Acoustic Attenuation Performance of Helicoidal Resonator Due to Distance Change from Different Cross-sectional Elements of Cylindrical Ducts

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Abstract: This work presents acoustic attenuation performance of helicoidal resonator due to distance change from different cross-sectional elements of cylindrical duct. The helicoidal resonator properties are described mainly in infinite long cylindrical duct. For practical applications it is important to know the limits of use of this solution. This paper describes how the helicoidal resonator should be placed inside acoustic systems and what distance from different cross-sectional elements like round silencer, expansion chamber, second helicoidal resonator, should be satisfied for its proper work.

Keywords: helicoidal resonator, silencer, cylindrical ducts, sound propagation, sound attenuation.

1. Introduction

Helicoidal resonator [1-4] is a newly developed acoustic band-stop filter, which can be constructed by inserting a helicoidal profile with proper dimensions to a cylindrical duct, in simple formulation - Figure 1.

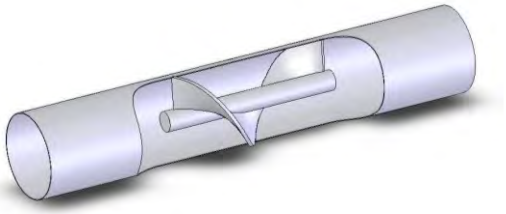

Figure 1. Helicoidal resonator as a part of simple acoustic system - cylindrical duct.

Acoustical properties, mainly attenuation of sound, as presented example transmission loss characteristics in Figure 3, due to an acoustical resonance of helicoidal resonators can be modified by doing a change in relations between its basic geometrical parameters (lead of the helicoidal turn *s*, thickness of the helicoidal profile *g*, diameter of a the mandrel d_m, diameter of the infinite long cylindrical duct *d*), which are shown in Figure 2.

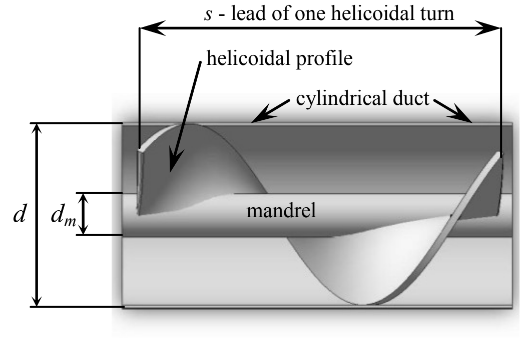

Figure 2. Basic elements of a helicoidal resonator placed inside a cylindrical duct.

Also very important parameter of helicoidal resonator is the number of helicoidal turns *n*, which strongly determines the character of acoustical resonance.

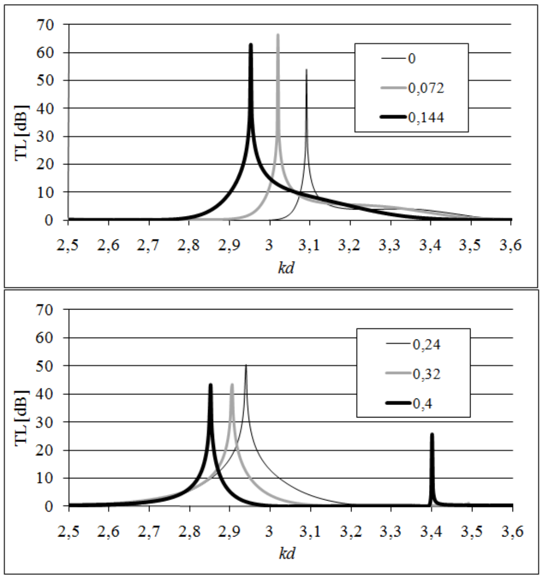

Figure 3. Example transmission loss characteristics of helicoidal resonators with different ratios d_m/d [3].

This paper considers acoustic attenuation performance of helicoidal resonator due to distance change from different cross-sectional elements like round silencer, expansion chamber and second helicoidal resonator placed inside cylindrical duct.

