Numerical Simulation of a Joule Heating Problem

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Outline

- Work in progress

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  - Modelling started w/ William Lee and Michael Vynnicky, Limerick - Ireland

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Numerical Simulation of a Joule Heating Problem

We consider a mathematical model that describes the combined processes of heat conduction and electrical conduction in a body which may undergo a phase change as a result of the heat generated by the current, the so called **Joule heating**.

Numerical Simulation of a Joule Heating Problem

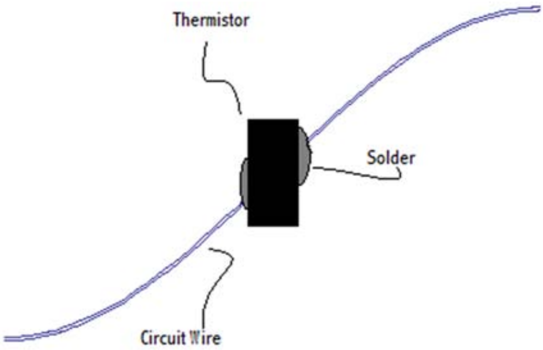

Figure: A Thermistor

Joule heating is generated by the resistance of materials to electrical current; it is present in any resistance of materials to electrical current and is present in any electrical conductor operating at normal temperatures.

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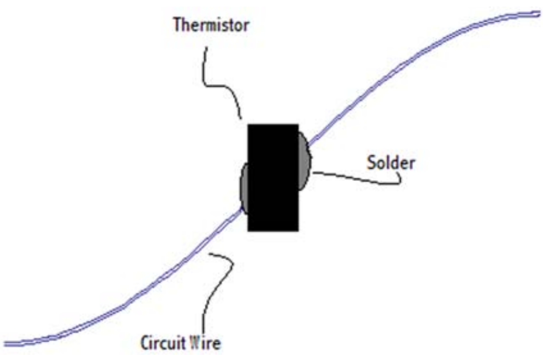

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Numerical Simulation of a Joule Heating Problem

The fuse is a type of sacrificial overcurrent protection device. Its essential component is a metal wire or strip that melts when too much current flows, which interrupts the circuit in which it is connected. Short circuit, overload or device failure is often the reason for excessive current.

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The melting of the conductor is useful in fuses and is the basis of the industrially important process of electrical welding.

Numerical Simulation of a Joule Heating Problem

For $-H \leq x \leq H$,

$$\rho_s c_{p,s} \frac{\partial T_s}{\partial t} = k_s \frac{\partial^2 T_s}{\partial x^2} + \sigma(T_s) \left(\frac{\partial \phi}{\partial x} \right)^2,$$

$$0 = \frac{\partial}{\partial x} \left(\sigma(T_s) \frac{\partial \phi}{\partial x} \right),$$

subject to boundary conditions

$$T_s = T_0, \qquad \phi = 0 \quad \text{at } x = -H,$$
$$T_s = T_0, \qquad \phi = V \quad \text{at } x = H,$$

where k is the thermal conductivity, ρ is the density, and initial conditions

$$T_s = T_0.$$

$\phi(x,t)$ and $T(x,t)$ are the electric potential and temperature, respectively, and $\sigma(T)$ the electrical conductivity.

Numerical Simulation of a Joule Heating Problem

Since this formulation is symmetric, we can consider just the region $0 \leq x \leq H$, subject to

$$\frac{\partial T_s}{\partial x} = 0, \qquad \phi = 0 \quad \text{at } x = 0,$$
$$T_s = T_0, \qquad \phi = V \quad \text{at } x = H.$$

As the solid fuse heats up, material will melt when the temperature exceeds the melting temperature, T_{melt}. Once this happens, a liquid region will form; this first appears at $x = 0$. A melting front then advances into $x > 0$; call it $x = s(t)$. The formulation then becomes:

$$\rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} + \sigma(T) \left(\frac{\partial \phi}{\partial x} \right)^2,$$
$$0 = \frac{\partial}{\partial x} \left(\sigma(T) \frac{\partial \phi}{\partial x} \right),$$

subject to boundary conditions

$$\frac{\partial T}{\partial x} = 0, \qquad \phi = 0 \quad \text{at } x = 0,$$
$$T = T_0, \qquad \phi = V \quad \text{at } x = H.$$

with the additional conditions at the interface $x = s(t)$:

$$T = T_{melt},$$
$$\left[k \frac{\partial T}{\partial x} \right]_-^+ = \rho_s \Delta H_f \frac{ds}{dt}.$$

where ΔH_f is the latent heat per unit mass.

Usually

$$\rho = 5.6x10\, kg\, m^{-3},$$
$$k = 2W\, K^{-1} m^{-1},$$
$$\Delta T = 100\, K,$$
$$c = 540J\, kg^{-1}\, K^{-1},$$
$$H = 10^{-3} m.$$

In our simulations we will take

$$\rho = 5600$$
$$k = 2$$
$$\Delta = 1 \; c = 1$$
$$H = 1$$

at $m = 0.2 \Rightarrow T_{melt}$.

Let $\sigma(T) = 1$. And ϕ was not time dependent.

Temperature Profile

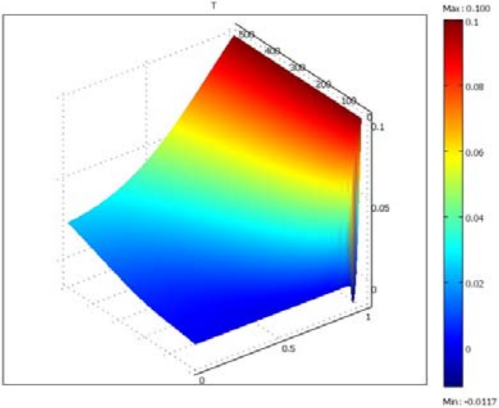

Figure: Temperature Profile

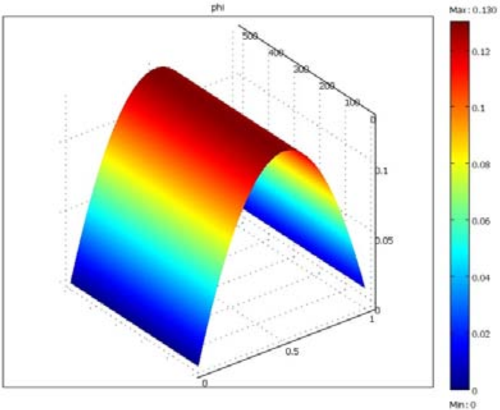

Figure: Φ Profile

Movie

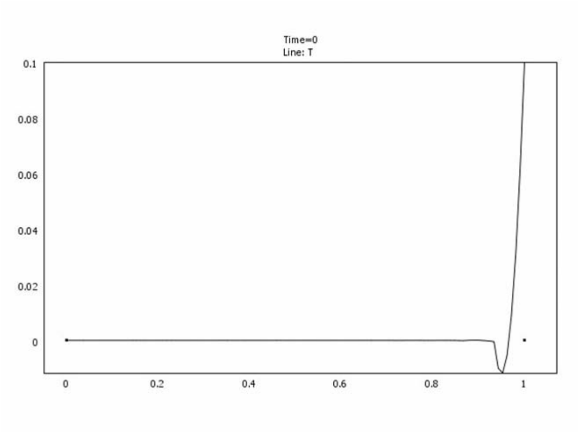

Figure: Movie