Magnetic control of deformation of a ferrofluid droplet in simple shear flow

Md Rifat Hassan[1], Cheng Wang[1]
1. Department of Mechanical and Aerospace Engineering, Missouri University of Science and Technology, Rolla, MO, USA

Abstract

This study investigates the effect of uniform magnetic field on the deformation of a ferrofluid droplet in a two dimensional (2D) simple shear flow by means of numerical simulation. The magnetic field is applied in a perpendicular direction to the flow direction. A numerical scheme called level set method in combination with laminar two phase flow under fluid flow module is used to solve the flow field both inside and outside of the droplet while level set method is required to track the dynamic motion of the droplet interface which is suspended in another immiscible medium. A constant shear rate is applied by using a velocity of the same magnitude but in opposite direction on the top and bottom wall. Magnetic field both inside and outside of the droplet is simulated using the AC/DC module and it is applied to the flow domain using volume force feature under laminar flow module. We found that at a low shear rate with increasing magnetic field strength, the magnetic field plays a dominant role on the deformation of the droplet and also the droplet is found to orient itself more along the direction of magnetic field. On the other hand, at high shear rate the deformation and orientation of the droplet is determined predominantly by shear flow although the magnetic field has a considerable effect at higher strengths. It is also found that the flow field inside and outside of the droplet changes at different conditions.

1. Introduction

Emulsions are liquid droplets dispersed in another immiscible phase. When subjected to shear flows, the droplets may break up. Understanding the dynamics of droplets in shear flows is of great importance to a variety of technological and industrial applications that utilize emulsions, including cosmetics, food production and polymer processing. For example, in blending molten polymers, the distribution of droplet size and shape is critical to the rheology and physical properties of the polymer system.

A single droplet in simple shear flow serves as an excellent model problem to understand droplet dynamics and can provide fundamental insights to more complex emulsion systems[1,2]. Following the pioneering work of Taylor[3,4], numerous experimental[5–7], theoretical[8], and numerical studies[9–12] have been carried out to investigate the deformation and breakup of a Newtonian droplet suspended in shear flow of another viscous Newtonian fluid.

Magnetic fields have also been demonstrated to control the dynamics of single droplets[13,14] or emulsion systems[15–17]. To use magnetic manipulation, either the droplet or suspending fluid needs to be a ferrofluid – a dispersions of magnetic nanoparticles (typical diameter around 10 nm, and typical volume fraction 5%). Multiphase ferrofluid droplets have promising biomedical applications due to their ability to be delivered at specific site with the help of proper manipulation of a magnetic field. A notable biomedical application is treatment of retinal detachment[18] by guiding a ferrofluid droplet inside retinal. Magnetic control of droplet formation has also been extensively used in microfluidics. Liu[19] and Wu[20] studied the ferrofluid droplet deformation under uniform magnetic field.

However, till now few have studied the deformation of ferrofluid droplets in a simple shear flow under the influence of a uniform magnetic field. Recently, Jesus[21] performed three-dimensional numerical analysis on the droplet dynamics and field induced deformation of a ferrofluid droplet in another Newtonian fluid. One significant advantage of using the magnetic fields in compared to electric fields is that magnetic field can be applied at arbitrary directions with ease, while the direction of electric fields is often limited by the placement of electrodes.

By using two-dimensional (2D) direct numerical simulations, this paper investigates the dynamics and deformation of a ferrofluid droplet in a simple shear flow under a uniform magnetic field that is perpendicular to the flow domain. For computational efficiency, we have chosen to use 2D simulations in order to study a wide range of parameter space i.e. capillary number, magnetic bond number. Our numerical simulation, built with commercial FEM solver, models the dynamic deformation of droplet interface by using the level-set method, and couples the magnetic and flow fields.

The remainder of the paper is organized as follows: in section 2, the mathematical model and numerical method with COMSOL settings are described. In section 3, we first present numerical results obtained from droplet deformation in simple shear flow only and validate our results against existing theory. We then examine the effect of magnetic field on the droplet deformation and orientation angles by considering the field direction perpendicular to the flow domain, magnetic bond number and capillary number. Finally, we conclude our major findings in section 4.

