Adaptive Structures Lab
NC STATE UNIVERSITY

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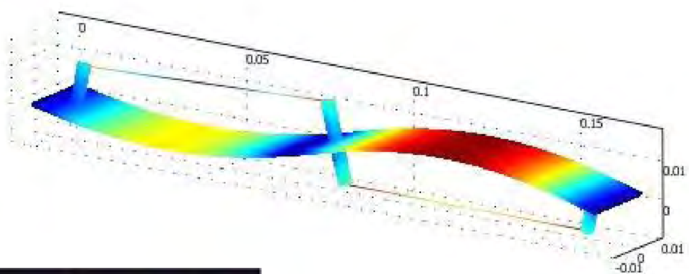
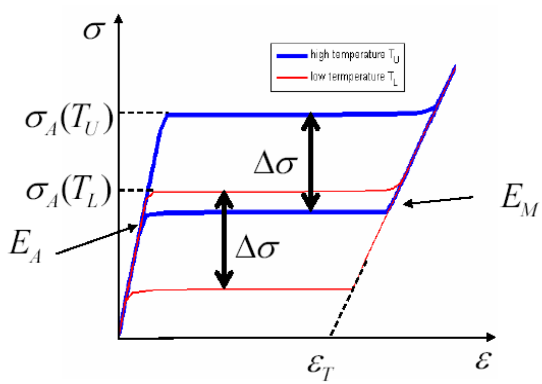
October 7th, 2008

FE Analysis of SMA-Based Adaptive Structures





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Introduction
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Future Work

Shape Memory Alloys (SMA) are characterized by two distinct features:

- **Shape memory effect**
- **Superelasticity**

Provides flexible, low cost, low weight ***actuation*** with highest work output of all known actuation mechanisms.

Can be used as fatigue-free solid state one-component ***joint*** based on flexible hinge mechanism.

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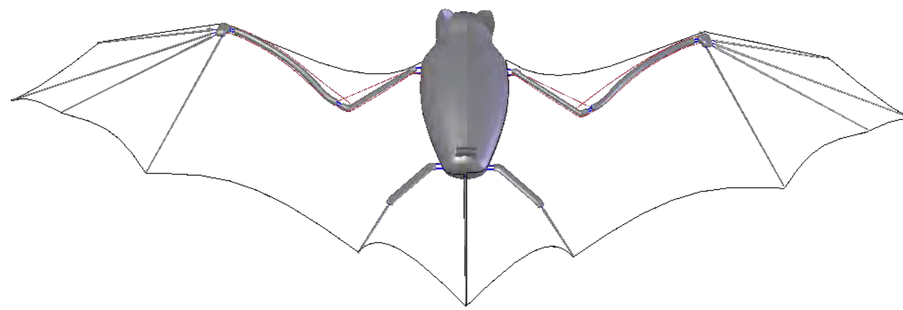
BATMAV Project

Design and Fabrication of a bio-inspired bat-like MAV platform with flapping flight capability

Phase 1: Design and fabrication of skeleton structure



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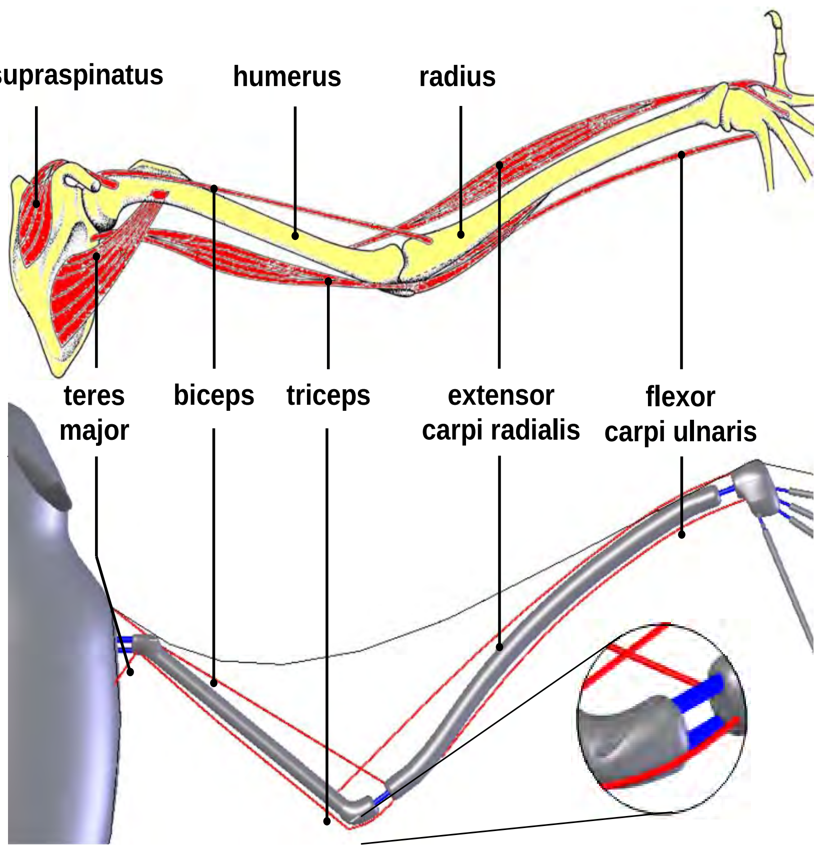
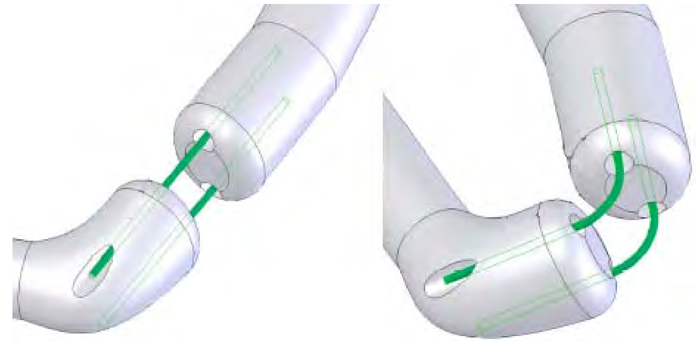
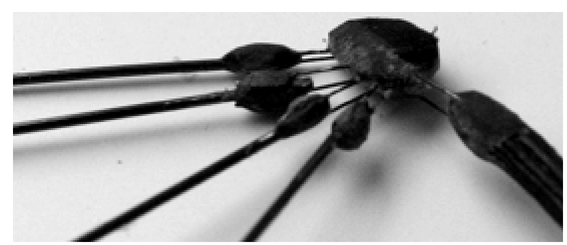
Muscle wire actuation and superelastic joints







