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# Thermally Induced-Noise Reduction Using an Electrostatic Force Feedback

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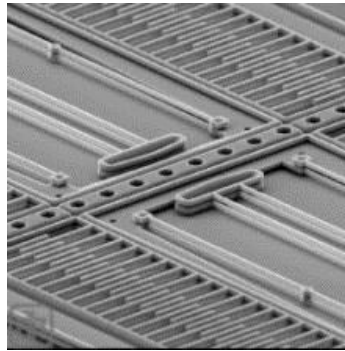
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# Introduction to MEMS

- MEMS gave versatile sensing solutions

- Gyroscope
- Accelerometer
- Bio-Sensors

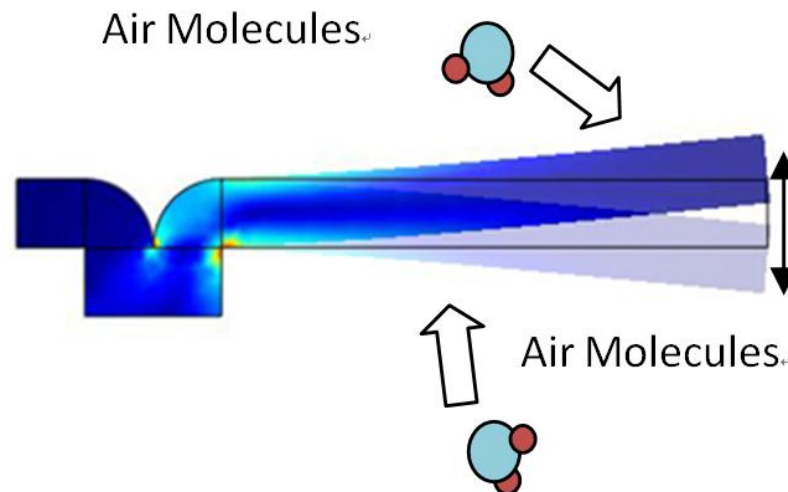


- MEMS have various advantages

- Low cost and high performance
- Small size

# Thermal Noise in MEMS

- Thermal Agitation
  - Caused by temperature fluctuation
  - Inconsiderable in macro-scale
  - Becomes significant in micro-scale



# Thermal Noise in MEMS

- Displacement of a mass-spring oscillator

$$\frac{1}{2}k\langle x^2 \rangle = \frac{1}{2}K_B T \quad [1]$$

k= spring constant

x= mean-square displacement

$K_B=1.38e-23$  J/K (Boltzmann's constant)

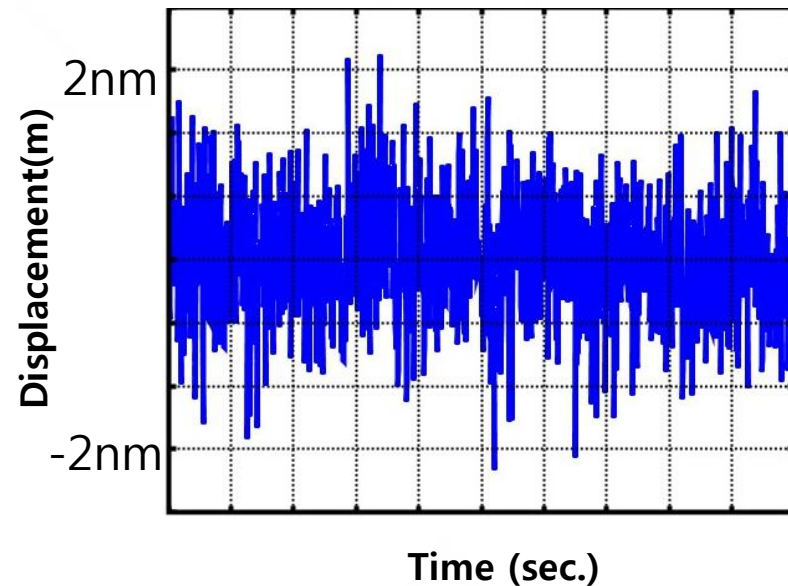
T= temperature

# Thermal Noise in MEMS

**Example)** At  $T=300\text{K}$ , a micro cantilever with an effective stiffness of  $k=1\text{e-}3[\text{N/m}]$  will have an expected displacement amplitude  $\langle x \rangle$  about  $\sim 2\text{nm}$ .