2. Numerical Simulation Method

2.1 Level set method

In our model, we have used the conservative level set method to track the dynamic evolution of the interface between the droplet and suspending medium. The level set function, ϕ is an auxiliary scalar function to represent the phases of the two fluids which has a value of zero in one domain and 1 in another domain. The value of ϕ varies smoothly from 0 to 1 across the interface and ϕ = 0.5 defines the position of the interface. The level set function ϕ which is advected by the velocity field by[22,23]:

$$\frac{\mathrm{d}\phi}{\mathrm{dt}} + \nabla\cdot(\mathbf{u}\phi) = \gamma \nabla\cdot\left(\varepsilon\nabla\phi - \phi(1-\phi)\frac{\nabla\phi}{|\nabla\phi|}\right) \quad (1)$$

where u, γ and ε determine the velocity field, amount of reinitialization and thickness of the interface respectively. The terms on the left hand side of the equation represents the motion of the interface while the terms on the right hand side is required for numerical stability. The level set function, ϕ can also be used to find the unit normal to the interface, n which is given by:

$$\boldsymbol{n} = \frac{\nabla\phi}{|\nabla\phi|} \quad (2)$$

With the level set method, the two immiscible fluids are treated as a single phase flow but the material properties vary according to the level set value. Here, a linear average is used to calculate the density (ρ), dynamic viscosity (η), magnetic permeability (μ) and magnetic susceptibility (χ) which are related to ϕ through the following equations:

$$\rho = \rho_c + (\rho_d - \rho_c)\phi, \qquad \eta = \eta_c + (\eta_d - \eta_c)\phi$$
$$\mu = \mu_c + (\mu_d - \mu_c)\phi, \qquad \chi = \chi_c + (\chi_d - \chi_c)\phi$$

where subscripts c and d represent the continuous and droplet phase respectively.

2.2 Governing Equations

The motion of an incompressible, immiscible ferrofluid droplet in another incompressible, immiscible medium under the effect of a uniform magnetic field is governed by the following continuity and momentum equations:

$$\nabla\cdot\mathbf{u} = 0 \quad (3)$$

$$\rho\frac{\mathrm{D}\mathbf{u}}{\mathrm{Dt}} = -\nabla p + \nabla\cdot\tau + \mathrm{F}_\sigma + \mathrm{F}_m \quad (4)$$

where, $\frac{D\mathbf{u}}{Dt}$ represents the total derivative of the velocity field, $\mathbf{u}$. The right hand side of the equation (4) represents the force terms due to pressure, viscosity, surface tension (F_σ) and magnetic field (F_m) respectively. The viscous stress tensor τ can be expressed as: $\tau = [\eta(\nabla\mathbf{u} + (\nabla\mathbf{u})^T)]$. The surface tension force, F_σ can be defined by:

$$\mathbf{F}_\sigma = \nabla\cdot[\sigma\{\mathbf{I} + (-\mathbf{n}\mathbf{n}^T)\}\delta] \quad (5)$$

where, σ is the surface tension coefficient, I is identity matrix, δ is the Dirac delta function and n is the unit normal to the interface which can be calculated using equation (2). The Dirac delta function, δ can also be approximated using the level set function as:

$$\delta = 6|\phi(1-\phi)||\nabla\phi| \quad (6)$$

Assuming linear and homogeneous material properties, the different magnetic properties i.e. magnetic induction **B**, magnetization **M** and magnetic field **H** can be related using Maxwell magneto-static relationship through the following equations:

$$\nabla\cdot\mathbf{B} = 0, \nabla\times\mathbf{H} = 0, \mathbf{M} = \chi\mathbf{H} \quad (7)$$

