# Modelling of Droplet Charge Dynamics during an Ink Jet Breakup using COMSOL Multiphysics®


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## Abstract

Continuous Ink Jet (CIJ) printing technologies are widely used in the field of industrial coding and marking. These technologies are based on the emission of a high-speed jet of ink droplets (20 m/s) onto the surface of a moving medium. To maintain sufficient print quality, it is essential to control the positions of the droplets on the printing medium. The position of each droplet depends on several factors, including the quality of the jet breakup, the charge carried by each droplet, the deflection of the droplets in an electric field, and their interactions in flight. Numerical modelling can provide detailed information at each step, allowing to predict the behavior of ink droplets, and ultimately helping to design CIJ printheads. This article focuses on modelling the droplet charging process between electrodes during ink jet breakup in COMSOL Multiphysics®. The dynamics of the ink jet surrounded by air is modelled using the two-phase flow Level Set interfaces from the Fluid Flow module. The spatial charge is modelled using the Electric Current (EC) physics from the AC/DC module. With a particular attention to the timestep, numerical simulations can be used to predict the charge embedded in each droplet just before the breakup, and therefore the embedded charge in each droplet that is deflected after breakup. This model also allows to apprehend the historical charge distributed along the following droplets. The prediction of the uncharged jet breakup dynamics is assessed thanks to experiments ran at MARKEM-Imaje, validating the CFD part of the model, and providing confidence in using the charging model for industrial use.



**Keywords:** Continuous inkjet printing, industrial printer, charged ink, CFD, electric currents, two-phase flow, level-set method, ink-air interaction, charge transport.


## 1 Introduction

In the field of industrial marking, continuous ink jet technology (CIJ) is based on the emission of a high-speed stream of ink drops (20 m/s, 100 kHz) propelled onto a moving medium to be printed [1] [2] [3] [4].

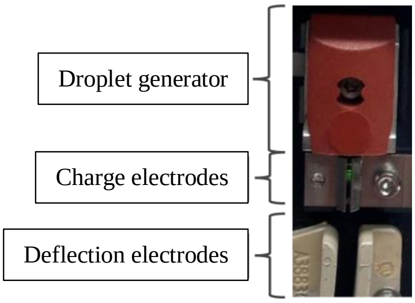



*Figure 1. Top part of a CIJ print head.*

The mechanism used and studied in this paper is pictured in Figure 1 and works as follows. A continuous ink jet is stimulated by a high frequency PZT stimulation within the droplet generator. Then, the jet oscillates and breaks between charge electrodes. A potential difference is applied between the ink jet and charge electrodes, making droplets embedding a certain amount of electrical charges at jet pinch-off. Finally, charged droplets pass through deflection electrodes, deviating them from their initial trajectory, which forms the print pattern. The printing quality then depends on the accuracy of the droplet breaking-charging process.

The purpose of this work is to model the coupled jet breaking-charging process, and accurately forecast the charge embedded by each droplet.

## 2 Numerical Model

The model is first based on a diphasic approach to, then, study the electrical behavior on one calculated breaking droplet.

### CFD Model: Droplet Formation

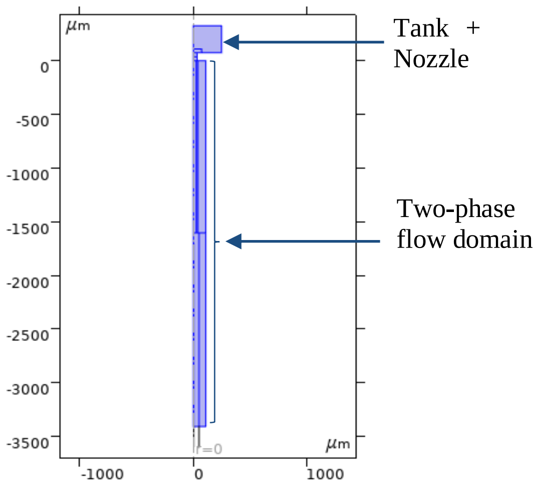



*Figure 2: 2D axisymmetric geometry of the CFD model.*

This model is based on chapter 2.2 of [5]. Assuming the tank and the nozzle have an axial symmetry revolution, the COMSOL Multiphysics model is fed with a 2D axisymmetric geometry as described in Figure 2.

The two-phase ink-air flow is modeled using the Level-Set method coupled with the Laminar Flow physics (Multiphysics Two-phase Laminar flow Level-Set). The flow is described by incompressible Navier-Stokes equations at transient state:

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} + \rho(\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u} = \boldsymbol{\nabla} \cdot [-p\boldsymbol{I} + \boldsymbol{K}] + \boldsymbol{F} \quad (1)$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0 \quad (2)$$

$$\boldsymbol{K} = \mu(\boldsymbol{\nabla u} + (\boldsymbol{\nabla u})^{\mathrm{T}}), \quad (3)$$

where $\boldsymbol{u}$ stands for the flow velocity, $\rho$ the fluid density, $p$ the pressure, $\mu$ the fluid dynamic viscosity, $\boldsymbol{K}$ the deviatoric stress tensor and $\boldsymbol{F}$ the sum of volume forces.