2. Computational environment - basic equations, calculated parameters and boundary conditions

The problem is solved in the frequency domain using the time-harmonic Pressure Acoustics application mode of COMSOL Multiphysics - Acoustic Module [5]. The final solving parameter is the acoustic pressure p [Pa], which can be computed by the use of slightly modified Helmholtz equation:

$$\nabla \cdot \left(-\frac{\nabla p}{\rho_0} \right) - \frac{\omega^2 p}{c_s^2 \rho_0} = 0 \qquad (1)$$

where ρ_0 is the density of air (ρ_0=1,23 kg/m^3), c_s is the speed of sound in air (c_s=343m/s), and ω gives the angular frequency.

As an acoustic attenuation performance parameter is used transmission loss (TL) [10]. For computational needs, the TL is expressed as the difference between the outgoing power at the outlet w_o and the incoming power at the inlet w_i,

$$TL = 10 \log_{10} \left(\frac{w_i}{w_o} \right), \text{ [dB]} \qquad (2)$$

Each of component quantities in equation (2) are calculated as an integral over the corresponding surface $\partial\Omega$ of inlet and outlet cylindrical ducts cross-sections S_n and S_1, respectively:

$$w_i = \int_{\partial\Omega} \frac{p_0^2}{2\rho_0 c_s} dS \qquad (3)$$

$$w_o = \int_{\partial\Omega} \frac{|p_c|^2}{2\rho_0 c_s} dS \qquad (4)$$

where p_0 is the source acoustic pressure at the inlet, [Pa], and p_c is the transmitted acoustic pressure at the outlet, [Pa].

Also for better analysis the acoustic attenuation performance can be presented as increase of transmission loss ΔTL [dB] as follows:

$$\Delta TL = TL_{diff} - TL_{ini}, \text{ [dB]} \qquad (5)$$

where:
TL_{diff} - transmission loss of different case of an acoustic system with helicoidal resonator, [dB],
TL_{ini} - transmission loss of initial acoustic system without helicoidal resonator, [dB].

For investigated models in this work the boundary conditions are of three types [5]. For acoustically hard walls at the solid boundaries, which are the walls of the helicoidal profile, mandrel, cylindrical duct, expansion chamber, outer walls of round silencer the model uses sound hard (wall) boundary conditions:

$$\left(\frac{\nabla p}{\rho_0} \right) \cdot \mathbf{n} = 0 \qquad (6)$$

The boundary condition at the inlet surface (sound source) of cylindrical ducts is a combination of incoming and outgoing plane waves:

$$\begin{aligned} &\mathbf{n} \cdot \frac{1}{\rho_0} \nabla p + ik \frac{p}{\rho_0} + \frac{i}{2k} \Delta_T p = \\ &= \left(\frac{i}{2k} \Delta_T p_0 + \left(1 - (\mathbf{k} \cdot \mathbf{n})\right) ik \frac{p_0}{\rho_0} \right) e^{-ik(\mathbf{k} \cdot \mathbf{r})} \end{aligned} \qquad (7)$$

where Δ_T denotes the boundary tangential Laplace operator, $k=\omega/c_s$ is the wave number, $\mathbf{n}$ is the natural direction vector for investigated circular duct, and wave vector is defined as $\mathbf{k}=k\mathbf{n_k}$, where $\mathbf{n_k}$ is the wave-direction vector. In equation (7), p_0 represents the applied outer pressure, and i denotes the imaginary unit [6]. The inlet boundary condition is valid as long as the frequency is kept below the cutoff frequency for the second propagating mode in the cylindrical duct.

At the outlet boundary is set as the radiation boundary condition which allows an outgoing wave to leave the modeling domain with no or minimal reflections:

$$\mathbf{n}\cdot\frac{1}{\rho_0}\nabla p+i\frac{k}{\rho_0}p+\frac{i}{2k}\Delta_T p=0 \qquad (8)$$

The numerical model is computed by the use of finite element method (FEM) by the terms of the element size [7] and maximum element size equals $h_e=0{,}2(c_s/f_{max})$, where f_{max} is the value of maximum investigated frequency (in this paper f_{max}=2kHz). The longitudinal dimension of circular ducts is calculated as infinite by the use of radiation boundary conditions.