Thermal Noise

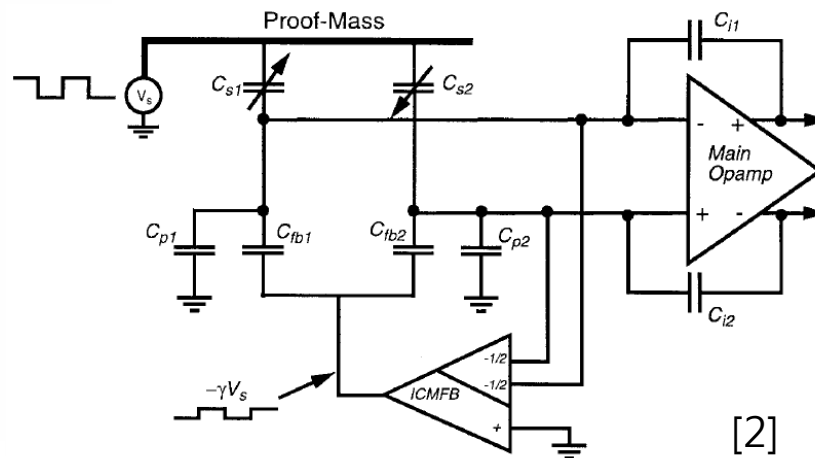
Not desirable for devices such as AFM which handles molecular scale measurements.



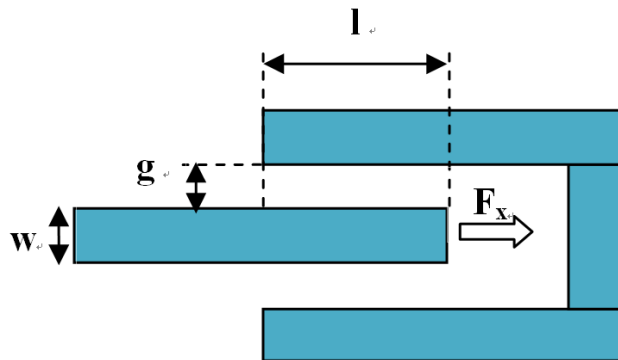
# Electrostatic Force Feedback

## Previous usages of force feedback

- Extend sensor bandwidth beyond  $\omega_0$ . [2]
- Nonlinearities in capacitive pickoff minimized [2]
- Decrease spring constant for high performance [2], [3]



# Electrostatic Force Feedback



$$F_{\text{electrostatic}} = N \epsilon_0 V^2 \frac{h}{g} \text{ [N]}$$

$$x = \frac{\epsilon_0 h N}{g \cdot k_x} V^2 \text{ [m]}$$

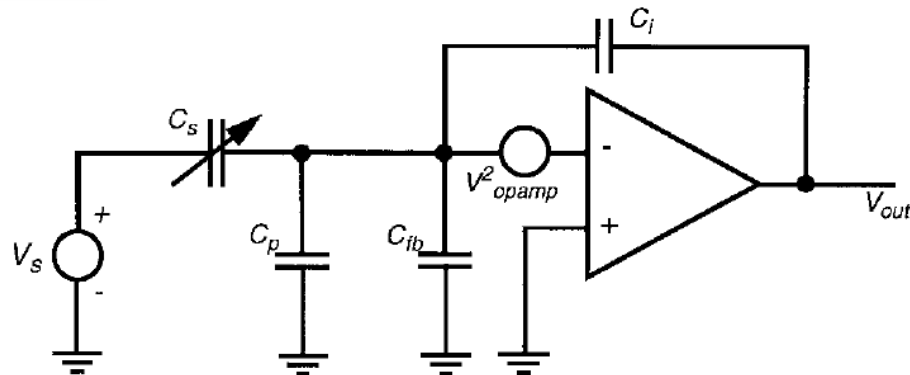
$$C = \frac{2 N \epsilon_0 h \cdot l}{g}$$

<b>l</b>	Overlap of the fingers
<b>g</b>	Gap between the fingers
<b>w</b>	Width of a finger
<b>h</b>	Thickness of the device
<b>N</b>	Number of fingers

**Table 1.** Important geometric variables for comb drive

# Electrostatic Force Feedback

- Single-ended sensing interface
  - position measurement by applying  $V_s$  pulse at capacitive half bridge[2].
  - Capacitive imbalance cause different amount of charge flow[2].