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu_0(1+\chi)\mathbf{H} \quad (8)$$

where μ_0 is the permeability of vacuum which is equal to $4\pi \times 10^{-7}\ N/A^2$. A scalar potential φ can be defined to satisfy the curl-free **H** i.e. $H = -\nabla\varphi$ which can be written as:

$$\nabla\cdot(\mu\nabla\varphi) = 0 \quad (9)$$

In addition, the total magnetic force can be calculated using the magnetic stress tensors as:

$$\mathbf{F}_\mathrm{m} = \nabla\cdot\boldsymbol{\tau}_\mathrm{m} = \nabla\cdot\left(\mu\mathbf{H}\mathbf{H}^\mathrm{T} - \frac{\mu}{2}H^2\mathbf{I}\right) \quad (10)$$

where, $\boldsymbol{\tau}_\mathrm{m}$ is the magnetic stress tensor for the applied magnetic field, H = $|\mathbf{H}|$ is the magnitude of the magnetic field and **I** is the second order identity tensor. The magnetic insulation on both the left and right walls are satisfied through the following equation:

$$\mathbf{n}\cdot\mathbf{B} = 0 \quad (11)$$

We introduced some dimensionless groups to reduce the number of variables and observe which dimensionless groups affect the droplet dynamics most. The dimensionless groups are defined as:

$$\mathrm{Re} = \frac{\rho_c R_0^2\dot{\gamma}}{\eta_c} \quad (12)$$

$$\mathrm{Ca} = \frac{\eta_c R_0\dot{\gamma}}{\sigma} \quad (13)$$

$$\mathrm{Bo_m} = \frac{R_0 \mu_0 H_0^2}{2\sigma} \quad (14)$$

where, Re, Ca and $\mathrm{Bo_m}$ represent Reynolds number, Capillary number and Magnetic bond number respectively.

2.3 Schematic of numerical model

Fig. 1 demonstrates the schematic illustration of a ferrofluid droplet suspended in another fluid medium in a simple shear flow under the application of a uniform magnetic field, $\mathbf{H}_0$. In this case, the magnetic susceptibility of the ferrofluid droplet was considered as 1 i.e. $\chi_d = 1$ while it was considered zero i.e. $\chi_c = 0$ for the suspending non-magnetic fluid. The subscript c and d represent the droplet and continuous phase respectively. The viscosity and density of both the phases are considered equal to each other i.e. $\eta_c = \eta_d$

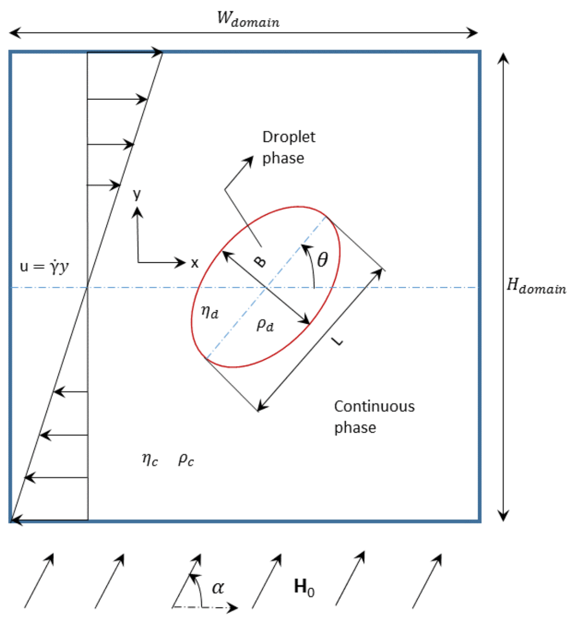

Figure 1. Schematic illustration of a ferrofluid droplet suspended in another medium in a simple shear flow under the application of uniform magnetic field, $\mathbf{H}_0$.