Stress-strain relation

$$\varepsilon = x^A \frac{\sigma}{E_A} + x^+\left(\frac{\sigma}{E_M} + \varepsilon_T\right) + x^-\left(\frac{\sigma}{E_M} - \varepsilon_T\right)$$

strain values at minima of Gibbs energy wells

phase fractions

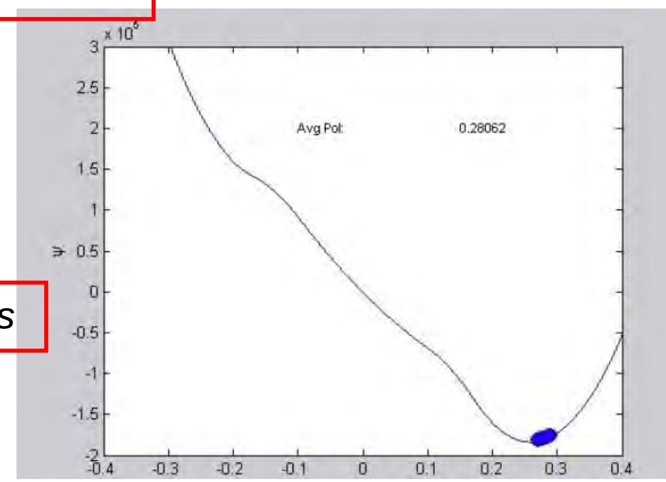
Kinetics of phase transformation

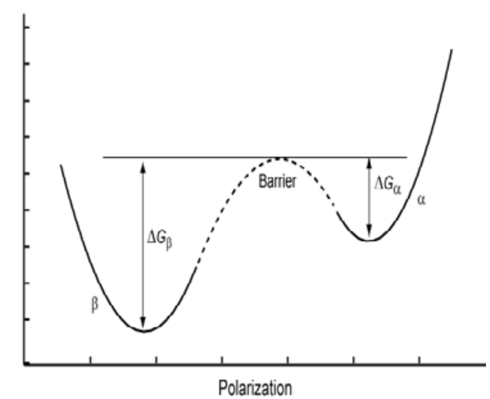
$$\dot{x}^+ = -x^+ p^{+A} + x^A p^{A+}$$
$$\dot{x}^- = -x^- p^{-A} + x^A p^{A-}$$

transition probabilities

$$p^{\alpha\beta} = \frac{1}{\tau_\alpha}\exp\left(-\frac{\Delta g_\alpha V_{LE}}{k_B T}\right)$$

Gibbs Free Energy





Energy balance

$$mc\dot{T} = -\alpha A_S\left(T - T_E(t)\right) + j(t) - \dot{x}^+ H^+(\sigma) - \dot{x}^- H^-(\sigma)$$

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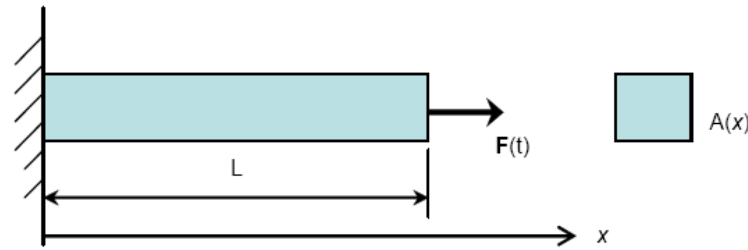
- SMA wire actuator COMSOL implementation

