Thermomagnetic Siphoning on a Bundle of Current-Carrying Wires

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Abstract: Using COMSOL Multiphysics 3.5a, thermomagnetic siphoning (TMS) was shown to be a sufficient manner of regulating the temperature of a bundle of current-carrying wires wrapped with a magnetorheological fluid (MRF) jacket. As the bundle heated up, cooler MRF on the outside of the jacket was drawn towards the center due to Curie's Law and the induced magnetic field. The process convected heat from the bundle as the MRF warmed up and was pushed out towards an isothermal jacket wall. Assuming an outer jacket diameter of 6 mm and a bundle diameter of 1 mm, COMSOL's Fluid Flow, Heat Transfer, and Magnetostatics modules showed a significant reduction in the steady-state bundle temperature. The passive thermal management technique would be beneficial in computer or space applications where temperature regulation is a concern, but actual fabrication would be difficult.

Keywords: Curie's Law, magnetic fluids, thermomagnetic siphoning.

1. Introduction

Similar to gravity-induced buoyancy, temperature differences in a paramagnetic fluid can result in bulk motion when in the presence of a magnetic field. The effect is the result of a change in its temperature-dependent intrinsic properties [1]; however, instead of density, thermomagnetic siphoning (TMS) results from a change in the magnetic susceptibility of the fluid. A cooler fluid is more susceptible to magnetism and is pulled towards an increasing field, just as a cooler, heavier fluid would be pulled in the direction of higher gravity.

The phenomenon has been used for regulating the temperature of voice coils [2] and other heat-producing devices, but with increasing demands on space and computing systems, TMS should also be investigated for its cooling effect on a bundle of current-carrying wires. The coupled magnetic, fluid, and thermal interactions are highly complicated; thus COMSOL Multiphysics 3.5a was used to study the temperature regulation under varying current loads for a theoretical magnetorheological fluid (MRF).

2. Theory

A current-carrying wire generates heat due to its resistance and, typically, wires are bundled due to the necessity of the application in which they are used. Each wire within the bundle contributes to the heat load and, if high current levels were drawn, would result in very high temperatures. However, the current also induces a magnetic field through electromagnetic induction, and, as distance from the bundle increases, the magnetic field strength decreases along with the temperature. A magnetic fluid jacket wrapped around the bundle of wires would benefit from TMS as a method for cooling the bundle through convection as shown in Figure 1.

Curie's Law states that the magnetic susceptibility, χ, of a paramagnetic fluid increases as its temperature, T, decreases, as shown in Figure 2. Considering a simple Curie type paramagnetic fluid, its volumetric susceptibility can be calculated as a function of the material's Curie constant, C, as,

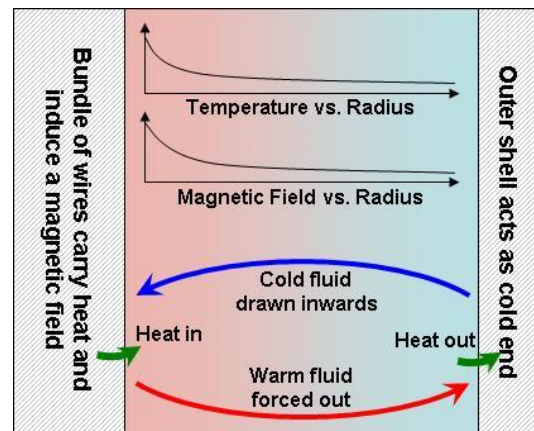$$\chi = \frac{C}{T} * \frac{\rho}{MW} * 4\pi, \qquad (1)$$

Figure 1. Thermomagnetic siphoning enhances cooling of a bundle of current-carrying wires by continually drawing in cool fluid.

where ρ is the fluid density and MW is molecular weight. The influence of susceptibility on the magnetic body force can be seen through a derivation of the Kelvin force density [3], f_m, as,

$$f_m = \mu_0 \, M \cdot \nabla \, H, \qquad (2)$$

where μ_0 is the permeability of free space, M is the magnetization vector, and H is the applied field vector. In the linear portion of the Langevin function, volumetric magnetic susceptibility is the ratio of the magnetization vector to the applied field vector,

$$\chi = M/H. \qquad (3)$$

By substituting for M, using the vector identity,

$$H \cdot \nabla H = \nabla \left(H \cdot H \right)/2 - H \times \left(\nabla \times H \right), \qquad (4)$$

and noting that Ampere's Law cancels out the curl of the applied field, Eq. (2) can be reduced to

$$f_m = \mu_0 \chi \nabla H^2 / 2; \qquad (5)$$

thus, a lower temperature results in a greater magnetic body force.