and $\rho_c = \rho_d$. The height and width of the computational domain are $H_{domain} = W_{domain} = 10R_0$ where R_0 is the radius of undeformed ferrofluid droplet which is equal to 50 μm. Initially the ferrofluid droplet was placed at the middle of the computational domain. The surface tension is considered as σ = 0.0135 N/m. Here, the top wall moves with a velocity, $\mathbf{u}_t = \frac{1}{2}\dot{\gamma}H_{domain}e_x$ and bottom wall moves with a velocity, $\mathbf{u}_b = -\frac{1}{2}\dot{\gamma}H_{domain}e_x$ thus leading to a simple shear flow with constant shear rate $\dot{\gamma}$. Periodic flow condition was applied to both left and right walls in the x-direction. A uniform magnetic field, $\mathbf{H}_0$ was applied at arbitrary directions which is denoted by angle, α. Here, the deformation and orientation angle of the droplet under the effect of magnetic field is studied. The deformation of the droplet is found out using dimensions L and B which are the lengths along the major and minor axes of the droplet respectively. Also, the orientation angle, θ is defined as the angle between the positive x-axis and major axis of the droplet when the droplet undergoes deformation under the effect of both shear flow and magnetic field.

2.4 COMSOL Settings

Two phase laminar flow, level set method in combination of transient with phase initialization feature was used to solve the flow domain and to track the deformable interface of the droplet. The velocity of top wall and bottom wall was set as 0.0125 m/s and -0.0125 m/s respectively using the moving wall feature which produces a constant shear rate of 50 s^{-1}. Another case was also studied using a different shear rate of 600 s^{-1}. Periodic boundary condition was applied to both left and right walls in the positive x-direction with zero pressure difference across them. The value of level set function, ϕ was assigned as 1 and 0 for the droplet phase and continuous phase respectively. The interface of the droplet was defined using initial interface condition. The reinitialization parameter, γ was equal to the maximum magnitude of the velocity in the flow domain and interface thickness, ε was of the order as the same size of the mesh elements. Additionally, magnetic field was applied to the flow domain and solved simultaneously using Magnetic fields, no currents interface from AC/DC module. In this case, the magnetic field was applied perpendicular to the flow domain i.e. α = 90°. Magnetic insulation was applied to both left and right walls. The value of magnetic field strength, $\mathbf{H}_0$ was varied from 25000 A/m to 45000 A/m using the parametric sweep feature. Then we investigated the deformation and orientation angle of the ferrofluid droplet for the above mentioned cases. For creating the mesh, we used free triangular elements in the computational domain. PARDISO solver with nested dissection multithread algorithm was used to solve our computational model.

3. Results and Discussions

3.1 Validation of Numerical Model

Before moving on to our intended study, we validated our model by comparing our results against Taylor's deformation theory in a simple shear flow for different capillary number, Ca. According to Taylor[3,4], the deformation of a ferrofluid droplet suspended in another medium with same viscosity ($\eta_c = \eta_d$) and density ($\rho_c = \rho_d$) can be calculated as:

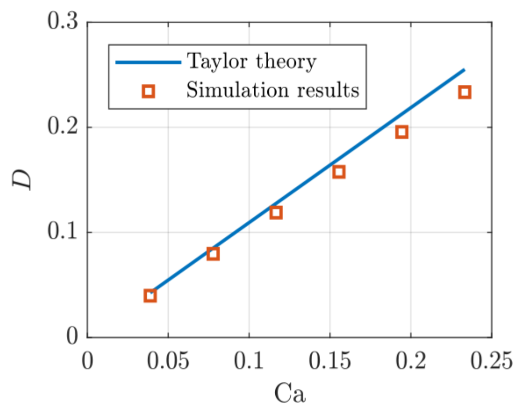

Figure 2. Validation of numerical model. Comparison of simulation results against Taylor's theory.