Stress-strain constitutive relation:



$$\sigma(\varepsilon)=\frac{E_M[\varepsilon-(x_+-x_-)\varepsilon_T]}{x_+ + x_- + \frac{E_M}{E_A}x_A} \qquad \varepsilon=\frac{\partial u}{\partial x}\equiv u_x$$

General form PDE: $$d_\alpha \frac{\partial U}{\partial t}+\nabla\cdot\Gamma=F$$

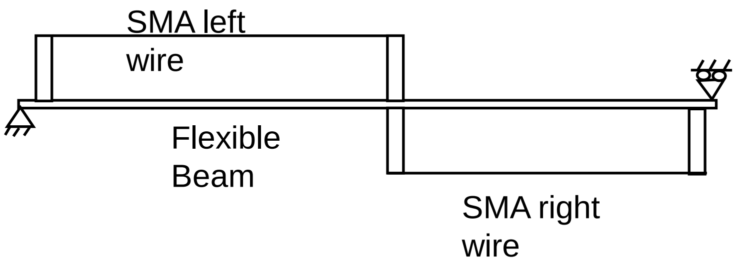
Solution variables: $$U=\begin{Bmatrix}u\\ x_+\\ x_-\\ T\end{Bmatrix}$$

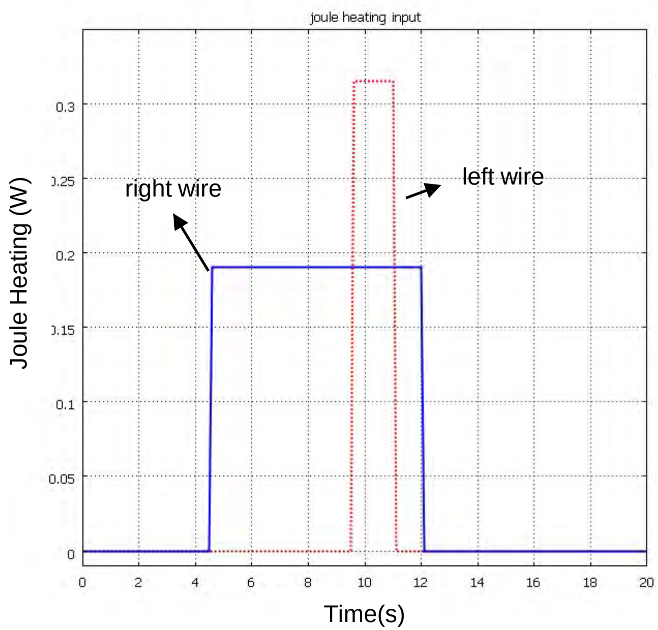
$$\begin{bmatrix}0&0&0&0\\0&1&0&0\\0&0&1&0\\0&(h_{M+}-h_A)&(h_{M-}-h_A)&\rho c\end{bmatrix}\begin{Bmatrix}\dot u\\ \dot x_+\\ \dot x_-\\ \dot T\end{Bmatrix}+\frac{\partial}{\partial x}\begin{bmatrix}\sigma(x)\\0\\0\\0\end{bmatrix}=\begin{bmatrix}0\\ -x_+p^{+A}(\sigma,T)+(1-x_+-x_-)p^{A+}(\sigma,T)\\ -x_-p^{-A}(\sigma,T)+(1-x_+-x_-)p^{A-}(\sigma,T)\\ -\alpha\frac{l_c(x)}{A(x)}(T-T_0)+j(t)\end{bmatrix}$$

