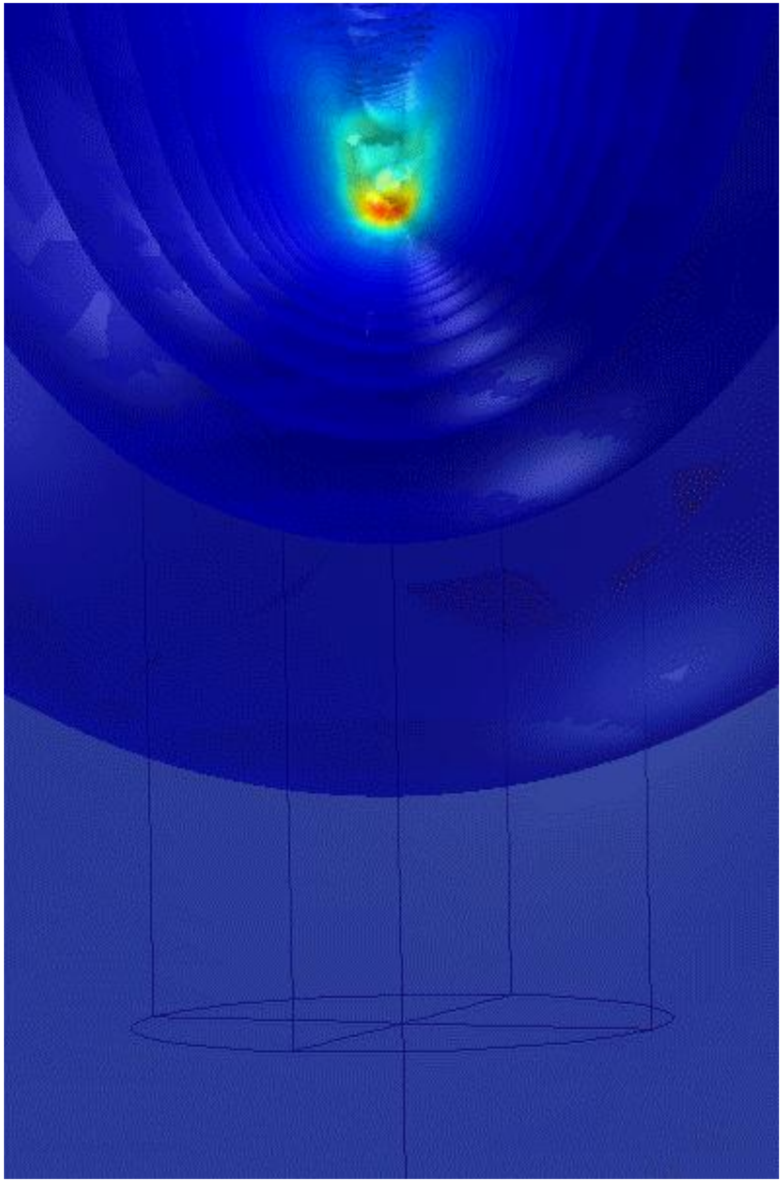


Streamer Branching

3D Simulation Results of Positive streamers

Three dimensional electric field magnitude distribution around the positive needle. Applied impulse voltage has a peak of **400 kV** and rise-time of **10 ns**.

Microscopic perturbation has been added to the spatial distribution of the oil permittivity.
i