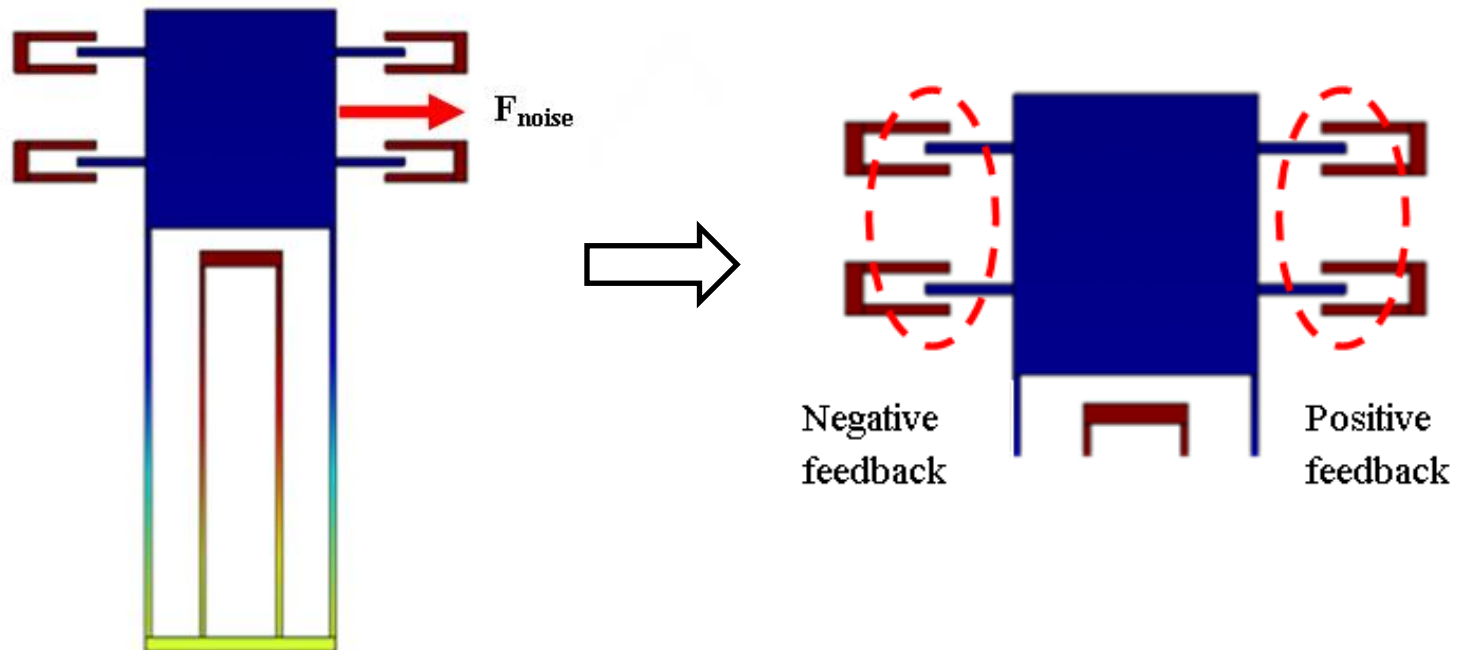


Single-ended representation for op-amp thermal noise analysis. [2]