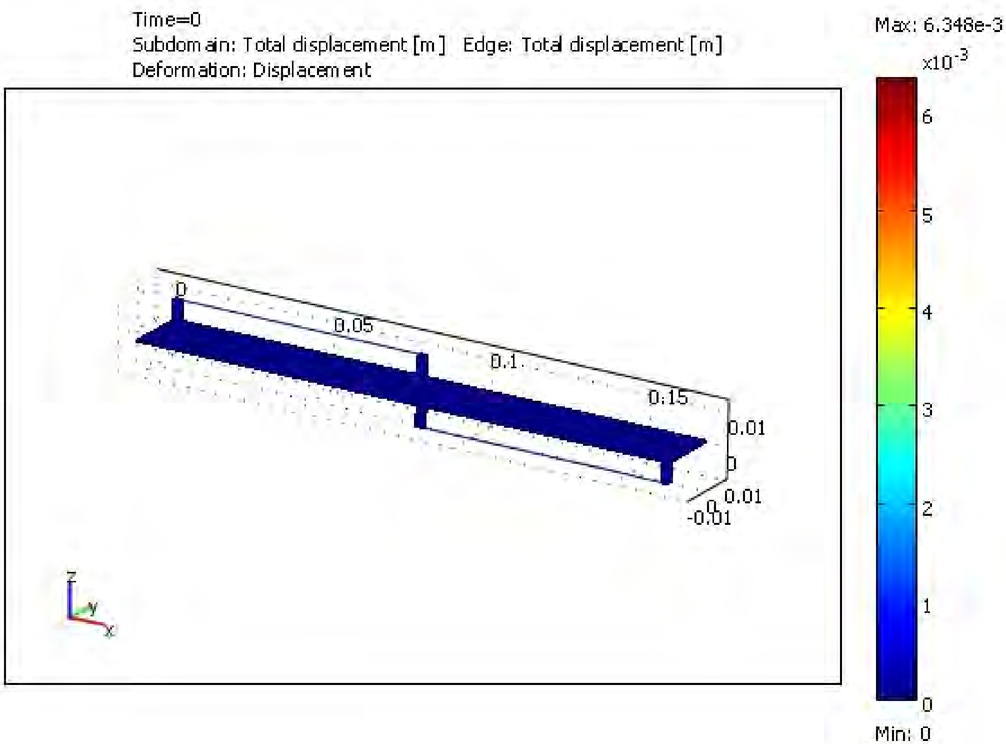
Boundary conditions: $u(0,t), u(L,t)$ or $\sigma(0,t), \sigma(\mathrm{L},t)$

$\mathrm{T}(0,t), \mathrm{T}(\mathrm{L},t)$ or $\frac{\partial \mathrm{T}(0,t)}{\partial x}, \frac{\partial \mathrm{T}(\mathrm{L},t)}{\partial x}$

Initial conditions: $u(x,0), x_\pm(x,0), \mathrm{T}(x,0)$


SMA left wire
Flexible Beam
SMA right wire


joule heating input
right wire
left wire
Joule Heating (W)
Time(s)


Time=0
Subdomain: Total displacement [m] Edge: Total displacement [m]
Deformation: Displacement
Max: 6.348e-3
Min: 0

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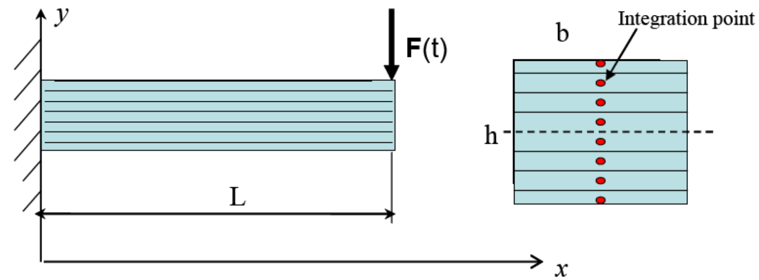
- Superelastic SMA beam implementation

Based on the small deformation Euler-Bernoulli beam theory, a SMA beam is treated as consisting of several layers.



Equilibrium equation

$$\frac{\partial Q}{\partial x} = -q(x)$$

The relation between the shear force Q(x) and the cross –section moment M(x)

$$\frac{\partial M}{\partial x} = Q$$

Stress-strain relation

$$\sigma(x,y) = \frac{E_M[\varepsilon(x,y) - (x_+ - x_-)\varepsilon_T]}{x_+ + x_- + \frac{E_M}{E_A}x_A}$$

$$\varepsilon(x,y) = -y\frac{\partial \theta}{\partial x}$$

$$\frac{\partial W}{\partial x} = \theta$$

The cross-section moment can be approximated using Gauss integration as

$$\begin{aligned} M(x) &= b\int_{-h/2}^{h/2} y\sigma(x,y)dy \\ &= \frac{bh^2}{4}\int_{-1}^{1} \xi\sigma(x,\xi)d\xi \qquad (y = \frac{h}{2}\xi) \\ &= \frac{bh^2}{4}\sum_{i=1}^{n} w_i\xi_i\sigma(x,\frac{h}{2}\xi_i) \end{aligned}$$

Solution variables $U = [Q \quad M \quad \theta \quad W \quad x_+^1 \quad x_-^i \quad \cdots \quad x_+^n \quad x_-^n]^T$

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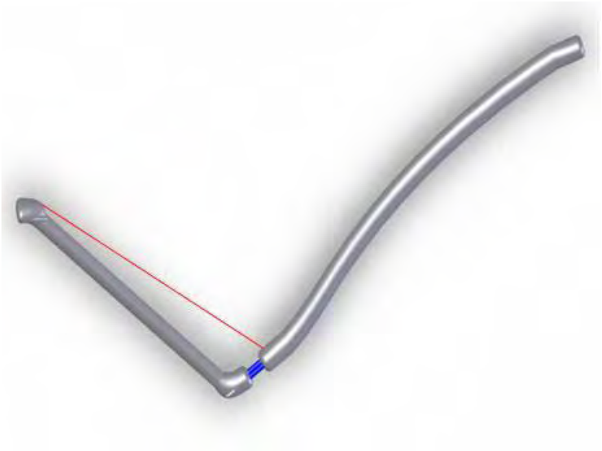