As the jet move at high velocity (20 m/s) and the generated droplets are very light ($\approx$ 1 µg), the gravity is neglected.

The ink/air phases and interface are modelled using the Level-Set equation:

$$\frac{\partial \phi}{\partial t} + \boldsymbol{u} \cdot \boldsymbol{\nabla}\phi = \gamma \boldsymbol{\nabla} \cdot \left(\epsilon_{ls} \boldsymbol{\nabla}\phi - \frac{\phi(1-\phi)\boldsymbol{\nabla}\phi}{|\boldsymbol{\nabla}\phi|}\right), \quad (4)$$

where $\phi$ is the phase function taking values 0 in ink and 1 in air and $\epsilon_{ls}$ and $\gamma$ are stabilization numerical parameters. The numerical parameters are purposely fitted to the flow velocity ($\gamma$ = 20 m/s) and model mesh ($\epsilon_{ls}$ = 2 µm).

Effects of surface tension between ink and air is taken into account in the $\boldsymbol{F}$ term of Eq. 1, *via* the capillary pressure:

$$\boldsymbol{F_{st}} = \sigma \delta \kappa \boldsymbol{n}_{int}, \quad (5)$$

where $\sigma$ stands for the surface tension between ink and air, $\delta$ the Dirac at the interface, $\kappa$ the curvature of the interface such as $\kappa$ is positive if the ink phase is convex and $\boldsymbol{n}_{int}$ the interior normal of the ink domain boundary. This force is implemented in the Multiphysics Two-phase Laminar flow Level-Set.

The boundary conditions closing the equations are summarized in Figure 3. The boundary condition on the top of the tank has two contributions. The constant value ($P_{out} + P_0$) drives the ink flow at the nozzle while the periodic perturbation ($P_1 sin(\omega t)$) induces instabilities to produce a jet break at a controlled height. The jet break leads to droplets formation. The objective of this model is to obtain a realistic droplet shape to study the charge phenomenon.

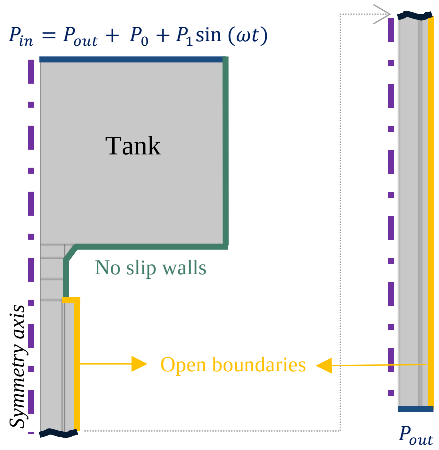



*Figure 3: Boundary conditions of the CFD model.*

### EC Model: Charge of a Breaking Droplet

The purpose of this second model is to forecast the electric field and charge distribution around the breaking droplet for a given electrode potential. In the CFD transient simulation, a snapshot of the phase function describing the geometry of a breaking droplet is selected and used to feed the electrical model. A 2D axisymmetric geometry is implemented in the COMSOL Multiphysics model schematized in Figure 4.

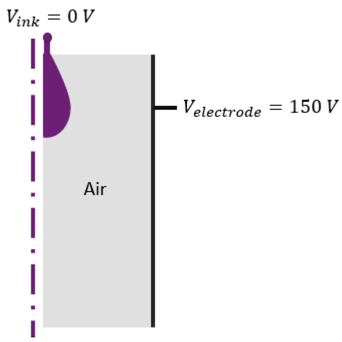
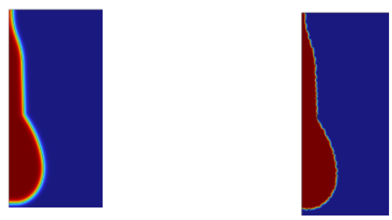



*Figure 4: Scheme of the 2D axisymmetric geometry of the EC model.*

As droplet charge should be influenced by the quality of the phase function interpolation, two different ones shown in Figure 5 are used as input for the EC model.



*(a)* *(b)*

*Figure 5 : Phases function of a droplet with a smooth interpolation (a) and a raw one (b).*

The droplet is simulated without the preceding inkjet. Indeed, at first order, the charge embedded in the droplet only depends on its shape just before the jet break and the potential of charge electrodes.