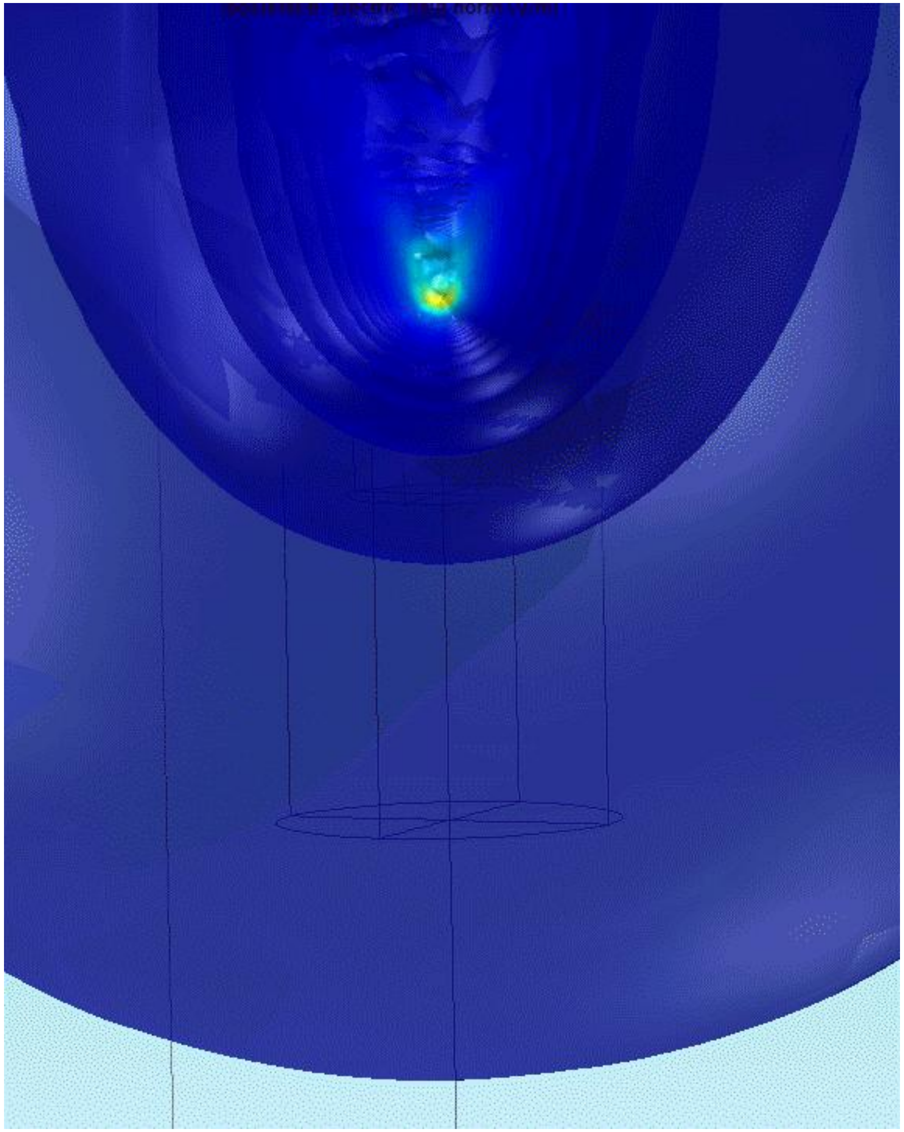


Streamer Branching

3D Simulation Results of Positive streamers

Three dimensional electric field magnitude distribution around the positive needle. Applied impulse voltage has a peak of **400 kV** and rise-time of **100 ns**.

Microscopic perturbation has been added to the spatial distribution of the oil permittivity.