Acoustic attenuation properties of absorptive materials inside round silencer are estimated by the use of the well-known model of Delany and Bazley [8] for complex impedance Z_c and complex wave number k_c:

$$Z_c=\rho_0 c_s\left[1+C_5\left(\frac{\rho_0 f}{R_f}\right)^{-C_6}-iC_7\left(\frac{\rho_0 f}{R_f}\right)^{-C_8}\right] \qquad (9)$$

$$k_c=\frac{\omega}{c_s}\left[1+C_1\left(\frac{\rho_0 f}{R_f}\right)^{-C_2}-iC_3\left(\frac{\rho_0 f}{R_f}\right)^{-C_4}\right] \qquad (10)$$

with original coefficients: C_1=0,0978, C_2=0,7, C_3=0,189, C_4=0,595, C_5=0,0571, C_6=0,754, C_7=0,087, C_8=0,732.

Whereas, for glass-wool-like materials the empirical correlation of Bies and Hansen [9]

$$R_f=\frac{3{,}18\cdot 10^{-9}\cdot\rho_{ap}^{1{,}53}}{d_{av}^{2}},\left[\frac{Pa\cdot s}{m^2}\right] \qquad (11)$$

is used to achieve flow resistivity R_f, where ρ_{ap} [kg/m^3] denotes apparent density of absorptive material, and d_{av} [m] is average fiber diameter.

4. Results

Three acoustic systems were executed to obtain acoustic attenuation performance of helicoidal resonator due to distance change from different cross-sectional elements of cylindrical duct: with expansion chamber, round silencer and second helicoidal resonator. The results are presented as the TL [dB] and ΔTL [dB] in the first frequency range from 10Hz to 2kHz with computational step of 5Hz, and the second range from 1,1kHz to 1,5kHz with the computational step of 1Hz. The helicoidal resonators ratios are constant in all cases and equals:

$$s/d=2, \quad g/d=0{,}04, \quad d_m/d=0{,}24. \qquad (12)$$

4.1 Expansion chamber

In Figure 4 is presented example view on investigated expansion chamber, which is 20cm long and it has 30cm in diameter, with helicoidal resonator at the inlet. The inlet and outlet cylindrical ducts have the same diameter d=12,5cm. Different cases of distance change between helicoidal resonator and expansion chamber are executed.

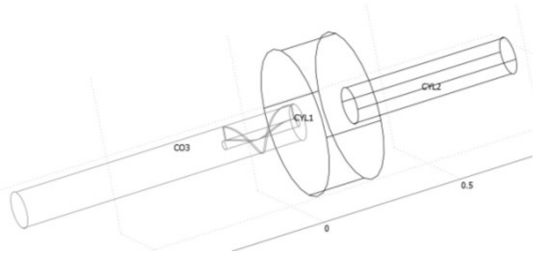

Figure 4. Example view on expansion chamber (50cm in diameter and 20cm long) with helicoidal resonator at the inlet.

In Figure 5 is presented TL characteristics of expansion chamber, which is the initial characteristics (TL_{ini}) to obtain increase of TL for different cases of acoustics systems with helicoidal resonator.

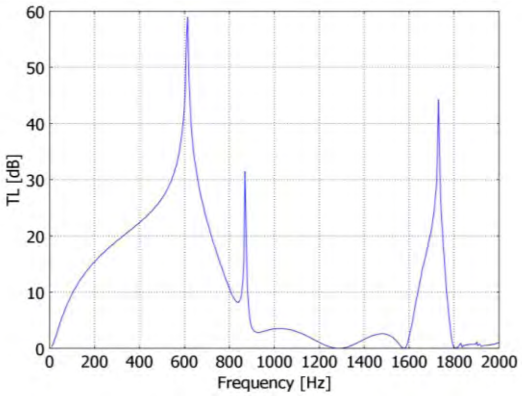

Figure 5. Transmission loss characteristics of expansion chamber - 20cm long and 30cm in diameter.

In Figure 6 are presented ΔTL characteristics for distances from 0cm to 90cm with the step of 10cm between helicoidal resonator and expansion chamber.

