Simulating Frequency Nonlinearities in Quartz Resonators at High Temperature and Pressure

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Outline

- Overview of Quartz Pressure Sensors and Purpose of Current Work
- Governing Equations of Piezoelectricity and the Incremental Method
- Use of COMSOL Multiphysics
- Simulation Results
- Conclusions

Quartz Resonators as Temperature and Pressure Sensors

G. Kirikera, W. Patton, and S. Behr, "Modeling Thickness Shear Mode Quartz Sensors for Increased Downhole Pressure and Temperature Applications," in *COMSOL Conference 2010*, Boston

- End Cap Pressure Sensor Design (current study)
  - end caps made of quartz
  - relates external hydrostatic pressure as biaxial stress to resonator
  - not temperature independent
- Used as downhole sensor in oil and gas Industry
  - Current study to 200°C/20kpsi

Purpose of Current Work

- Create an accurate finite element model of a quartz resonator that captures the nonlinearities in frequency response associated with changes in pressure and temperature
  - Doing so will aid in the design of quartz sensors for the oil and geothermal industries by enabling efficient exploration into the effects of untested cuts, geometry, and environmental variables
  - The process will also shed light on the material science behind the complex anisotropic and nonlinear nature of quartz’s electro-mechanical properties, as well as their interaction with design parameters at elevated temperature and pressure

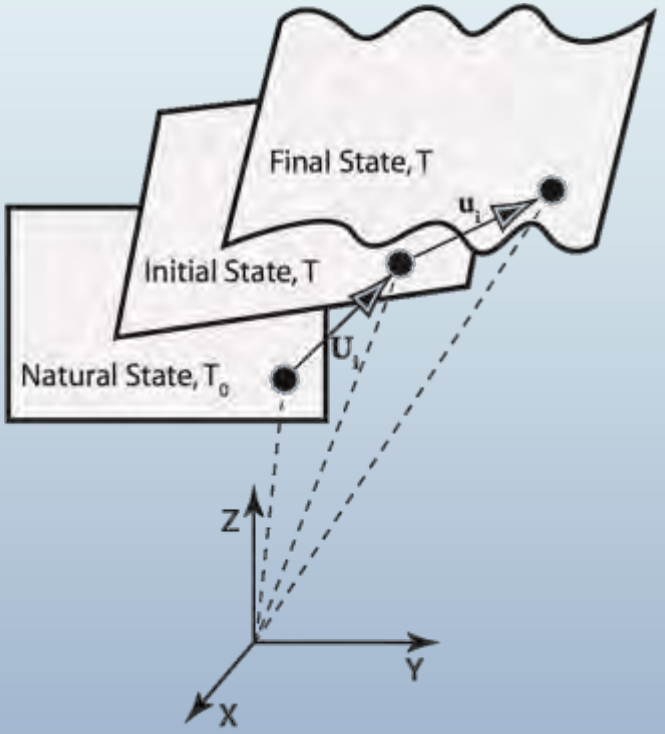
Previous Related Work

- Mihir S. Patel [1]: Developed a three-dimensional finite element model of quartz frequency response for looking at general trends and new glass packaging techniques
  - does not examine response to high external pressures
- Errol P. EerNisse [2]: Utilized superposition of empirical data to accurately predict the frequency response of a pressure sensor similar to the one in the present work
  - empirical method requires that good experimental data already exist for the cut angle and base geometry in question; method was not a direct finite element model of frequency response

[1] M. S. Patel, *Nonlinear Behavior in Quartz Resonators and its Stability*. Dissertation, Rutgers University, 2008.
[2] E. P. EerNisse, “Theoretical modeling of quartz resonator pressure transducers,” in *41st Annual Symposium on Frequency Control*. 1987, pp. 339–343, 1987.