$$D = \frac{L - B}{L + B} = \frac{19\eta_d + 16\eta_c}{16\eta_d + 16\eta_c}\mathrm{Ca} = \frac{35}{32}\mathrm{Ca} \qquad (15)$$

where the capillary number, Ca can be found from equation (13). Fig. 2 represents the comparison of simulation results against Taylor's theory for different capillary numbers, Ca. It shows that, with increasing shear rate, the deformation of the ferrofluid droplet increases. Also, it can be seen that both the results agree well with each other although at high capillary number, Ca there is little discrepancy between the two results. The reason maybe due to two dimensional nature of the simulation and also Taylor's theory works well for small capillary number, Ca. But the results are still satisfactory since even at a higher capillary number (Ca = 0.233), the error of the deformation of ferrofluid droplet between the simulation results and Taylor's theory was approximately 8%. Fig. 3 illustrates the shape of the droplet for the different cases which clearly proves our argument that the deformation of the droplet increases

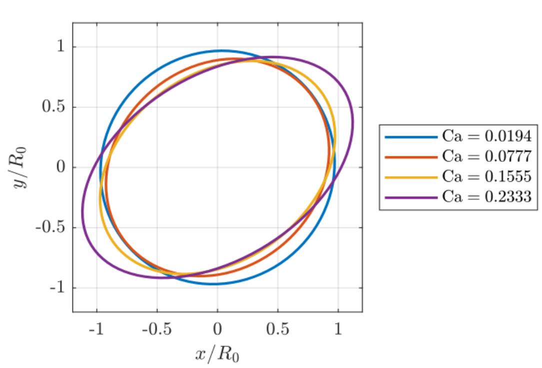

Figure 3. Outline of the droplet for different capillary number, Ca.

with increasing shear rate. Furthermore, it can be seen that the droplet follows the direction of shear flow more with increasing capillary number, Ca.

3.2 Droplet in simple shear flow with a magnetic field perpendicular to the flow (α = 90°)

When a magnetic field is applied to a ferrofluid droplet suspended in a simple shear flow, the droplet deforms even more due to the additional effect of magnetic field strength on the droplet interface. In order to see the effect of magnetic field we plotted the deformation of the droplet at various capillary number at a fixed magnetic bond number. In this case, the strength of the magnetic field was considered as $2.5{\times}10^4$ A/m.

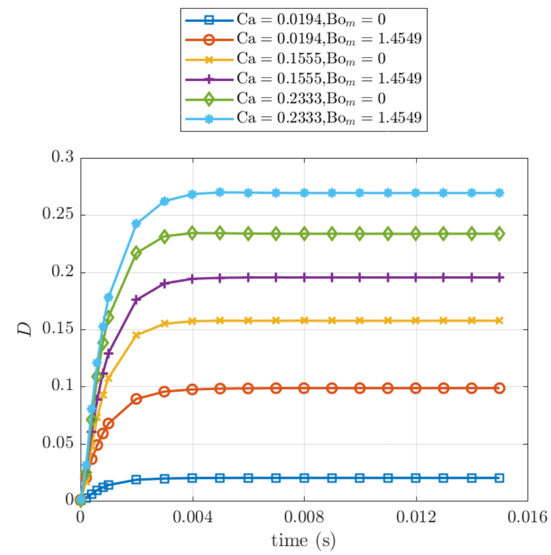

Figure 4. Effect of magnetic field in a simple shear flow on the deformation of ferrofluid droplet. Deformation, D vs time.

Fig. 4 represents the effect of magnetic field in a simple shear flow on the deformation of the droplet. It can be seen that at all capillary numbers, Ca when we apply the magnetic field the deformation of the droplet increases even more than shear flow without magnetic field. But for the low shear rate case, when we apply the magnetic field the deformation of the droplet increased about 5 times in compared to the shear flow only while for the high and moderate shear flow cases, the deformation of the droplet increased about 1.2 times after the application of magnetic field. The reason is that at low shear rate the magnetic field takes total control on the deformation of the droplet. On the other hand, for the high shear rate case the deformation of the droplet is predominantly controlled by the shear flow although the magnetic field has considerable effect on the deformation of the droplet at higher values. Fig. 5 depicts the outline of the droplet at various capillary numbers, Ca after the application of magnetic field and it clearly proves our argument stated above. We also studied the effect of magnetic field on the orientation of the droplet.