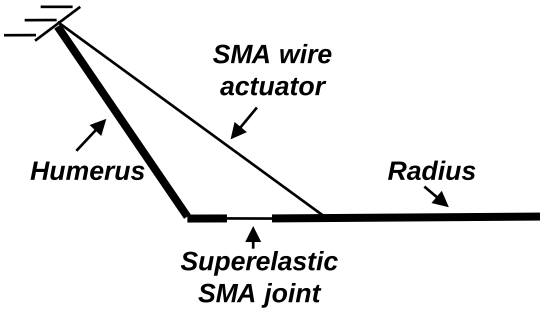

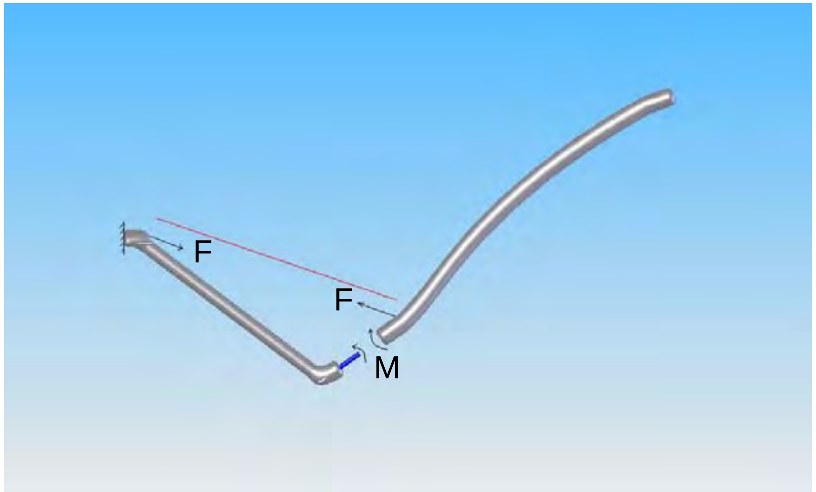

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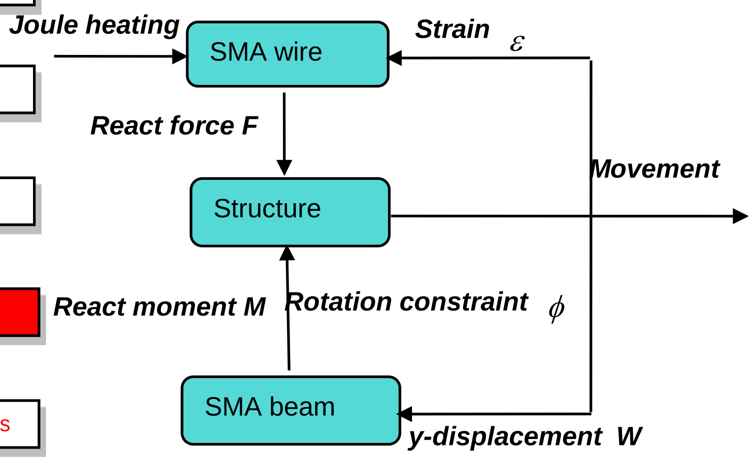
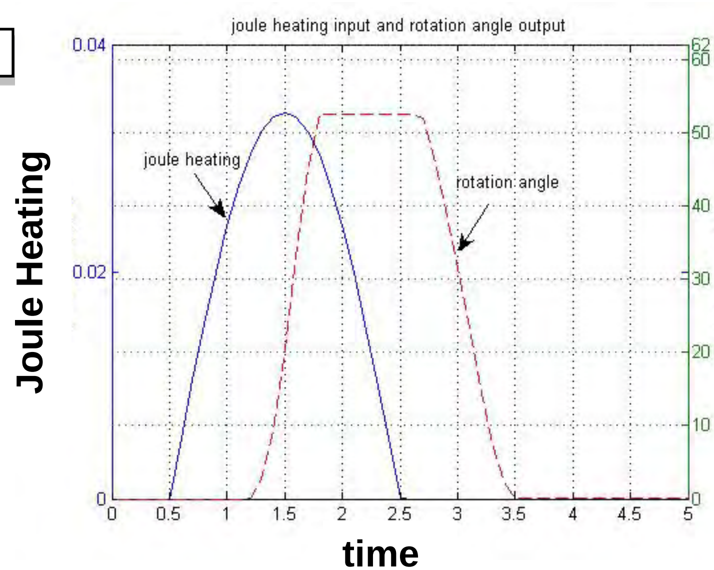
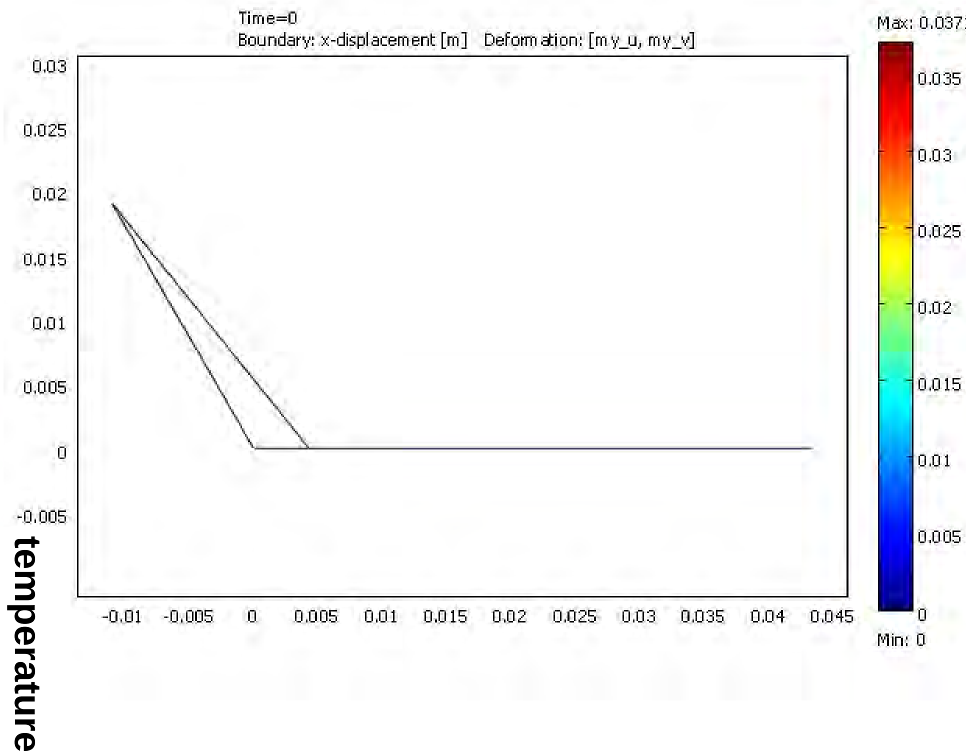
Bone-joint system