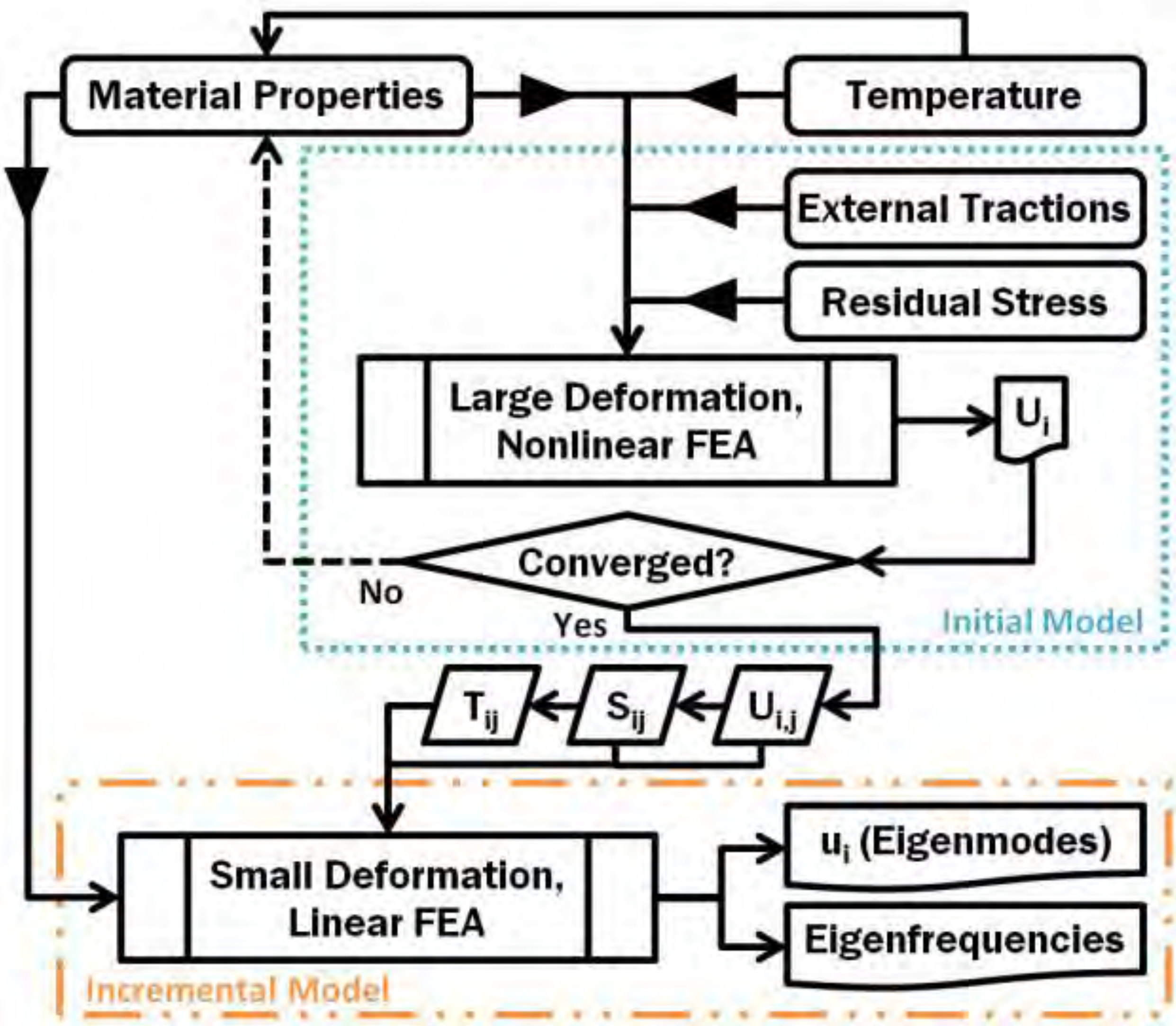
Lee-Yong Incremental Method

- Instead of defining a single displacement defined between two states (natural and final), displacement is divided into two stages between three states
- Defines total displacement as sum of "initial" (U_i) and "incremental" (u_i) parts
- Nonlinear response to pressure and temperature confined to "initial" displacement; piezoelectric vibrations confined to "incremental" displacement
- Linear, small-deformation waves are superposed as "incremental" displacements on the "initial" response



P. C. Y. Lee and Y. K. Yong, "Frequency-temperature behavior of thickness vibrations of doubly rotated quartz plates affected by plate dimensions and orientations," *Journal of Applied Physics*, vol. 60, no. 7, pp. 2327–2342, 1986.