Within the jacket, the MRF magnetizes due to the applied field and generates a magnetic flux density as,

$$B = \mu_0 \left(H + M \right). \qquad (6)$$

COMSOL solves for a magnetic vector potential, A, such that the magnetic flux density can also be defined as the curl of the magnetic vector potential, $B = \nabla \times A$. In two dimensions, A only has a z-component; thus, its curl can be written in index notation as,

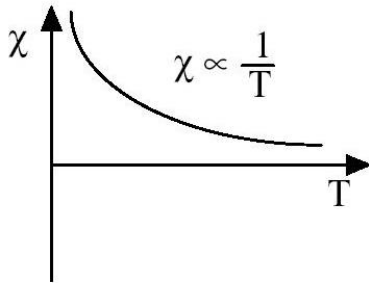**Figure 2**. Curie's Law explains that susceptibility of a paramagnetic fluid is inversely proportional to temperature.

$$\nabla \times A = \begin{matrix} A_{zy} \\ -A_{zx} \\ 0 \end{matrix} . \qquad (7)$$

Using Eqs. (3), (6), and (7), H can be written in terms of A as,

$$H = \frac{\nabla \times A}{\mu_0 \left(1 + \chi\right)} = \frac{1}{\mu_0 \left(1 + \chi\right)} \begin{matrix} A_{zy} \\ -A_{zx} \\ 0 \end{matrix} . \qquad (8)$$

Finally, when H is substituted into Eq. (5), the Kelvin force density becomes,

$$f_m = \frac{\chi}{2\mu_0 \left(1 + \chi\right)^2} \begin{matrix} \frac{d}{dx}\left(A_{zy}^{\;2} + A_{zx}^{\;2}\right) \\ \frac{d}{dy}\left(A_{zy}^{\;2} + A_{zx}^{\;2}\right) \\ 0 \end{matrix}, \qquad (9)$$

and, after performing the differentiation, a form of the equation is found that can be directly input into COMSOL as,

$$f_m = \frac{\chi}{\mu_0 \left(1 + \chi\right)^2} \begin{matrix} A_{zy}A_{zxy} + A_{zx}A_{zxx} \\ A_{zy}A_{zyy} + A_{zx}A_{zxy} \\ 0 \end{matrix} . \qquad (10)$$

The magnetic force is actually a body force per volume and can be added to the Navier-Stokes equations that COMSOL solves in its Weakly Compressible Navier-Stokes (WCNS) mode. The equation set has been used in a COMSOL tutorial [4] and actual application [5].

3. Use of COMSOL Multiphysics

COMSOL Multiphysics 3.5a provides a useful means to couple the necessary equations of the complicated TMS process. The magnetic, thermal, and fluid interactions could be studied in two dimensions for a variety of current loads without significant difficulty for the user.

3.1 Initial and Boundary Conditions

A bundle of current-carrying wires is at risk of overheating, but could be cooled with TMS if a magnetic fluid jacket is wrapped about it. The scenario studied uses a circular core to simulate a bundle of 16 wires of 32 gauge each. Rather than model each wire, the core is assumed to be uniform with the equivalent area giving it a diameter of 1 mm. The outer diameter of the magnetic fluid jacket is 6 mm and assumed

isothermal at 300 K. The problem studied employed a theoretical MRF whose properties are the same as water, but with a non-negligible Curie constant.