SMA wire actuator

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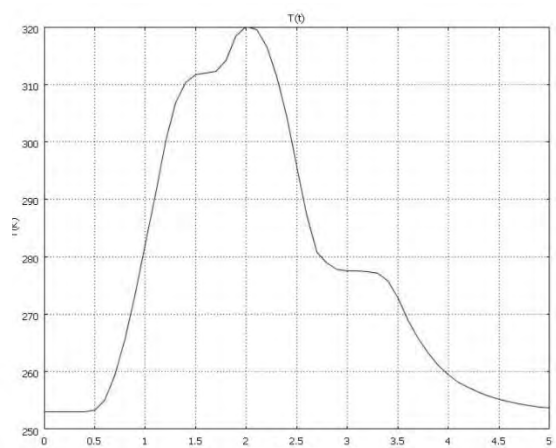
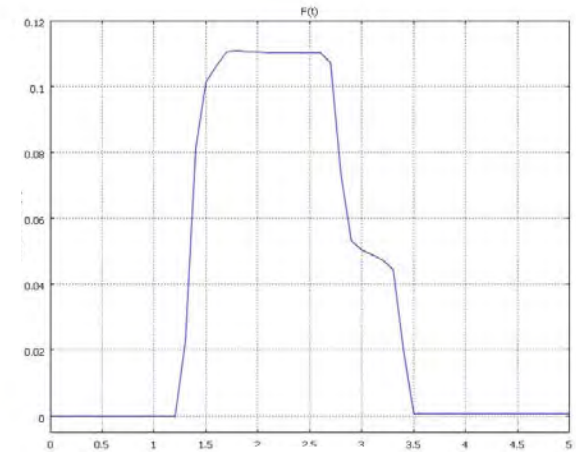
temperature