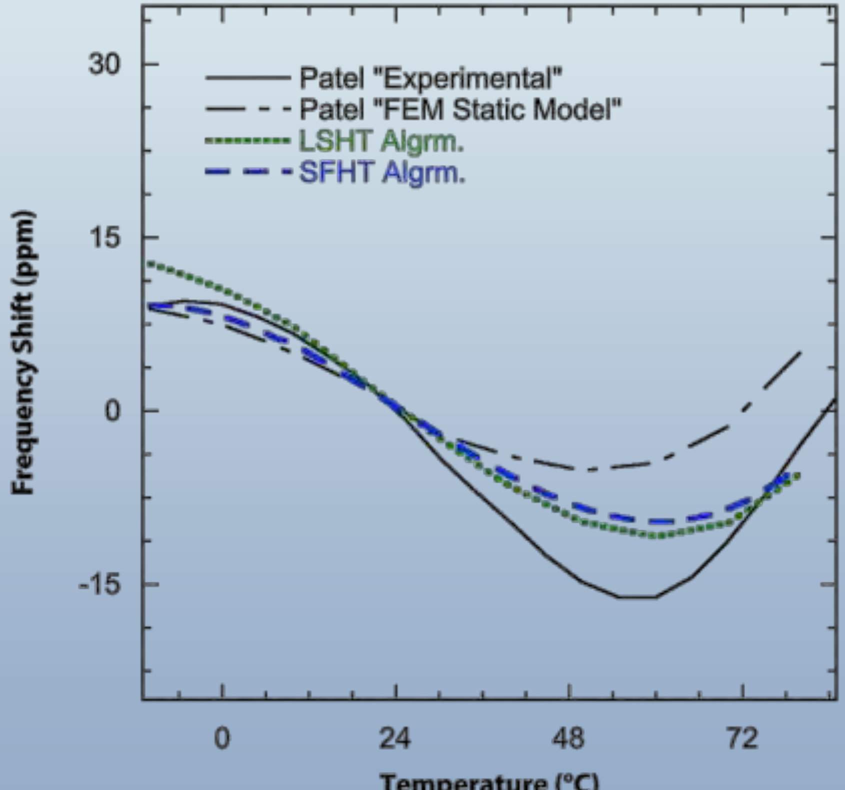
Nonlinear Stressed Homogeneous Temperature (NSHT) Algorithm



Benchmarking

Stress-free frequency-temperature response of rectangular AT-Cut quartz resonator

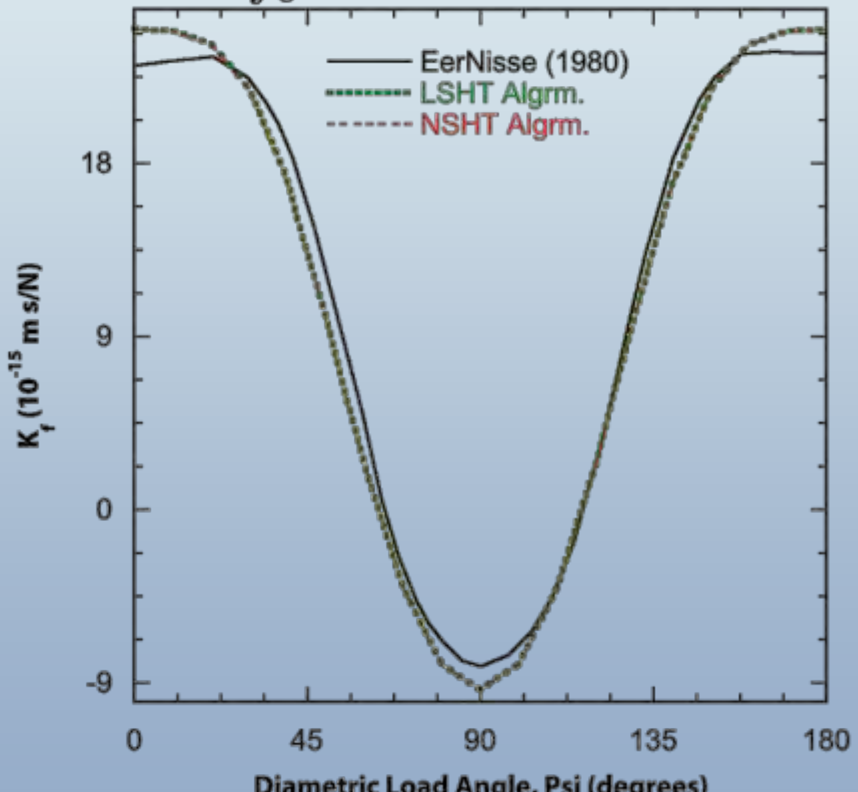
$$\text{Frequency Shift} = \frac{f - f_0}{f_0}$$



Baseline data from M. S. Patel, *Nonlinear Behavior in Quartz Resonators and its Stability*. Dissertation, Rutgers University, 2008.

