COMSOL CONFERENCE
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Modelling of Viscoelastic Phenomena in Concrete Structures

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Outline

- Introduction
- Modelling sensors in concrete
- Viscoelastic modelling of concrete
- Shrinkage strain modelling of concrete
- Analysis of a sensor in concrete
- Conclusions

Introduction

Market demand:

Structural Health Monitoring (SHM): Understand the health of the structure and the needs for maintenance intervention

e.g. →

Monitoring the pressure in various strategic points of the structure and its evolution over time

http://livesicilia.it/2014/07/07/crolla-il-ponte-fra-ravanusa-e-licata-auto-nel-vuoto-feriti_513263/

OUR OBJECTIVE:

Develop a method that allows us to model concrete mechanical properties and their variation over time, and to model the behaviour of a mechanical sensor embedded in it when external forces are applied to the concrete structure.

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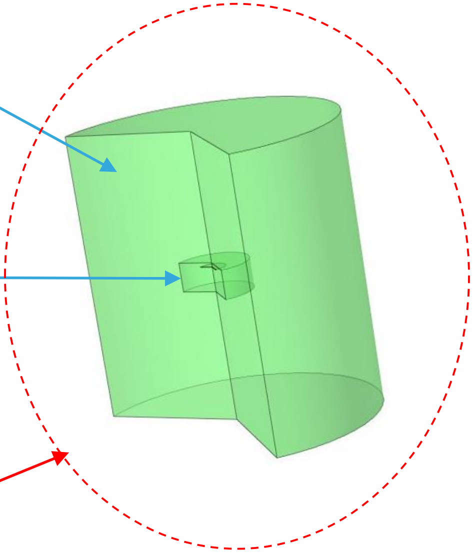
COMSOL for modelling sensors in concrete environments

For a reliable design of an electromechanical sensor in a concrete structure:

- Appropriate modelling of main concrete properties that can impact on an embedded sensor response
- Modelling of the sensor features
- Modelling of the total system and of the combined effect of an external force and of the concrete-induced effects on the sensor



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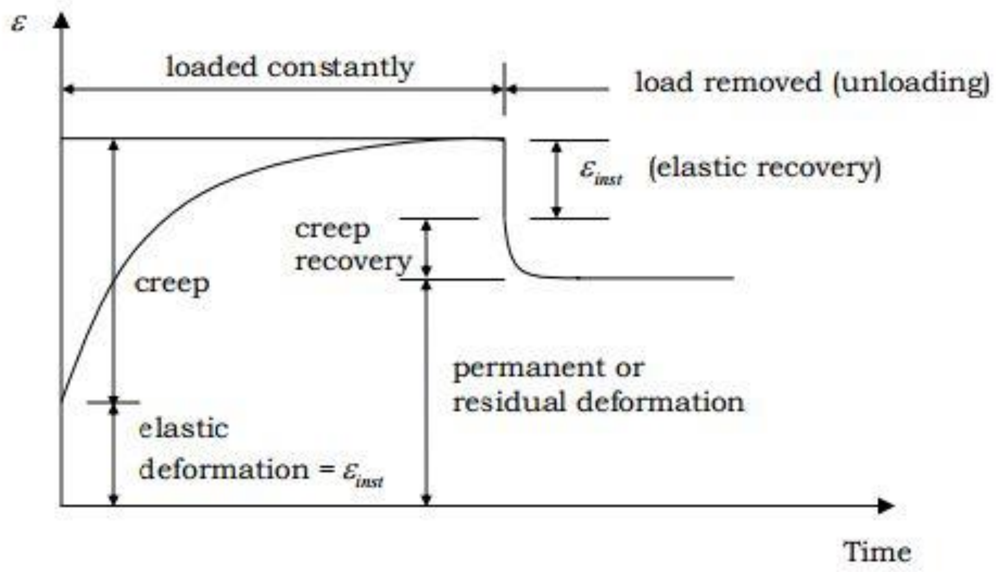
Concrete - Creep

- **Creep** is the property of materials by which they continue deforming over considerable length of time under sustained stress.
- **Concrete** under stress undergoes creep (gradual increase of strain with time)
- In concrete, **creep deformations are generally larger than elastic deformation** and thus creep represents an important factor affecting the deformation behavior.