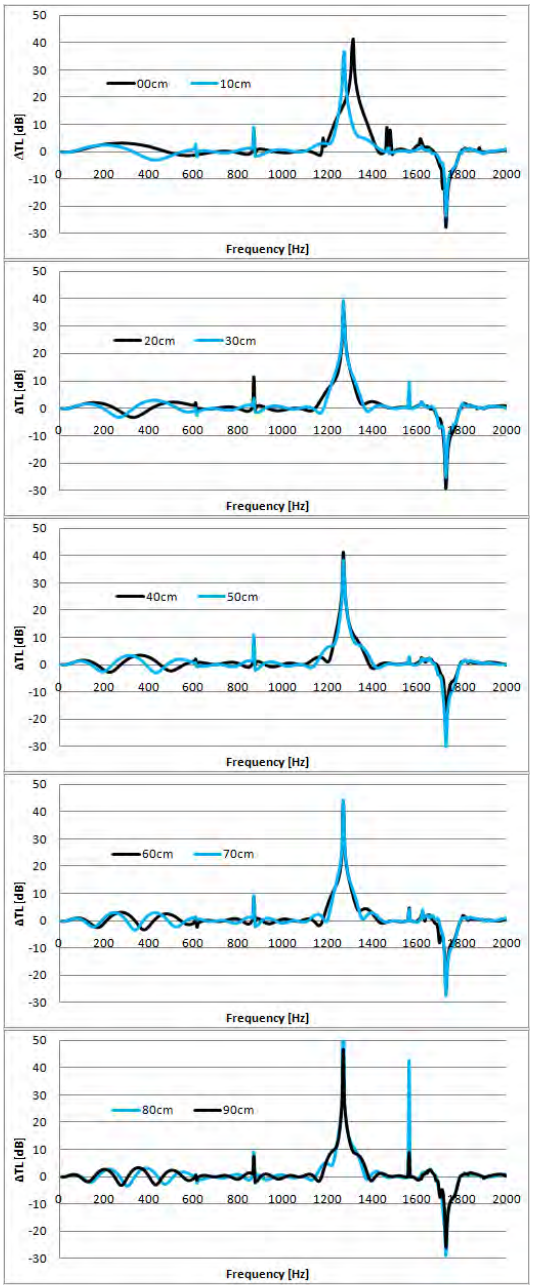

Figure 6. Increase of transmission loss characteristics for different distances from 0cm to 90cm with the step of 10cm between helicoidal resonator at the inlet and expansion chamber .

4.2 Round silencer

The acoustic attenuation performance of helicoidal resonator due to distance change from round silencer is investigated. In Figure 7 is presented example view on round silencer (58cm long, 34cm of external diameter, 12,5cm of internal diameter) with helicoidal resonator at the inlet. The inlet and outlet ducts have diameter *d*=12,5cm. Also the internal diameter of round silencer and diameter of inlet and outlet ducts are identical.

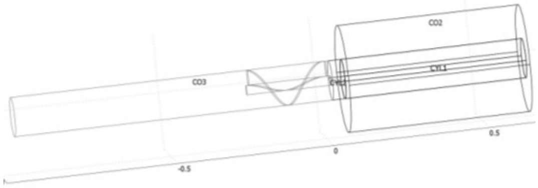

Figure 7. Example view of round silencer with helicoidal resonator at the inlet.

Figure 8 shows the initial transmission loss characteristics of a round silencer with glass-wool like material (filled in volume between diameters 12,5cm and 34cm) of apparent density ρ_{ap}=75 kg/m^3 and average fiber diameter d_{av}=8*10^{-6} m. The acoustic attenuation of this porous absorptive material were obtained by the use of equations (9), (10) and (11).

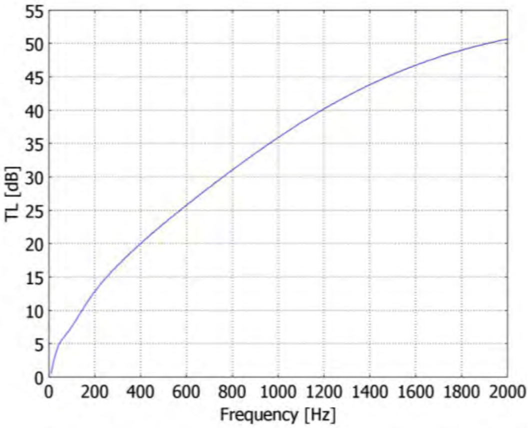

Figure 8. Initial transmission loss characteristic of a round silencer.

In case that the main changes of TL characteristics obtain near the resonance frequency f_r [Hz] of helicoidal resonator the range between 1,1kHz and 1,5kHz with the step of 1Hz is investigated here.