Force frequency coefficient, K_f, for the round AT-Cut under diametric load

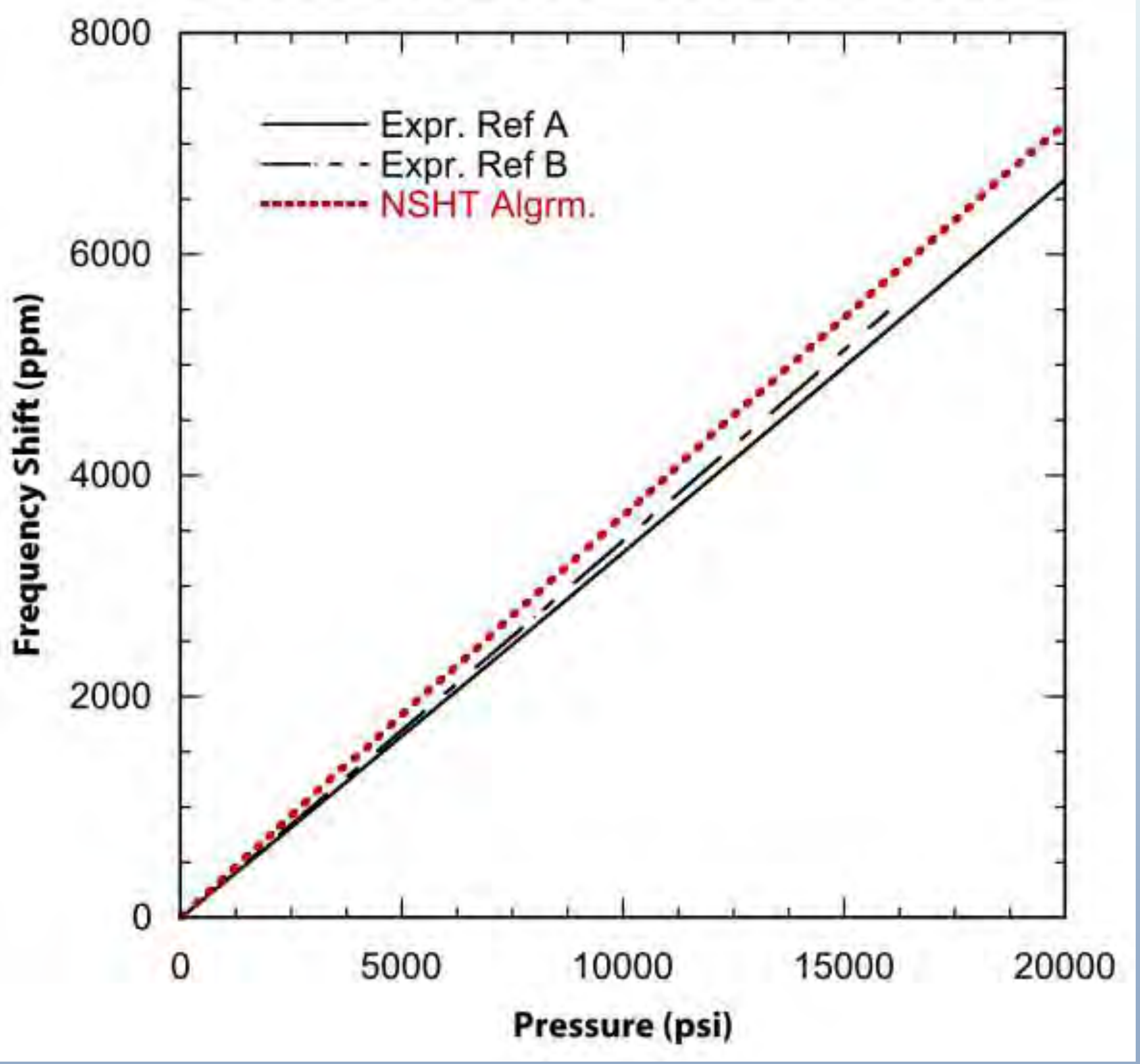
$$\frac{f - f_0}{f_0} = K_f \frac{F \cdot N_0}{d \cdot \tau}$$



Baseline data from E. P. EerNisse, "Temperature dependence of the force frequency effect for the at- , fc-, sc-, and rotated x-cuts," in *34th Annual Symposium on Frequency Control. 1980*, pp. 426–430, 1980.