Concrete under constant axial compressive stress

MIT: Mechanics and design of Concrete structures. 2004

Modelling viscoelasticity in concrete (1/2)

Concrete viscoelasticity is classically **described by means of the Creep Function**, representing the stress dependent strain per unit stress

Extract from **ModelCode2010**:

Unless special provisions are given the relations are valid for ordinary structural concrete (15 MPa ≤ f_{cm} ≤ 130 MPa) subjected to a compressive stress $|\sigma_c| \leq 0.4 f_{cm}(t_0)$ at an age at loading t_0 and exposed to mean relative humidities in the range of 40 to 100 % at mean temperatures from 5 °C to 30 °C. The age at loading should be at least 1 day.

f_{cm} is the mean compressive strength in [MPa] at an age of 28 days

The stress dependent strain at time *t* (in days) may be expressed as:

$$\varepsilon_{c\sigma}(t,t_0) = \sigma_c(t_0)\left[\frac{1}{E_{ci}(t_0)} + \frac{\varphi(t,t_0)}{E_{ci}}\right] = \sigma_c(t_0) J(t,t_0)$$

The Creep Function $J(t,t_0)$ of concrete can be calculated according to the concrete equation theory, for given concrete class, sample size and humidity conditions, and after specifying the concrete age at the loading instant.

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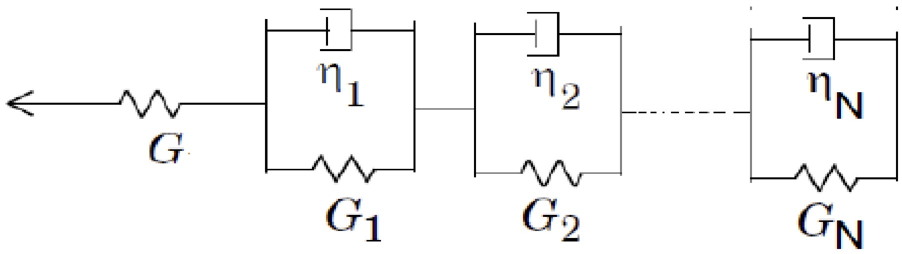
Modelling viscoelasticity in concrete (2/2)

STRATEGY: creep in concrete can be more easily analysed modelling the material by means of a so-called **Kelvin chain**



an elastic spring to represent the instantaneous stiffness plus n Kelvin-Voigt branches connected in series

Kelvin chain parameters for Concrete Modelling can be obtained by means of appropriate fitting of the Creep curves.

"Creep function" for a loading instant τ:

$$J(t-t_0)=\frac{1}{G_0}+\sum_{i=1}^{n}\frac{1}{G_i}\left[1-e^{-\frac{t-t_0}{\tau_i}}\right]$$

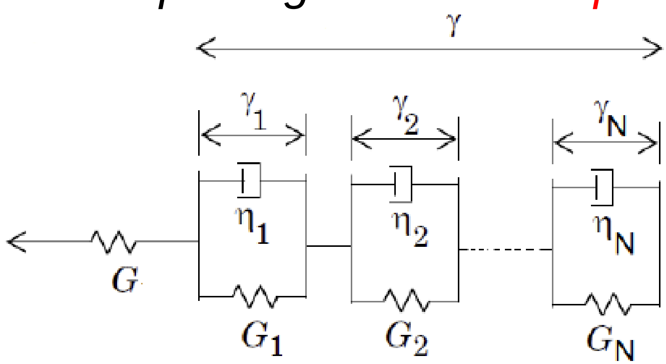
$$\tau_i=\frac{\eta_i}{G_i}$$

retardation time per branch (for each branch, estimates the time required for the creep process to approach completion)