By assuming all the power, P, in the wires was converted to heat, and each wire had a finite resistance, R, as,

$$R = \frac{\rho L}{\pi r^2},\qquad(11)$$

where ρ is electrical resistivity, L is the length of the wires, and r is the radius of a single wire, the total heat load could be calculated as the combined specific heat load from each of the 16 wires in the bundle,

$$P = 16 * I^2 R,\qquad(12)$$

where, I is the current through a wire. Eq. (12) can be converted into a term for heat generation per volume, q, by dividing by the cross-sectional area and length of the entire bundle as,

$$q = \frac{P}{16\pi r^2 L} = \frac{I^2 \rho}{\pi r^2{}^2},\qquad(13)$$

The expression for heat generation in Eq. (13) is used in the convection and conduction mode for the entire bundle.

The magnetic contribution of the wires is less influenced by the size of the bundle. The current density of each wire is

$$J = \frac{I}{\pi r^2},\qquad(14)$$

As seen, the current density is proportional to the square of the heat generation per unit volume. This relation is common throughout the bundle and is used in the Magnetostatics module. The MRF is considered to have a low dielectric constant and electrical conductivity; therefore, it cannot carry an electric charge, and Lorentz forces are negligible. Furthermore, because the magnetic permeability is relatively low compared to solids, the fluid motion can be considered to have a negligible effect on the disturbance of the field. This allows for steady-state computation of the magnetic field even when the fluid and heat transfer equations are solved as transient.

The fluid and bundle properties were set using the built-in Materials Library for water and copper. The fluid was non-isothermal; therefore, the velocity field was affected by the temperature

dependency of the fluid's density, viscosity, conductivity, and specific heat capacity. When the MRF was simulated, the Curie constant was given a non-negligible value; when the non-magnetic fluid was simulated, it was set to 0.

3.2 Mesh

The bundle and fluid jacket were meshed using the Advancing Triangle method with a maximum element size of 1.5e-4 along the bundle diameter. Figure 3 shows the resulting mesh and boundary conditions mentioned in the previous section

Table 1 lists the mesh statistics and number of degrees of freedom, using the three application modes mentioned previously.

3.3 Solver

With the non-magnetic fluid, the magnetostatics mode and WCNS mode are no longer necessary, as no magnetic coupling or fluid motion is instigated. The radial conduction calculation was solved for a range of currents from 1-10 A using the Parametric Sweep function to directly output the steady-state temperature of the bundle. The PARDISO linear solver proved to be effective and efficient at solving the model in an appropriate amount of time with a relative tolerance of 0.001.
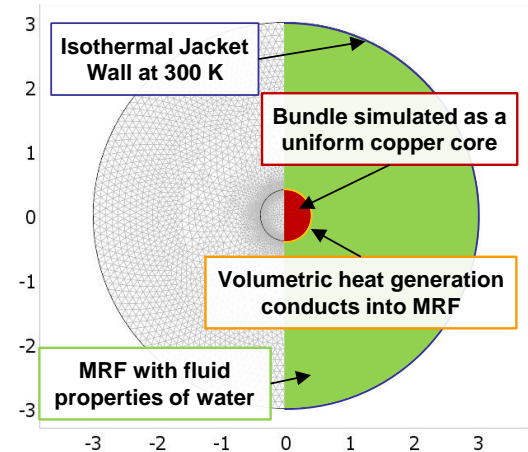

Figure 3. The copper core and MRF jacket are meshed with advancing triangles and given boundary conditions. Axes are in millimeters.
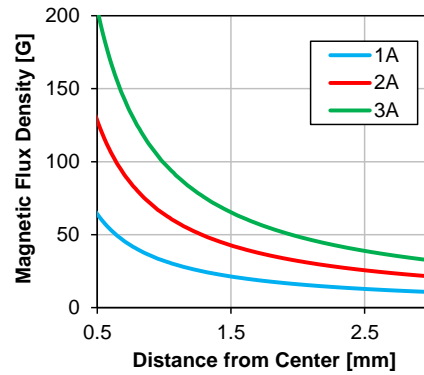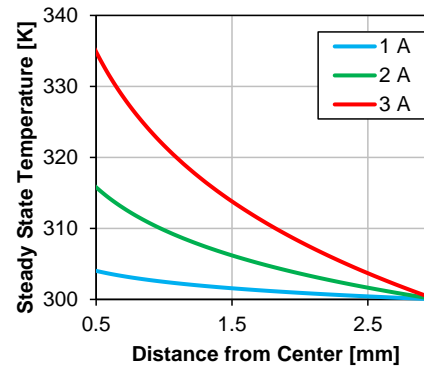
Table 1: Mesh and Solver Settings

Mesh Type:	Advancing Triangle
# of DOF's:	81593
# Elements:	10294
Min Element Qual.:	0.8326
Max Element Size:	1.47e-4
Min Element Size:	1.8e-5
Application Modes:	Magnetostatics
	Weakly Compressible Navier Stokes
	Convection and Conduction

Figure 4. Magnetic flux density decreases with distance from the bundle.