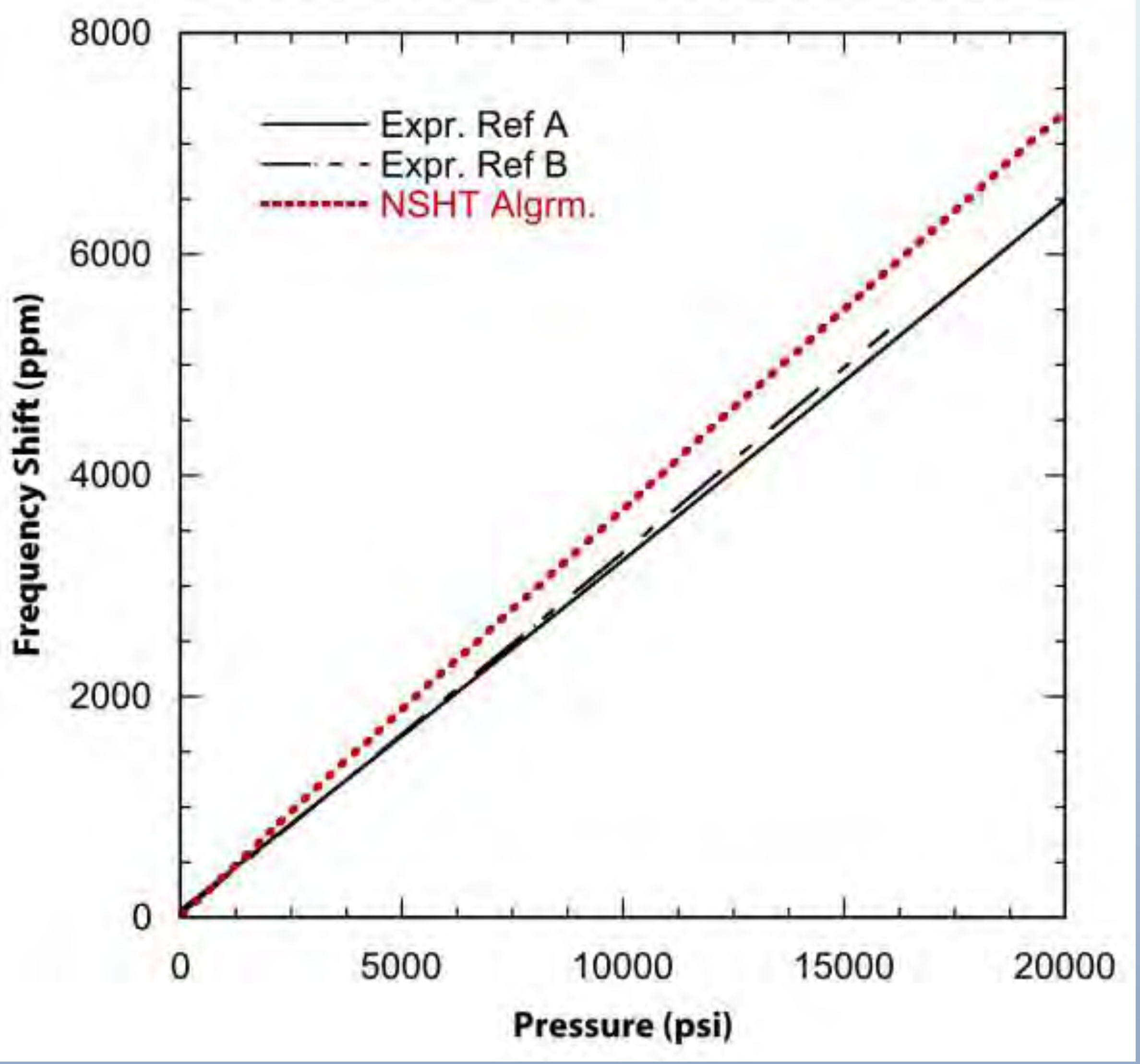
Results: Isothermal Frequency-Pressure Response