# Modeling in COMSOL

- MEMS > 2D-Plane Stress & Electrostatics

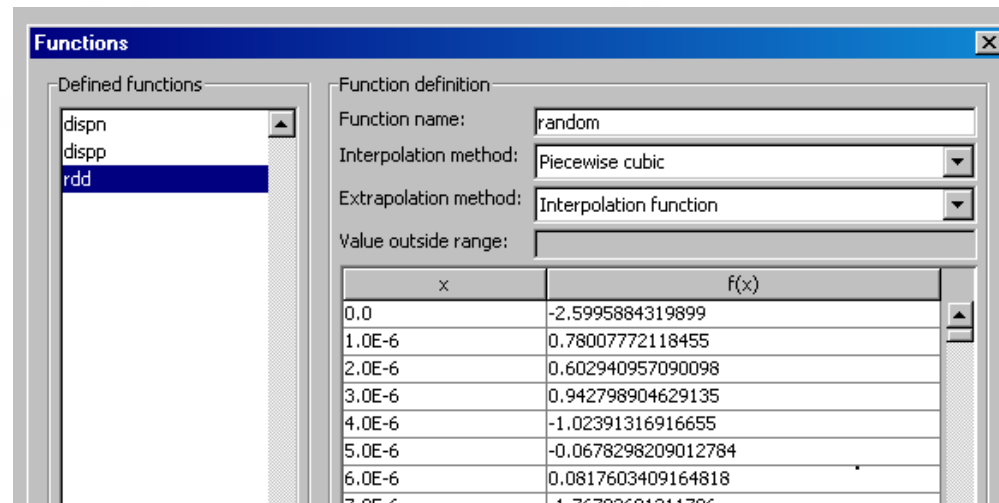


# Modeling in COMSOL

- Modeling Random Noise

Option > Functions > New > File

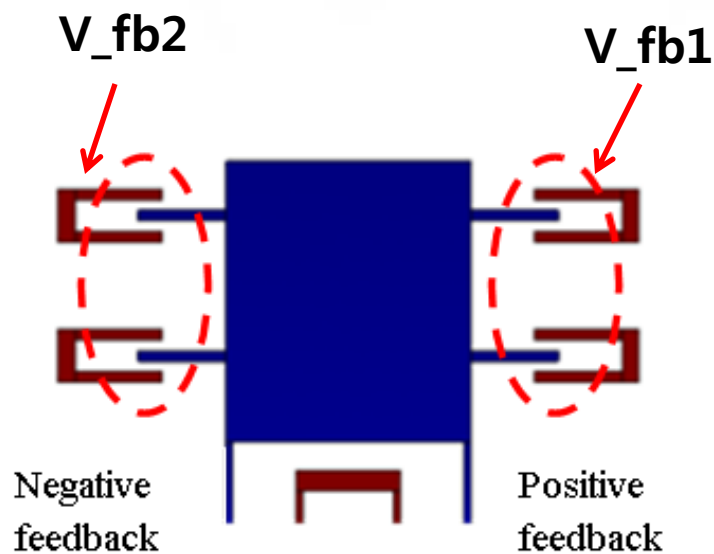
Random arrays of numbers were created using MATLAB.



Global Expression >  $F_{\text{noise}} = (\text{amplitude}) * \text{random}(t)$

# Modeling in COMSOL

- Feedback Voltage Expression



$$F_x = F_{\text{Electrostatic}}$$

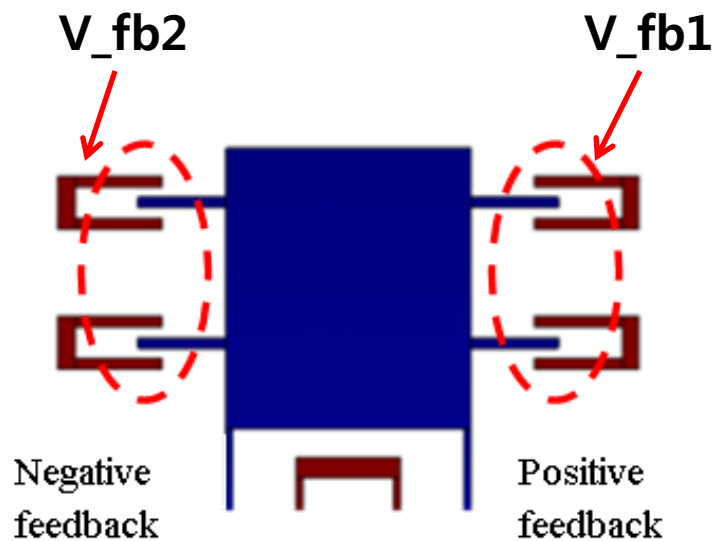
$$x \cdot k_x = N \epsilon_0 V^2 \frac{h}{g}$$

$$V_{fb1} = \sqrt{\frac{g \cdot k_x}{\epsilon_0 h N} \cdot \frac{d(\text{disp}_n)}{dt}}$$

$$V_{fb2} = \sqrt{\frac{g \cdot k_x}{\epsilon_0 h N} \cdot \frac{d(\text{disp}_p)}{dt}}$$

# Modeling in COMSOL

- Feedback Voltage Expression



$$F_x = F_{\text{Electrostatic}}$$

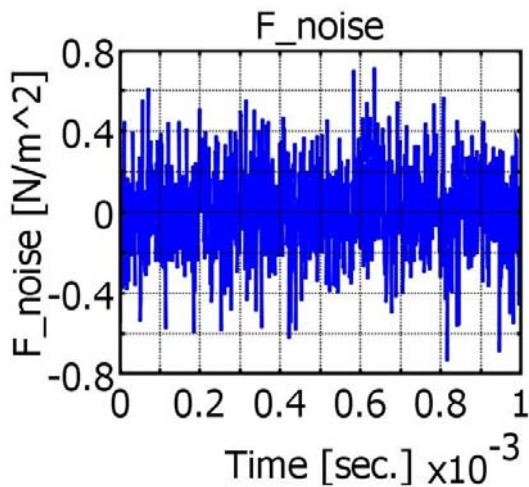
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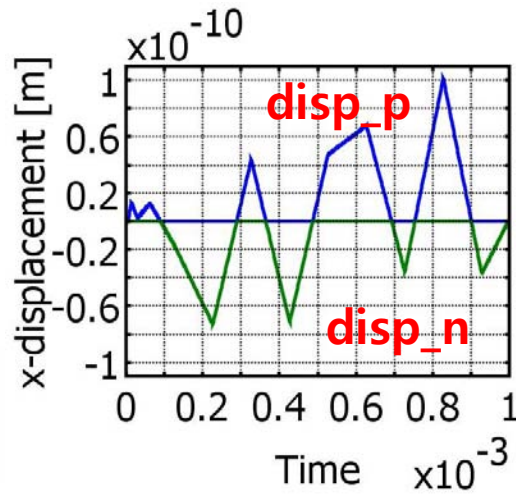
# Modeling in COMSOL