Figure 5. Steady-state temperature of a non-magnetic fluid decreases with distance from the bundle.

With the MRF, the computations are far more complex. Although the steady-state temperature of the bundle was the desired output, a steady-state analysis could not be performed. In such case, the fluid experienced an axisymmetric compression towards the center, negating any benefits of convective heat flow. Instead, a transient calculation was required and allowed to run for 300 sec to simulate a steady-state temperature. This method proved successful at generating eddy currents within the fluid, as would be expected in a real, physical experiment. Due to the large number of degrees of freedom, the magnetostatic equations were calculated first and saved as a stored solution. From this stored solution, a segregated time-dependent analysis was performed for the fluid and heat equations. Each segregated group still used the PARDISO linear solver with a relative tolerance of 0.001.

Higher currents generated more complicated fluid dynamics and required more time for analysis. For a 300 s run, the calculation time was approximately 20-100 minutes on a Pentium 4, 2.4 GHz machine with 1 GB of RAM.

4. Results

The magnetic flux density and temperature versus radius is shown within the fluid jacket for 1, 2, and 3 A of current for a non-magnetic fluid in Figs. 4-5. As seen, the temperature and magnetic field gradients correlated to the trends necessary for TMS shown in Fig. 1. When the non-magnetic fluid was replaced by a MRF, however, the fluid responded through bulk motion. A surface plot of the temperature and velocity vectors of the MRF is shown over time in Fig. 6 for 5 A of current.

As seen, thermal and velocity jets were formed within 6-9 s and maintained their structure indefinitely. With an isothermal boundary condition at the outer diameter of the jacket, the heat was transported from the bundle by convection. For the 5A case in Fig. 6, the maximum temperature of the bundle occurred at around 6 s, just before TMS was instigated. Fig. 7 shows the temperature over time for current loads of 5-10 A.

In each case, the temperature rose until its gradient caused enough of a difference in the MRF susceptibility and instigated TMS. Comparing the 5 A case in Fig. 7 with the surface plots in Fig. 6, the increase in temperature up to 6 s correlates to a purely conductive mode of heat transfer until the onset of TMS. From 6 s to 9 s, the drop off in temperature was due to TMS generating forced convection to the 300 K wall. The maintained structure of the jets continually rejected heat and the bundle rose to a steady state temperature.
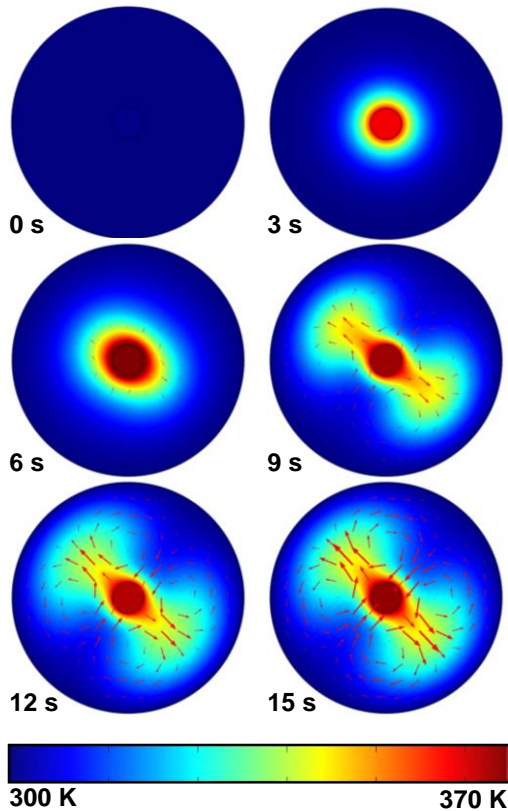
Figure 6. Cooling of a bundle of wires can be enhanced through thermomagnetic siphoning.
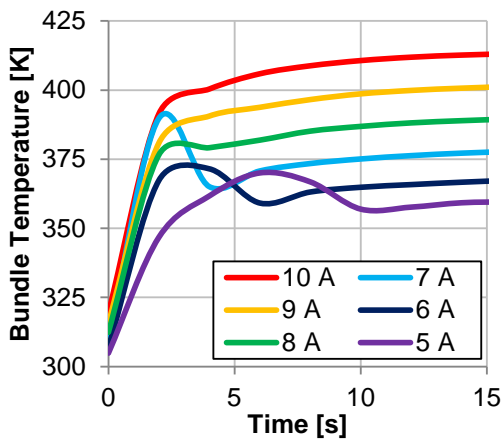