Temperature = 50°C



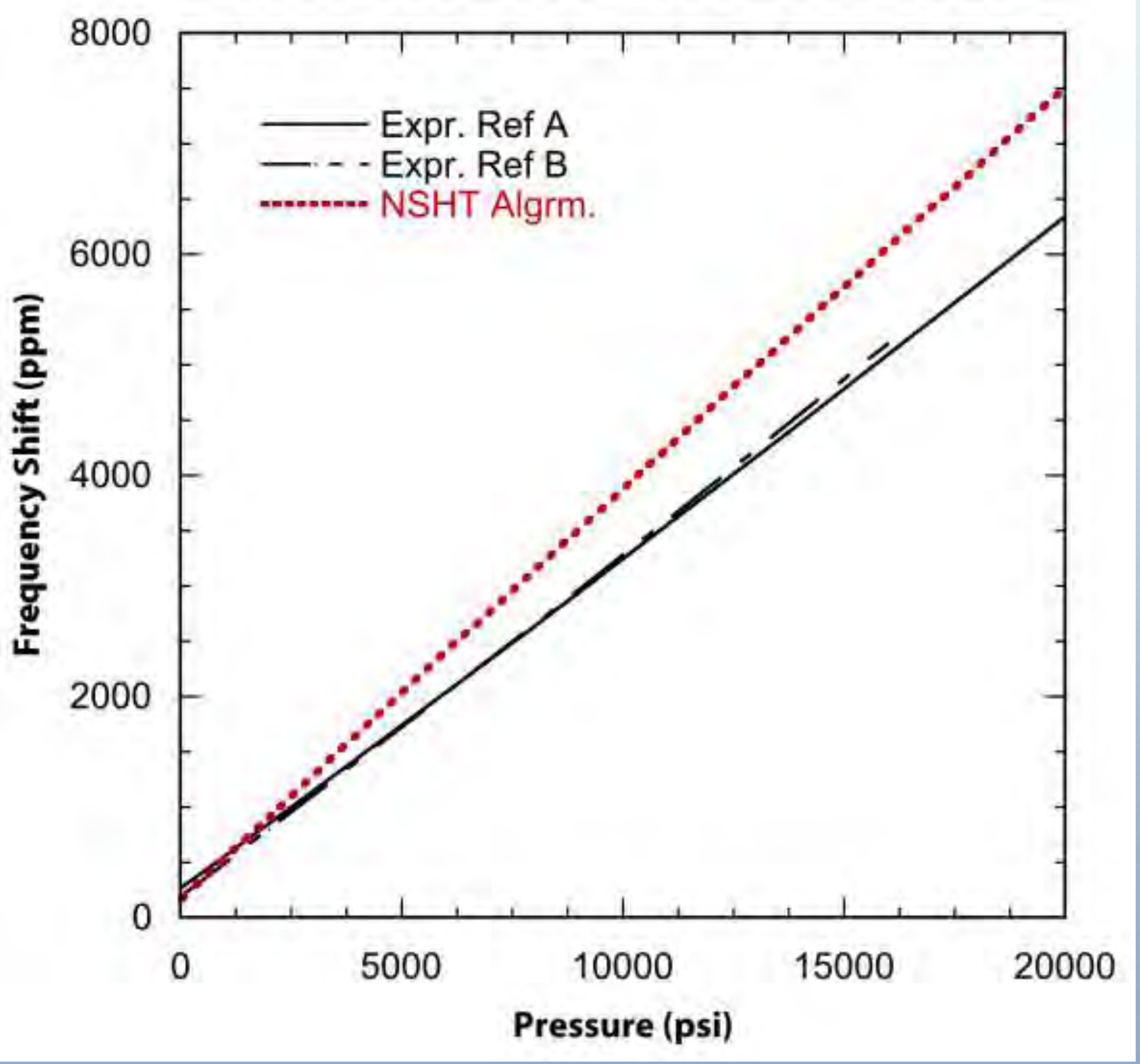
Results: Isothermal Frequency-Pressure Response