- Summary of Simulation



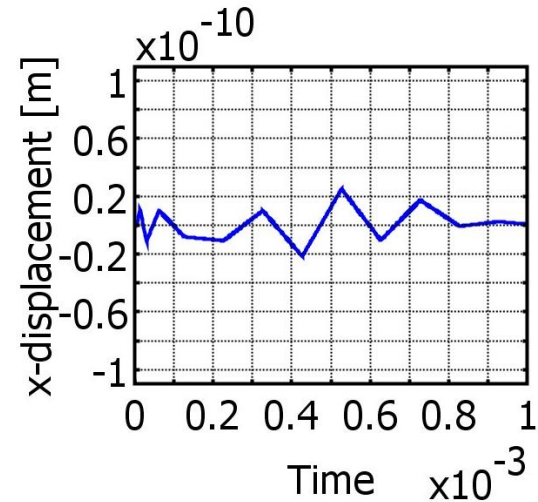
①

Apply Noise



②

Obtain disp.



③

Apply Feedback V

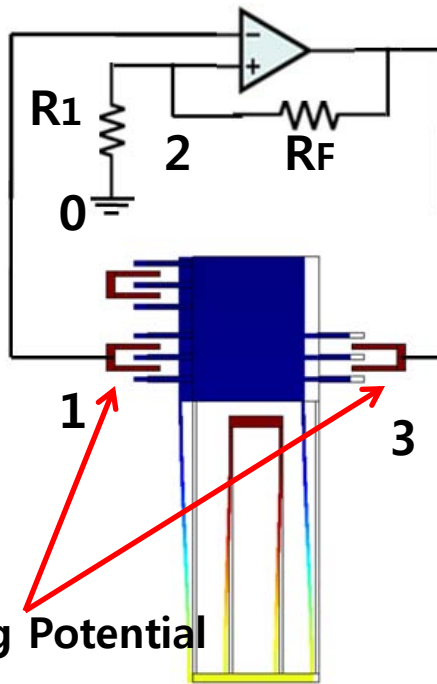
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# Challenges in COMSOL

- Modeling sensing interface with SPICE

Physics > SPICE Circuit Editor



SPICE Circuit Editor

SPICE netlist:

```
XM1 1 3 mems
R1 2 0 1MEG
RF 2 3 1
XOP1 2 1 3 OPAMP1
*
* OPAMP MACRO MODEL, SINGLE-POLE
* connections: non-inverting input
*              | inverting input
*              | | output
*              | | |
.SUBCKT OPAMP1 1 2 6
* INPUT IMPEDANCE
RIN 1 2 10MEG
* gain bandwidth product = DCGAIN x POLE1 = 10MHz
* DCGAIN=100K AND POLE1=100HZ
EGAIN 3 0 1 2 100K
```

Force AC analysis

OK Cancel Help

# Conclusion

- Electrostatic force feedback reduces the amplitude of noise induced displacement
- More careful modeling necessary for more significant reduction
  - Randomized noise
  - Realistic Geometry
  - Sensing Interface

# References

- [1] T.B. Gabrielson, "Mechanical-Thermal Noise in Micromachined Acoustic and Vibration Sensors", IEEE Transactions on Electron Devices, Vol.40, 5 May 1993, pp. 903-909
- [2] M. Lemkin, B. E. Boser, "A Three Axis Micromachined Accelerometer with a CMOS Position-Sense Interface and Digital Offset-Trim Electronics", IEEE J. Solid-State Circuits 34(4) (1999) 465-456.
- [3] M. Handtmann, R. Aigner, A. Meckes, G. K. M. Watchutka. "Sensitivity Enhancement of MEMS Inertial Sensors Using Negative Springs and Active Control", Sensors and Actuators A 97-98(2002) 153-160.
- [4] [http://pakogom.files.wordpress.com/2008/06/main\\_accelerometer20080609.png](http://pakogom.files.wordpress.com/2008/06/main_accelerometer20080609.png)
- [5] <http://mems.sandia.org>