Figure 7. A peak in temperature indicates the onset of TMS.

For currents below 5 A, the heat generated was not enough to have a significant effect on the magnetic susceptibility and was only rejected by conduction through the MRF (this was confirmed by comparing it to the temperature versus time of the non-magnetic fluid). As the current increased, the onset of TMS occurred earlier as seen through the peaks of the cases in Fig. 7, except for the 7 A case which may have had a delayed onset due to meshing or other solver parameters. At 10 A, the peak and drop-off of temperature were less distinguishable due to the high heat generation. Despite this, significant benefits to the steady-state bundle temperature were still present.

Running a stationary case with the MRF resulted in an axisymmetric pressure profile which did not generate TMS. Instead, the simulations were run as transient to 300 s when the change in temperature of the bundle over time was less than 0.001%. This represented a steady-state approximation as the bundle temperature was very near its asymptotic value. Fig. 8 compares the steady-state temperature of the center of the bundle for the MRF and a non-magnetic (NM) fluid as the current increased.

As mentioned, currents of 1-4 A did not generate enough heat, and the cooling was achieved purely by conduction, identical to a NM fluid. As the current increased, however, the benefits of TMS were clear. Higher currents generated higher velocity jets, and the convective cooling was increased. Thus, while the steady-state temperature using the NM fluid increased parabolically, the MRF fluid maintained a seemingly linear profile. At 10 A, the difference in the bundle steady-state temperatures is 325 K between the NM fluid and the MRF and would be even greater at higher currents.
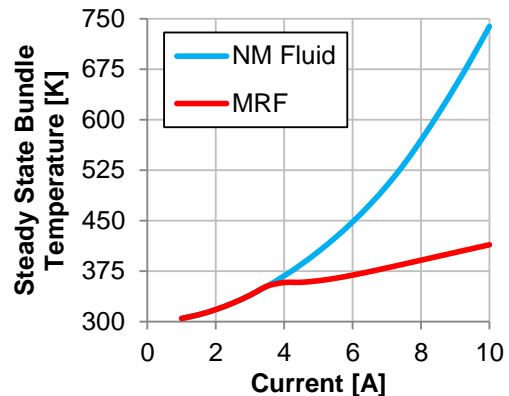

Figure 8. TMS becomes more beneficial at higher current loads.

While the studies performed suggest that TMS should be used for high power cables, actual application of a physical model would be extremely difficult due to operability, manufacturing, and affordability. The analytical model presented is primarily intended to demonstrate the benefits TMS could have on heat-producing components and does not factor in phenomena such as boiling or electrostatic discharge. Nonetheless, TMS could be useful to space applications where gravitational buoyancy is impossible and to computer applications where high heat and current loads are present.

5. Conclusions

Thermomagnetic siphoning was studied for its cooling performance on a bundle of current-carrying wires using COMSOL Multiphysics 3.5a. The software efficiently solved the magnetic, thermal, and fluid equations in two dimensions and allowed for a thorough comparison of a magnetorheological fluid versus a non-magnetic fluid. The benefits of TMS were shown through a significant reduction of the steady-state temperature of the bundle for high current loads. While the results suggest that TMS should be employed for space and computer applications, actual fabrication of a magnetic fluid jacket may negate any benefits due to additional cost and complexity.

6. References

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7. Acknowledgements