Temperature = 100°C



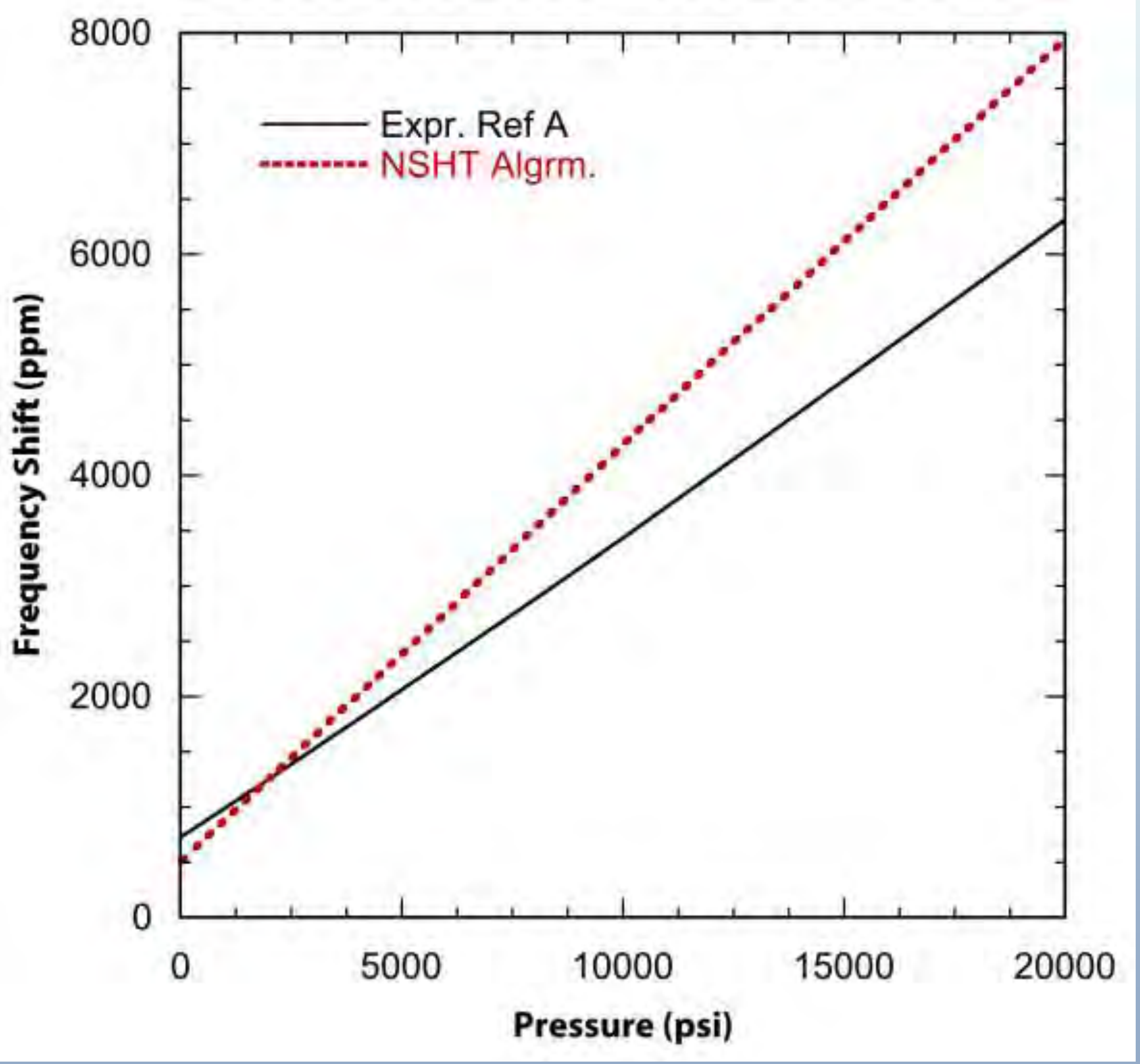
Results: Isothermal Frequency-Pressure Response

Temperature = 150°C



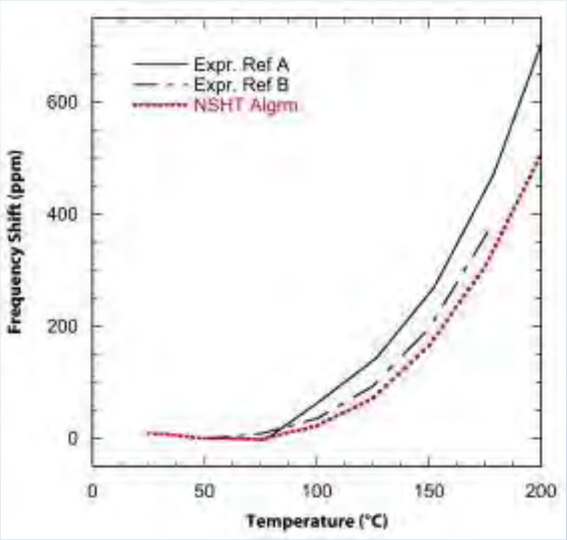
Results: Isothermal Frequency-Pressure Response