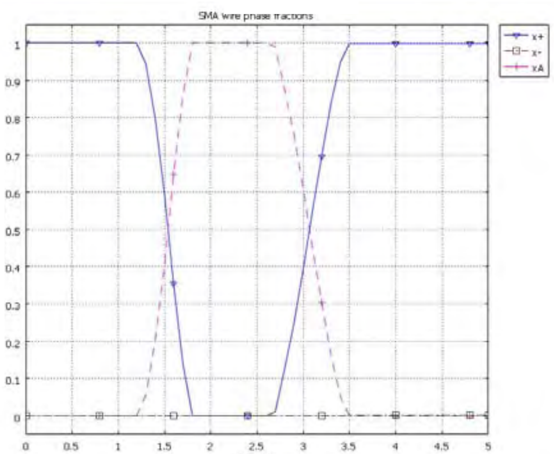
time

actuator force



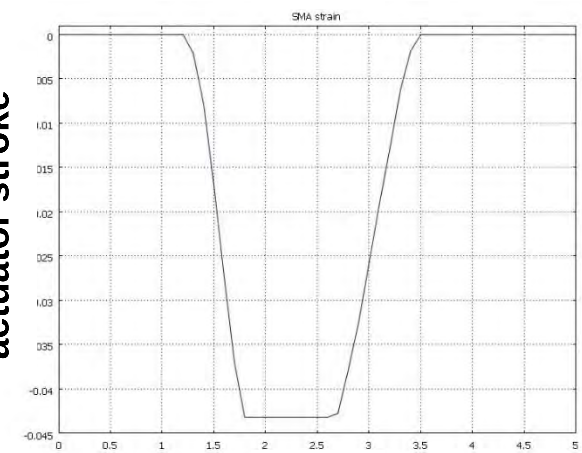
time

phase fractions



time

actuator stroke



time

SMA joint

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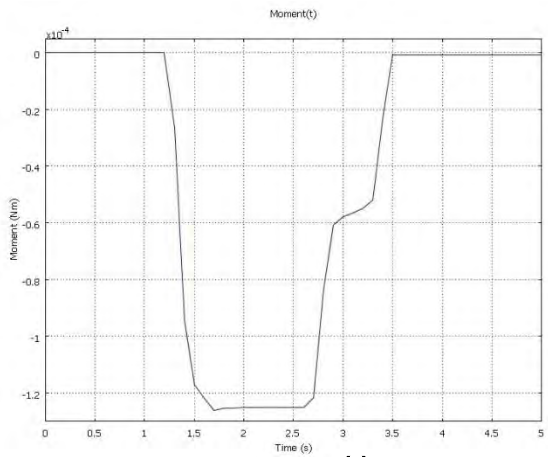
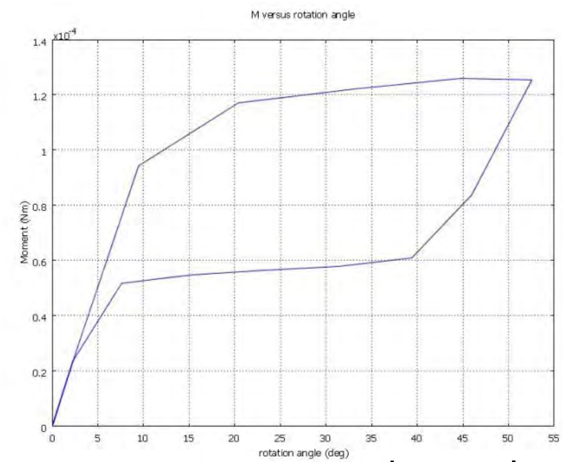
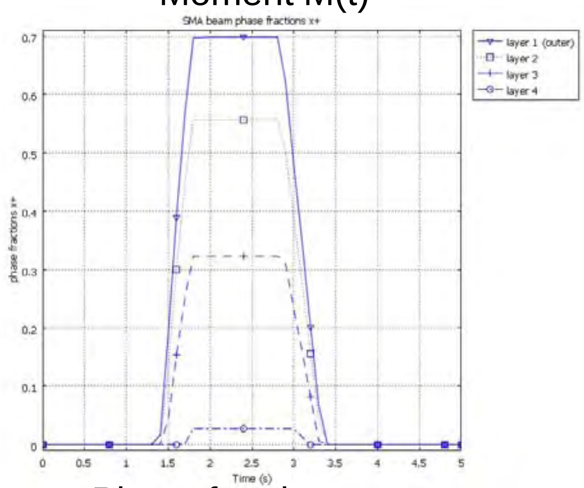


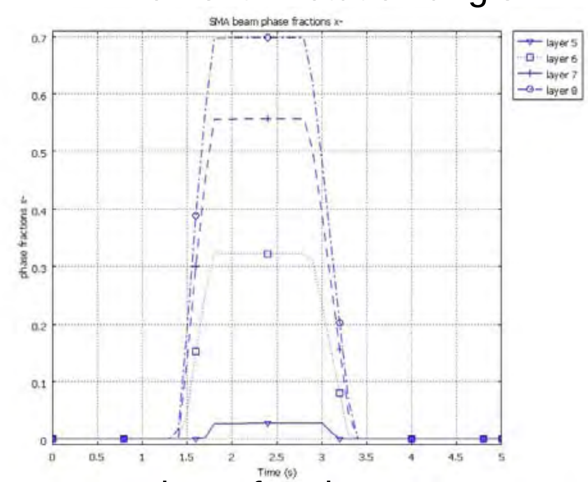
Moment M(t)



Moment ~ rotation angle



Phase fraction x+



Phase fraction x-

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- FE analysis of an adaptive implementing SMA in two different ways
    - Thermally activated actuator wire
    - Superelastic flexible hinge
- Implementation in COMSOL through general PDE mode
- Coupling to Structural Mechanics Module
- Example: BATMAV joint actuation