The TL characteristics of acoustic systems with distance change between helicoidal resonator and round silencer are presented in Figure 9.

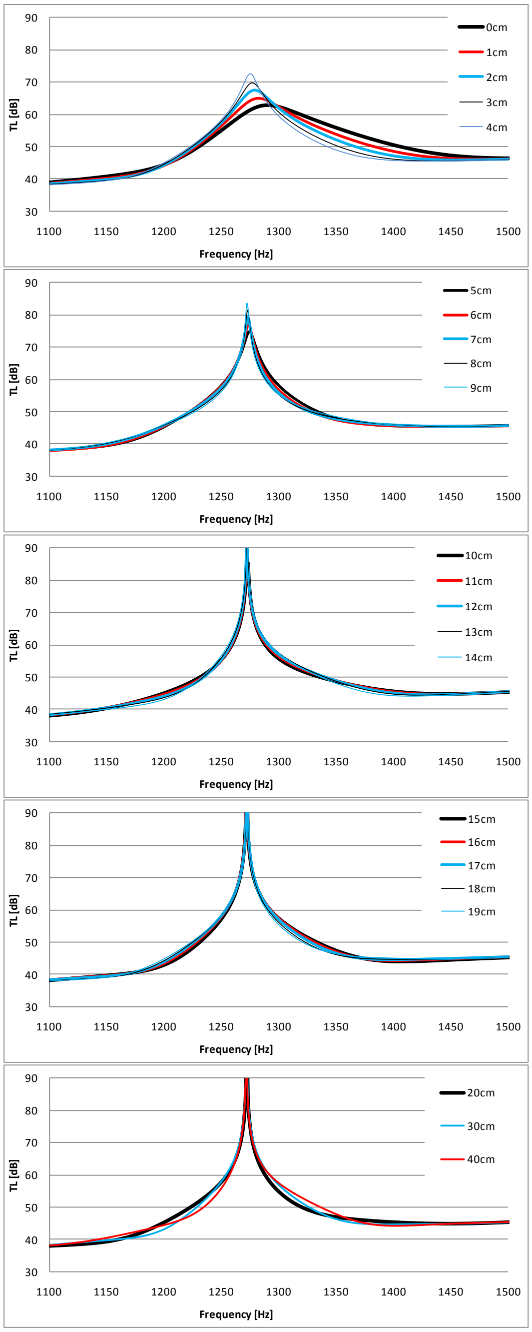

Figure 9. Transmission loss characteristics of acoustic systems with distance change in [cm] between helicoidal resonator at the inlet and round silencer.

4.2 Second helicoidal resonator

The acoustic attenuation performance of acoustic systems consisting two helicoidal resonators due to distance change between them is investigated. In Figure 10 is presented example view of two helicoidal resonators placed inside cylindrical duct of diameter *d*=12,5cm.

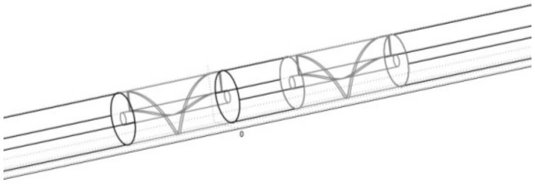

Figure 10. Example view of two helicoidal resonators placed inside cylindrical duct.

In Figure 11 are presented TL characteristics of acoustic systems with two helicoidal resonators due to distance change between them in [cm].