Temperature = 200°C

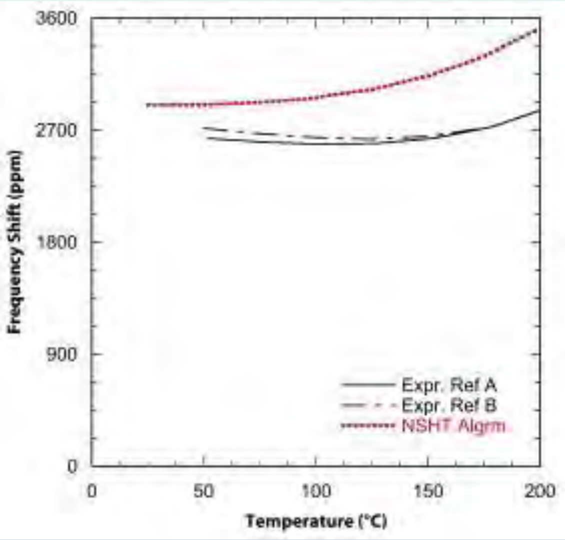


Results: Isobaric Frequency-Temperature Response

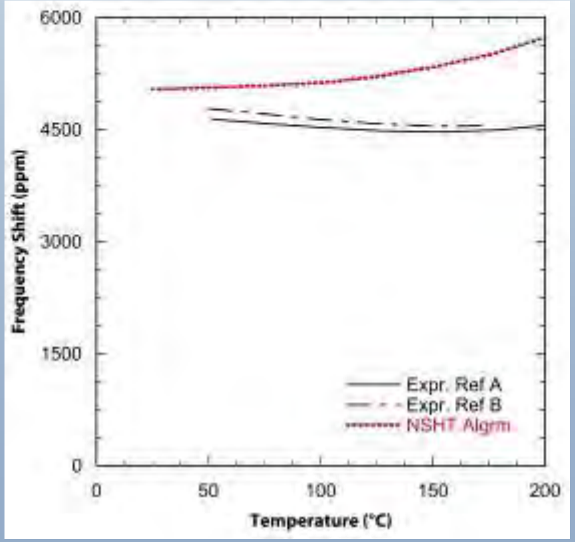
Pressure = 14 psi



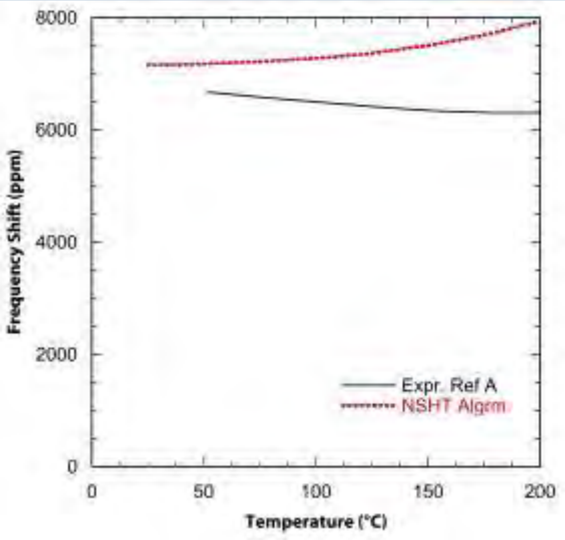
Pressure = 8,000 psi



Pressure = 14,000 psi



Pressure = 20,000 psi



Results Discussion

- The linear isothermal frequency-pressure response agrees very well with experimental data for low temperatures, but starts to diverge as temperatures increase
  - Maximum error compared to experimental values on a single isothermal plot grows from 7.5% at 50°C and 20,000 psi to 25% at 200°C and 20,000 psi
- Similarly for the isobaric plots, frequency-temperature response at low pressures shows the characteristic curve of the AT-Cut and is near experimental values, but at higher temperatures this likeness deteriorates
  - the experimental trends actually show inverted temperature response for high pressures, while the NSHT Algorithm never shows this negative slope

Conclusions

- On the whole the model performed fairly well, especially at low temperatures or low pressures; as temperature and pressure were increased, the shape of the experimental frequency curves changed in ways that the simulation could not predict.
- The temperature derivatives of the nonlinear third-order elastic coefficients, which are not currently available in literature, could be the primary source of the error in both temperature and pressure response.
- This idea is based on the fact that pressure response is almost entirely attributed to the third-order (or nonlinear) elastic coefficients.
- The results highlight the potential benefit to the material model of quartz that could be realized if future experimental work to define the temperature derivatives of third-order elastic coefficients is achieved.

Thank you!

Questions ?

Acknowledgement

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