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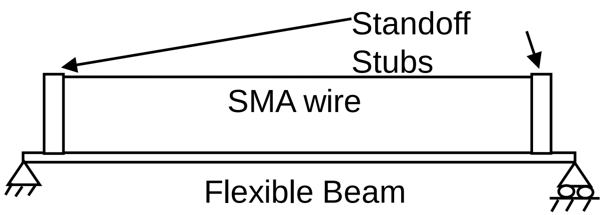
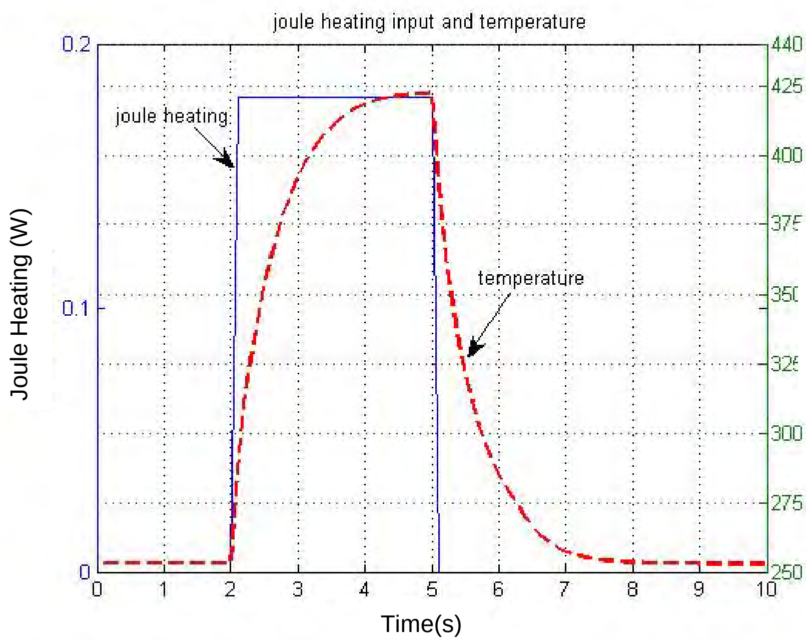
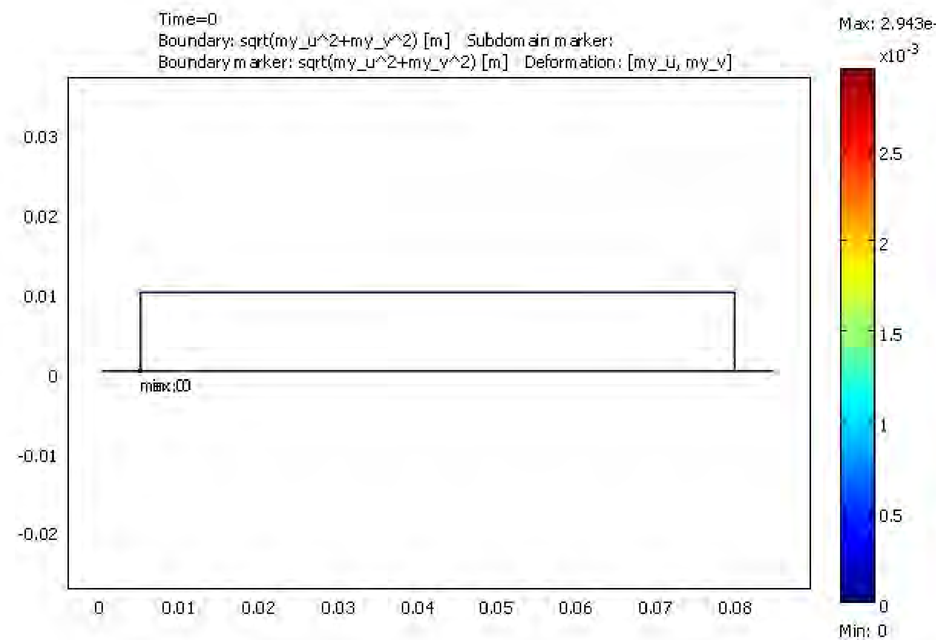
- Extend joint model to 3D
- Two-beam joint
- Simulate entire wing
- Implement control algorithm through Simulink coupling

Thank you for your attention!


Standoff
Stubs
SMA wire
Flexible Beam


joule heating input and temperature
joule heating
temperature
Joule Heating (W)
Temperature (K)
Time(s)
0.2
0.1
0
440
425
400
375
350
325
300
275
250
0 1 2 3 4 5 6 7 8 9 10


Time=0
Boundary: sqrt(my_u^2+my_v^2) [m] Subdomain marker:
Boundary marker: sqrt(my_u^2+my_v^2) [m] Deformation: [my_u, my_v]
Max: 2.943e-3
x10-3
2.5
2
1.5
1
0.5
0
Min: 0
0.03
0.02
0.01
0
-0.01
-0.02
0 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08

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- Superelastic SMA cantilever beam bending