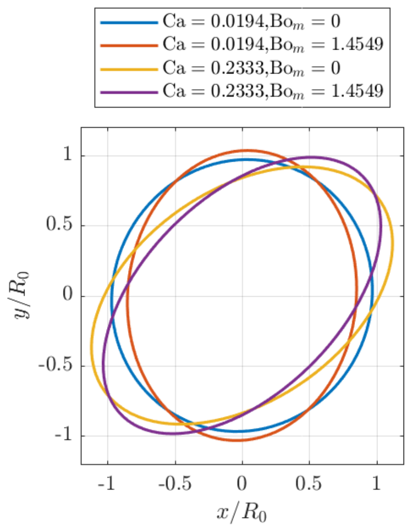

Figure 5. Outline of the droplet for different capillary number, Ca after the application of magnetic field.

Fig. 6 represents the orientation angle of the droplet at steady state for different capillary numbers, Ca with and without the application of magnetic field. It can be clearly seen that when magnetic field is applied the orientation of the droplet increases at all shear rates. Also, at low shear rate the droplet orientation is totally controlled by the magnetic field alone and that's why the droplet orientation angle is closer to 90° in this case. But at high and moderate shear rates, the shear flow becomes dominant and the orientation angle of the droplet is predominantly determined by the shear flow. As a result, the orientation angle is found closer to 45° than 90° in this case.

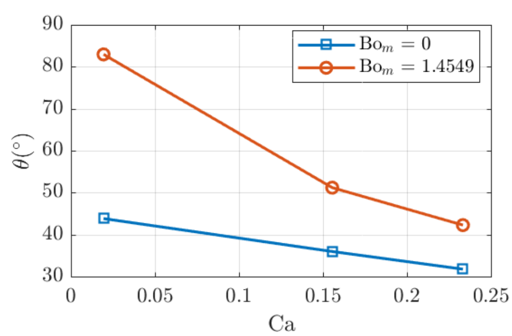

Figure 6. Effect of magnetic field in a simple shear flow on the orientation of ferrofluid droplet. Orientation angle, θ vs Ca.

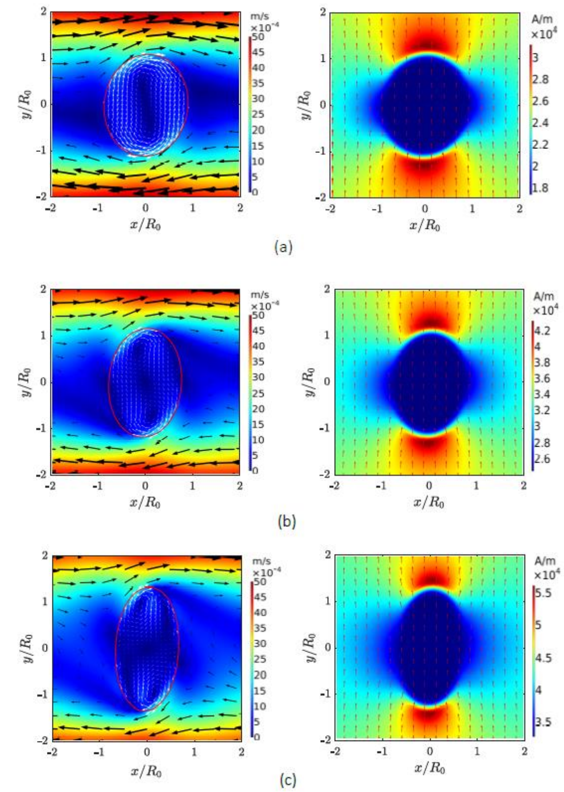

Figure 7. Velocity field and magnetic field strength for Ca = 0.0194, Re = 0.0015 at α = 90°. (a) $\mathrm{Bo_m}$ = 1.4549; (b) $\mathrm{Bo_m}$ = 2.8515; (c) $\mathrm{Bo_m}$ = 4.7138.