Assuming that the travelling time of the droplet between electrodes is many times longer than the transient electrical effects, the electric field is computed at quasi-static state. To avoid a boundary condition at the ink-air interface, the air is modelled as a conductor with *zero* conductivity and the same physics (EC) is applied on the entire geometry. The governing equation is electrical charge conservation one:

$$\nabla \cdot (\sigma_{elec} \boldsymbol{E}) = 0 \tag{6}$$

$$\boldsymbol{E} = -\nabla V \tag{7}$$

$$\sigma_{elec} = \begin{cases} 0.1 \text{ if } \phi < 0.5 \text{ (ink)} \\ 10^{-15} \text{ if } \phi > 0.5 \text{ (air)} \end{cases}, \tag{8}$$

where $\sigma_{elec}$ is the electrical conductivity in S/m, $\boldsymbol{E}$ the electric field and $V$ the electric potential. In order to ensure that the stationary problem is well-defined, the conductivity of the air is considered very low but nonzero ($10^{-15}$ S/m).

As shown in Figure 4 there are two Dirichlet boundary conditions: $V_{ink} = 0$ V on top of the droplet and $V_{electrode} = 150$ V on the electrode.

**EC Model: Charge Measurement**

With a geometric interface, the charge of the droplet is evaluated by integrating the normal projection of the displacement field ($\boldsymbol{D} = \epsilon_0 \epsilon_r \boldsymbol{E}$) on the droplet surface. Using a phase function, the normal is given by the Level-Set physics. Numerically, the phase function may not be smooth enough, resulting in biases in the numerical interface normal vectors. The interpolated phase function shown in Figure 5.b allows to study that case. This first method is further called "Normal method".

To ensure robustness of charge measurement, a new method is implemented. Partitioning the domain in boxes and using the divergence theorem with the local Gauss's law, one is able to deduce the charge of any jet part from the displacement field on others box boundaries (Figure 6). This second method is further called "Box method".

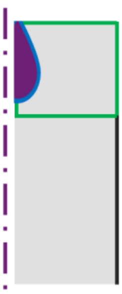


*Figure 6: Box used to measure one droplet charge.*

The model has only one droplet with a known position. Thus, the box is defined with geometric lines but the method is extendable to arbitrary boxes with linear projections.

**Numerical Aspects**

Both models are discretized in space using the finite element method on a linear triangular mesh.

CFD equations are solved using stabilized P1+P1 discretization. The element size is refined from 10 to 2 µm as we get closer to the symmetry axis where the most phase changes are expected. The model have 620 000 DOFs and uses 8 Gb of RAM. Using 4 cores cadenced at 3 GHz and for a 1 ms physical time simulation, equivalent to approximately 100 jet periods, the computation time is about 100 hours. The model is solved using a time-dependent study with a direct linear solver and a fixed maximal timestep 300 times lower than the pressure $P_{in}$ period.

EC equation are solved using a P2 discretization. The element size is fixed to 2.5 µm. The model has 702 000 DOFs and uses 2.5 Gb of RAM. The computation time of the stationary solver is around 20 seconds with 4 cores cadenced at 3 GHz. The model is solved using a stationary study with default COMSOL Multiphysics 6.1 settings.

## 3 Simulation Results

**CFD Results**

Figure 7 illustrates a snapshot of the phases predicted by the CFD model from the tank to the bottom of the air domain.

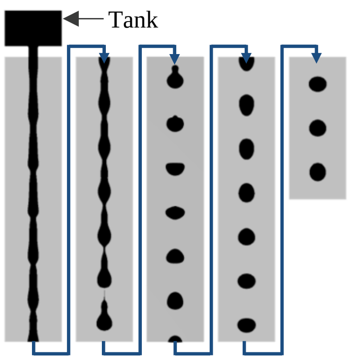
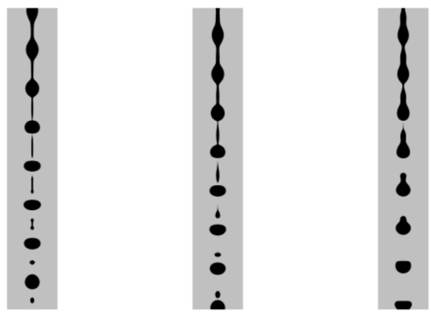



*Figure 7: Snapshot of a phase function simulated with the CFD model at final time.*



*(a)* *(b)* *(c)*

*Figure 8: Snapshots of two-phase flow simulations for multiple $P_1$ values.*

Tuning the value of the pressure oscillation amplitude at the entry of the tank $P_1$, the model predicts different shapes of jet around the break length as shown in Figure 8.

The jet shape of Figure 8.c is used in the following results. The part of the phase function highlighted in Figure 9 is then used to feed the EC model.