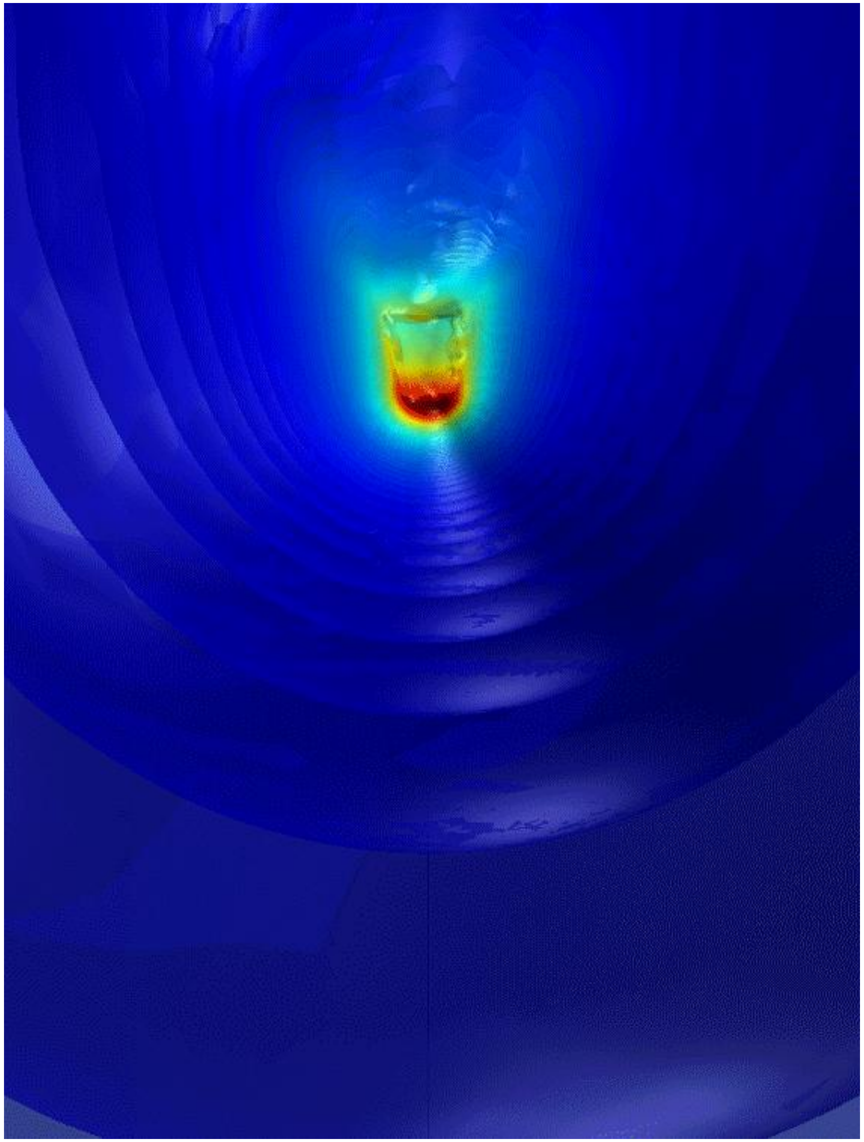


Streamer Branching

3D Simulation Results of Positive streamers

Three dimensional electric field magnitude distribution around the positive needle. Applied impulse voltage has a peak of **450 kV** and rise-time of **10 ns**.

Microscopic perturbation has been added to the spatial distribution of the oil permittivity.