Fig. 7 represents velocity and magnetic field strength profiles at steady state for Ca = 0.0194, Re = 0.0015 at different Magnetic bond numbers, $\mathrm{Bo_m}$ while Fig. 8 represents velocity and magnetic strength profiles at steady state for Ca = 0.2333, Re = 0.018 at different magnetic bond numbers, $\mathrm{Bo_m}$. From the velocity fields it is seen that there is circulation inside the droplet due to the flow sorrounding the droplet in the domain which contributes considerably in defing the shape of the droplet. The arrow surface denotes the direction of velocity in the domain. The droplet experiences maximum shear stress on the poles of the droplet. Also, the shape of the droplet from the velocity figures gives us a clear illustration that with increasing magnetic field strength, the droplet deformation increases and the droplet tends to orient itself more along the direction of the magnetic field. From the magnetic field strength profiles it is clear that the magnetic field is uniform both inside and outside of the droplet. The magnetic field lines are also parallel to each other although the lines gets slightly deflected at the interface of the droplet due to change in magnetic susceptibility at the interface of the droplet.

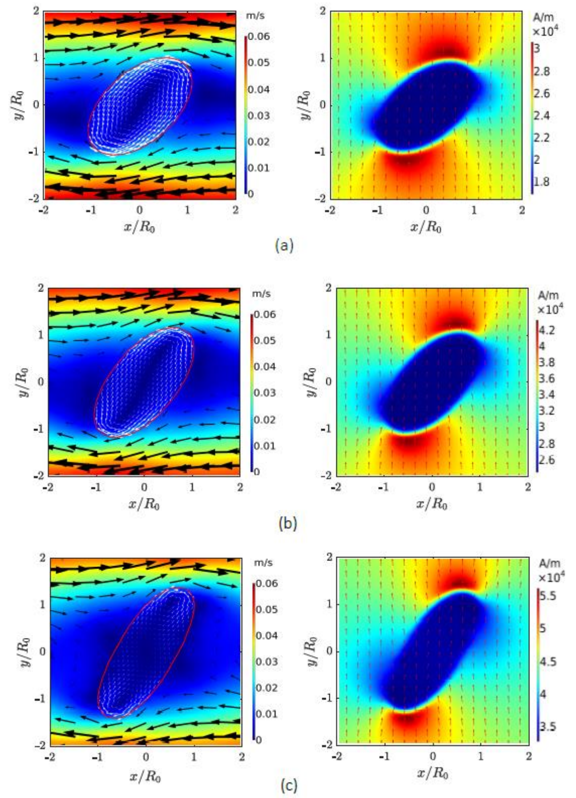

Figure 8. Velocity field and magnetic field strength for Ca = 0.2333, Re = 0.018 at $\alpha = 90°$. (a) $Bo_m = 1.4549$; (b) $Bo_m = 2.8515$; (c) $Bo_m = 4.7138$.

The droplet also experiences maximum magnetic field strength at the poles of the droplet while it is least in magnitude along the equator of the droplet.

4. Conclusions

The influence of magnetic field perpendicular to the flow domain on the droplet deformation and orientation angle in a simple shear flow is systematically studied in this paper. At a low shear rate ($Ca \cong 0.02$), the deformation and orientation angle of the ferrofluid droplet is determined by the magnetic field strength. With increasing magnetic field strength, the deformation was found to increase and the droplet orientation angle was found to be closer to 90°. But at a high shear rate ($Ca \cong 0.25$), the deformation and orientation angle is controlled by the dominant shear flow although at higher values of magnetic field strength it has considerable effects on the deformation and orientation angle of the droplet. For example, at a very high magnetic bond number ($Bo_m \cong 5$), the orientation angle of the droplet was found to be closer to 60° than 45° due to the additional effect of the magnetic field strength. Furthermore, the flow field inside and outside of the droplet has also a great contribution in defining the shape of the ferrofluid droplet.

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