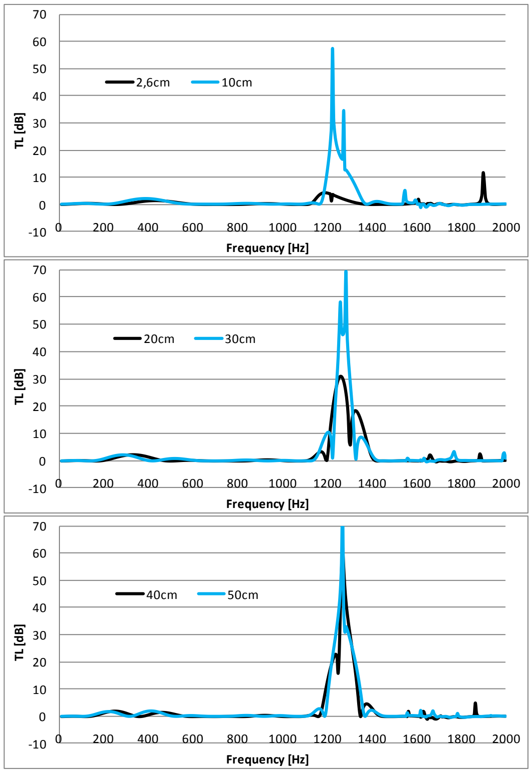

Figure 11. Transmission loss characteristics of acoustic systems with two helicoidal resonators inside cylindrical duct due to distance change between them in [cm].

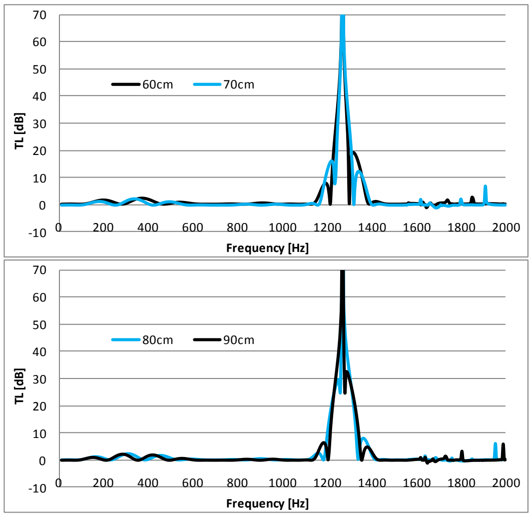

Figure 11. Continuation.

It can be observed from Figure 11 that the helicoidal resonators can't be placed close to each other because they can lost their acoustic attenuation properties. If the distance between them grows up the sound attenuation becomes higher. But also we can observe the specific influence on sound attenuation for different distances between helicoidal resonators. Also there can be observed the influence on low frequency sound attenuation. When the distance between helicoidal resonators is small we observe small changes, lower than 5dB, in low frequencies. When the distance grows up we observe more changes of attenuation at lower frequencies. It can be compared to results in first case, expansion chamber, where we observed similiar changes in low frequencies due to distance change between cross-sectional elements of cylindrical duct.

5. Conclusions

The acoustic attenuation performance of helicoidal resonator due to distance change from three different cross-sectional elements of cylindrical duct was considered: first case - expansion chamber, second case - round silencer, third case - second helicoidal resonator.

In first case, it can be observed in Figure 5 that the TL of investigated expansion chamber in frequency range from 900Hz to 1600Hz is small. The helicoidal resonator placed at the inlet of expansion chamber mainly increases the sound attenuation. This increase is high especially in range of frequency from about 1200Hz to about 1400Hz. Also the resonance frequency f_r [Hz] of helicoidal resonator changes due to distance change from expansion chamber. The highest f_r=1315Hz can be achieved with the distance of 0cm, due to change of 10cm it equals 1275Hz. In distance 20cm it is 1270Hz and there is visible small change till 90cm (about 1265Hz). It can be also observed, in Figure 6, that the distance change of helicoidal resonator from expansion chamber influences on attenuation in lower frequencies, but the attenuation change is not higher than ±5dB, and the longer distance it is the more changes in attenuation of lower frequencies are obtained.

At the second case, the acoustic attenuation performance of round silencer with helicoidal resonator at the inlet due to distance change between them strongly differs at the distance from 0cm to 5cm. There is visible in Figure 9 that the helicoidal resonator can't strongly resonate if placed directly at the inlet of a round silencer. But in this case the range of attenuated sounds in the frequency domain is larger than for example 20cm before the silencer. Also it depends on what is more needed, stronger resonance or wider range of attenuated sounds.

For third case, there is visible strong minimization of sound attenuation of two helicoidal resonators when placing them one by one. The range of attenuated sounds in frequency domain doesn't change a lot in fact that two helicoidal resonators are placed inside the duct. But it could change when the helicoidal resonators would have different s/d ratios.

Presented results shows that helicoidal resonator can be an effective additional sound attenuation element for ducted systems. This research work doesn't present all spectrum of possible use of this solution, also further research work should be undertaken.

6. References

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