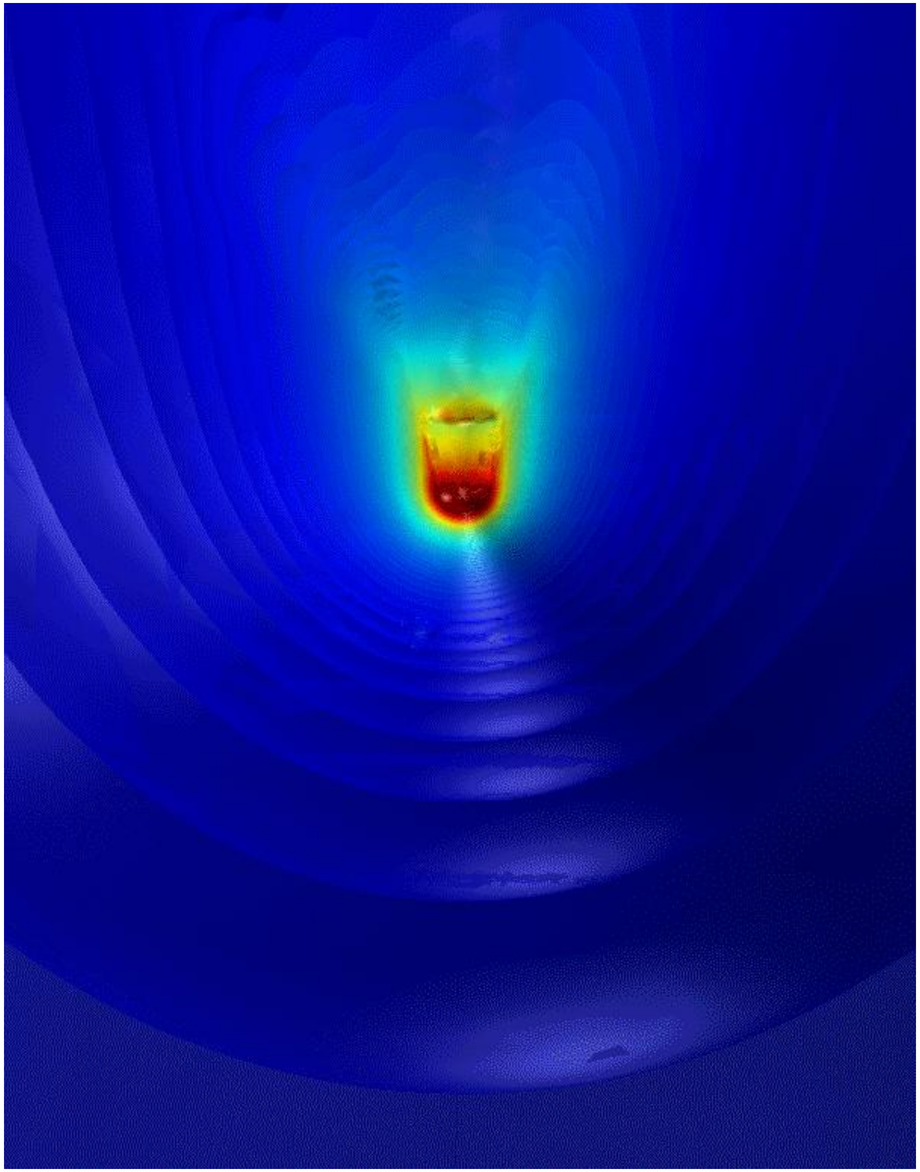


Streamer Branching