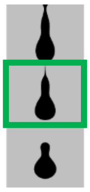


*Figure 9: Zoom on the selected droplet shape from the Figure 8.c.*

**EC results**

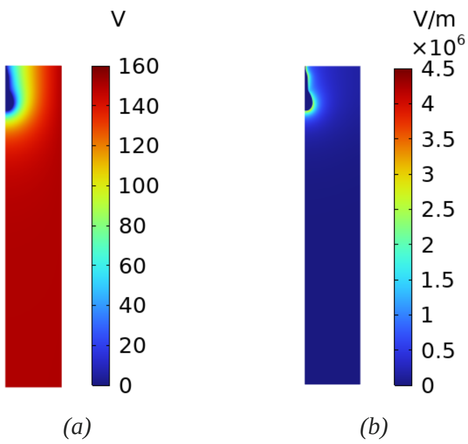



*Figure 10: Results from the EC model using Figure 5.a phase function, Electric potential (a) and Electric field norm (b).*

Figure 10 illustrates the electric potential and the electric field norm around the droplet. As expected for a conductor at electrostatic state, the potential is homogeneous, equals to zero in the ink.

Using phase functions from Figure 5, the evaluated total charges of the droplet from Box and Normal methods are given in Table 1.

*Table 1: Charge in pC obtained with the two measurement methods applied on phase functions presented in Figure 5.*

| | Box | Normal |
|---|---|---|
| Smooth function (5.a) | - 1.2481 | - 1.2485 |
| Raw function (5.b) | - 1.2745 | - 0.31189 |

The results of Table 1 show that the Box method is the most accurate because it is less impacted by the quality of the phase function. In fact, more the phase function is sharp, less the numerical interface normal is representative of the real surface. Then, information about the charge is biased during the measurement.

With a smooth interface, using Figure 5.a phase function, it is possible to plot the charge density distribution over the droplet surface as shown in Figure 11.

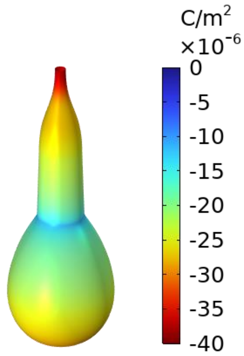
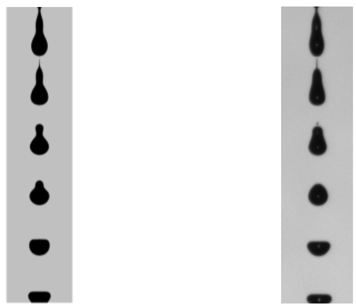



*Figure 11: Charge surface density on the droplet surface (smooth interface case).*

## 4 Results Discussion

An experimental bench makes it possible to take snapshots of the droplets under a stroboscopic light calibrated to the droplet emission frequency. The simulated droplet shapes match pretty well with effective observed shapes as shown in Figure 12.



*(a) Model.* *(b) Experiment.*

*Figure 12: Snapshot from CFD model (a), Snapshot from laboratory experiment (b).*

These results increase our confidence in the capacity of the model to reproduce real behaviors of droplets breaking and charge.

The axisymmetric hypothesis in EC model introduces a bias on the charge repartition compared to the industrial charging system using plates as electrodes: the real geometry is 3D. Then, more charges should accumulate on the electrode sides and less on the others.

The air conductor hypothesis should impact the charge embedded by the droplet. In fact, the impact of the value of the air conductivity has been checked and the relative error committed on the charge is lower than 0.1% under the Eq. 8 condition:

$$\frac{\sigma_{elec\,air}}{\sigma_{elec\,ink}} \leq 4 \times 10^{-7}, \quad (9)$$

strengthening the confidence on the numerical results.

Physically, negative charges repulse themselves and tend to accumulate in places where their mean distance is the higher. That is why the surface density is higher on the top and the bottom of the droplet. Nevertheless, as the negative charges repulse each other they must be pushed away from the jet restriction before breaking, then it may be assumed that the charge embedded in a broken droplet is close to the charge measured on the breaking droplet. Our model differs from this statement because the domain does not represent the jet above the droplet. That is why charges accumulates on the top boundary in Figure 11.

## 5 Conclusions

In CIJ printing, controlling the charge embedded by droplets is important to ensure the printing quality. In this article, a 2D axisymmetric two-phase flow model is built to forecast the shape of the droplets. Then, a static electric field is simulated on a snapshot of a droplet just before breaking, described by a phase function, in order to predict its charge.

Results show a good agreement between experimental and numerical droplet shapes, giving confidence in the capacity of forecasting the electric field between the ink jet and charge electrodes. Using this electric field, the embedded charge is measured using an accurate method based on the divergence theorem.

Future works include solving the electric field at each timestep of the two-phase flow model, allowing to study the impact of already charged droplets on breaking ones.

## References

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