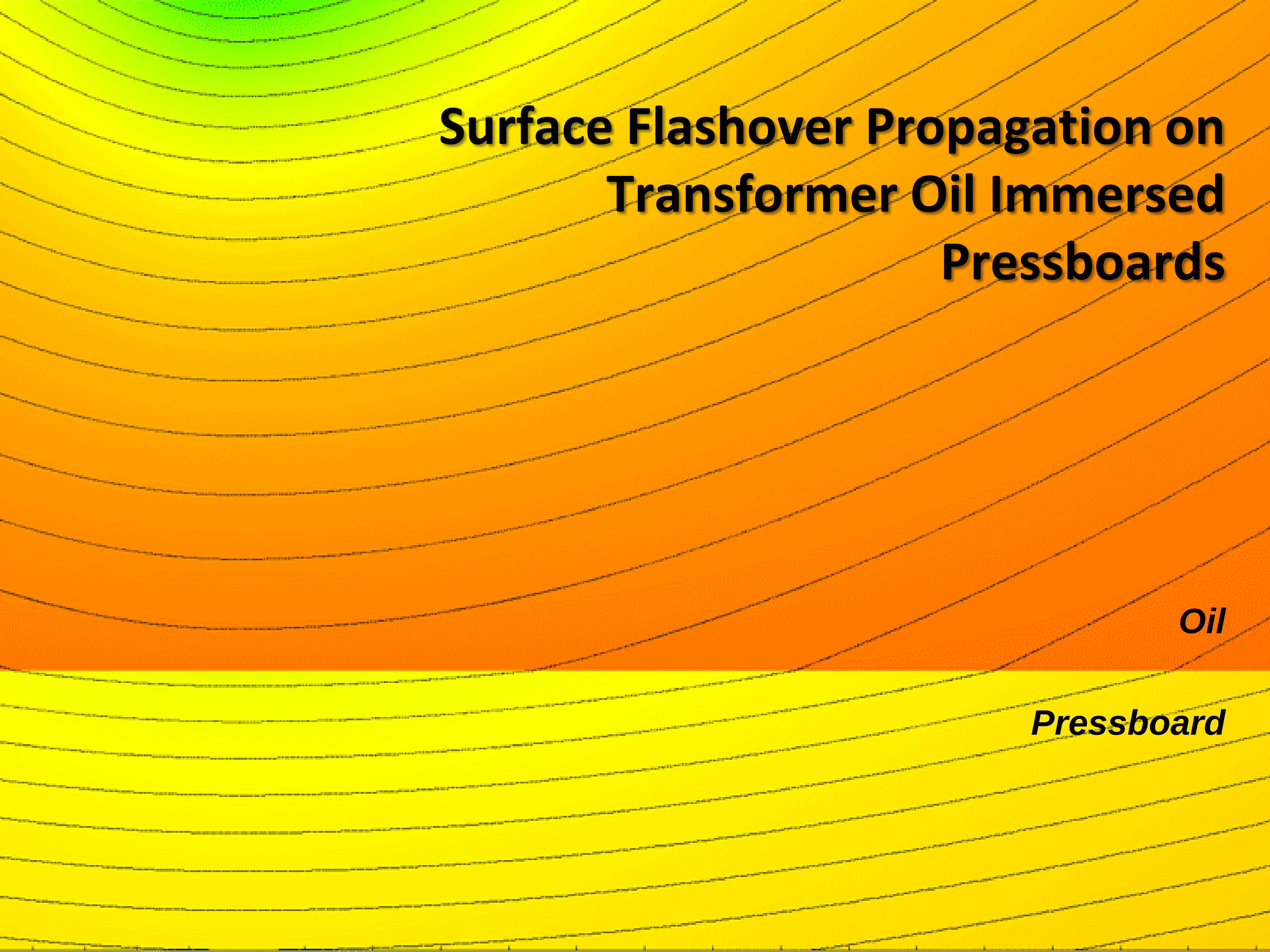
3D Simulation Results of Positive streamers

Three dimensional electric field magnitude distribution around the positive needle. Applied impulse voltage has a peak of **500 kV** and rise-time of **10 ns**.

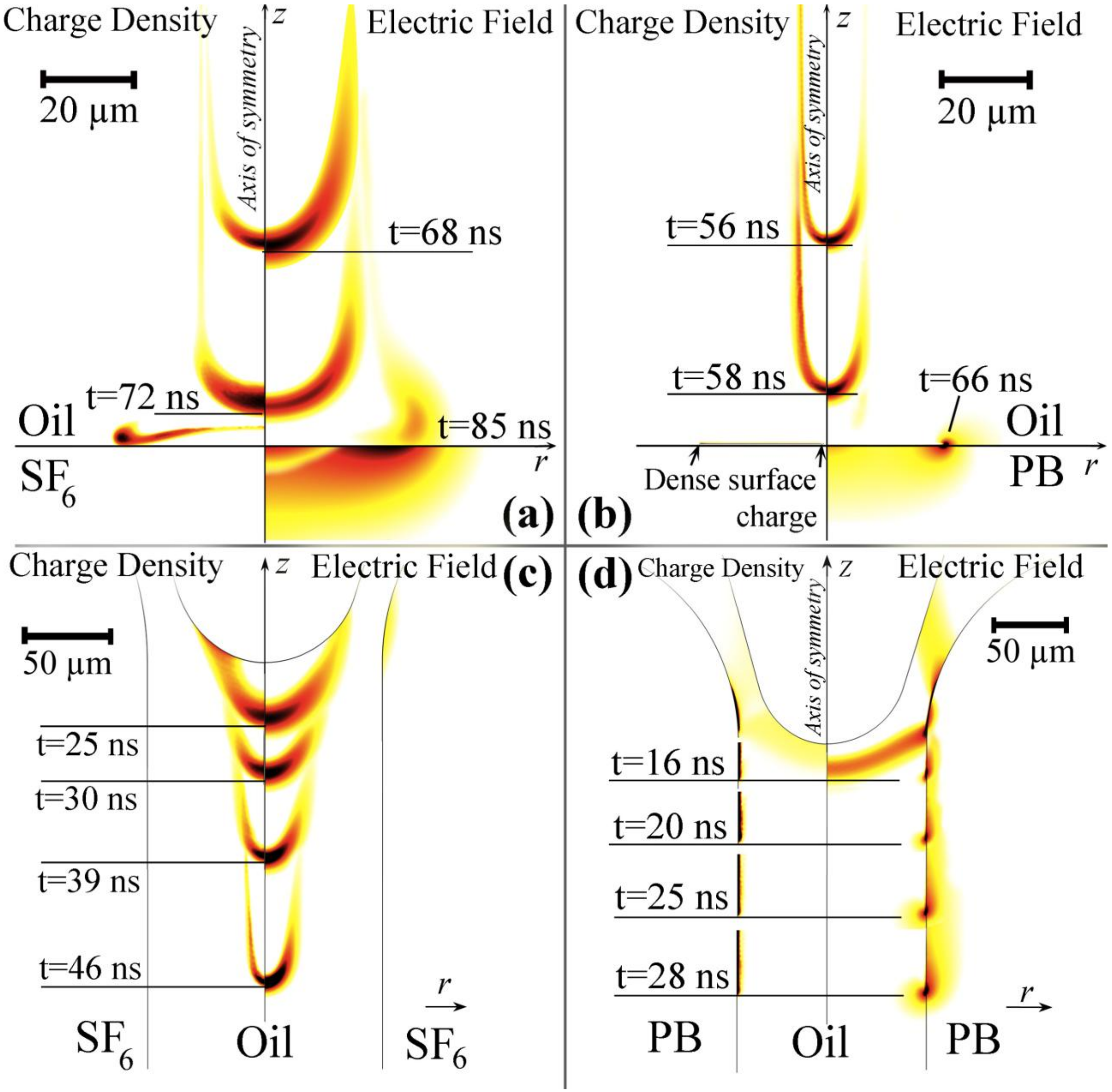
Microscopic perturbation has been added to the spatial distribution of the oil permittivity.



Surface Flashover Propagation on Transformer Oil Immersed Pressboards



Effect of Pressboard Permittivity:



Acknowledgements and References

- The project is technically and financially supported by
  - ABB Corporate Research, Västerås, Sweden.
  - IEEE Dielectrics and Electrical Insulation Society.
- For references and more readings please go to:
  - http://web.mit.edu/~jouya/www/

Published Papers:

1. J. Jadidian, M. Zahn, N. Lavesson, O. Widlund, K. Borg, “Impulse breakdown delay in liquid dielectrics,” Applied Physics Letters, 100, 192910, 2012.
2. J. Jadidian, M. Zahn, N. Lavesson, O. Widlund, K. Borg, “Surface Flashover Breakdown Mechanisms on Liquid Immersed Dielectrics,” Applied Physics Letters, 100, 172903, 2012.
3. J. Jadidian, M. Zahn, N. Lavesson, O. Widlund, K. Borg, “Effects of Impulse Voltage Polarity, Peak Amplitude and Rise-Time on Streamers Initiated from a Needle Electrode in Transformer Oil,” IEEE Transactions on Plasma Science, Vol. 40, No. 2, pp. 909 – 918, March. 2012.
4. J. Jadidian, M. Zahn, “Unipolar Charge Transport in Oil-Pressboard Systems with Planar, Coaxial Cylindrical and Concentric Spherical Electrode Geometries,” Invited paper presented as Inuishi Memorial Lecture at 2011 International Symposium on Electrical Insulating Materials (ISEIM 2011), Kyoto, Japan, 2011.
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7. J. Jadidian, G. J. Hwang, M. Zahn, N. Lavesson, O. Widlund, K. Borg, “Streamer dynamics in transformer oil: Influence of applied voltage rise time,” 38th International Conference on Plasma Science, Chicago, IL (25 and 27 June 2011 in both oral and poster formats).
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