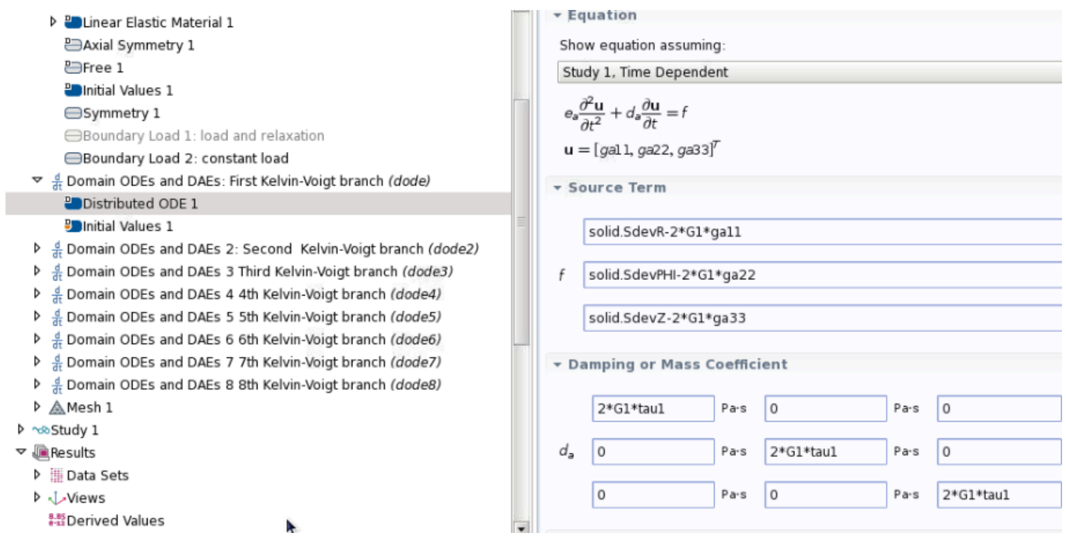
We built a **new mathematical model for Viscoelasticity**, exploiting Equation-based Modelling capabilities of COMSOL.

Kelvin chain model of viscoelasticity in COMSOL (1/2)

We built in COMSOL a ***new mathematical model for Viscoelasticity****,*
exploiting COMSOL Equation-based Modelling capabilities:



Implementation of Kelvin chain model by introducing differential equations in a “Linear Elastic Material” framework:



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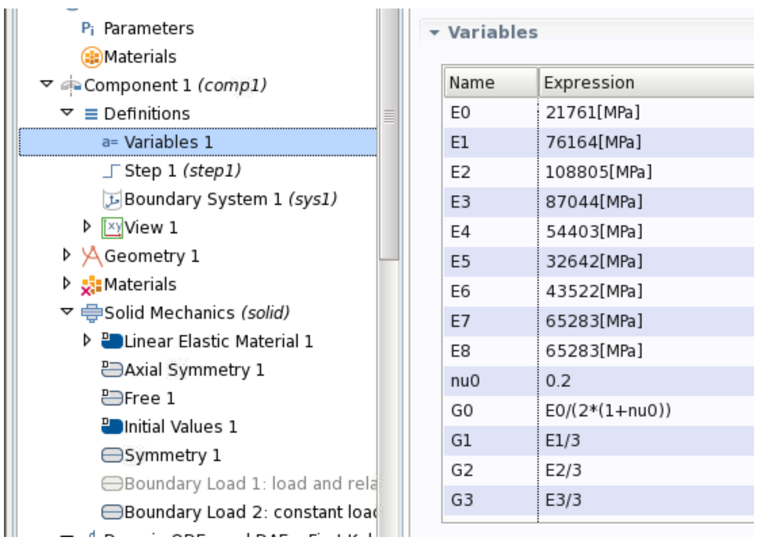
Kelvin chain model of viscoelasticity in COMSOL (2/2)

Viscoelastic material is described as a domain obeying a set of equations, and all the material parameters are manually introduced as “Variables”:



Values used for Kelvin chain:

(Values had been obtained by fitting of the exact creep function calculated for a given creep sample, using Kelvin-chain parametric equation)

E0	21761[MPa]
E1	76164[MPa]
E2	108805[MPa]
E3	87044[MPa]
E4	54403[MPa]
E5	32642[MPa]
E6	43522[MPa]
E7	65283[MPa]
E8	65283[MPa]
tau1	5e2[s]
tau2	5e3[s]
tau3	5e4[s]
tau4	5e5[s]
tau5	5e6[s]
tau6	5e7[s]
tau7	5e8[s]
tau8	5e9[s]

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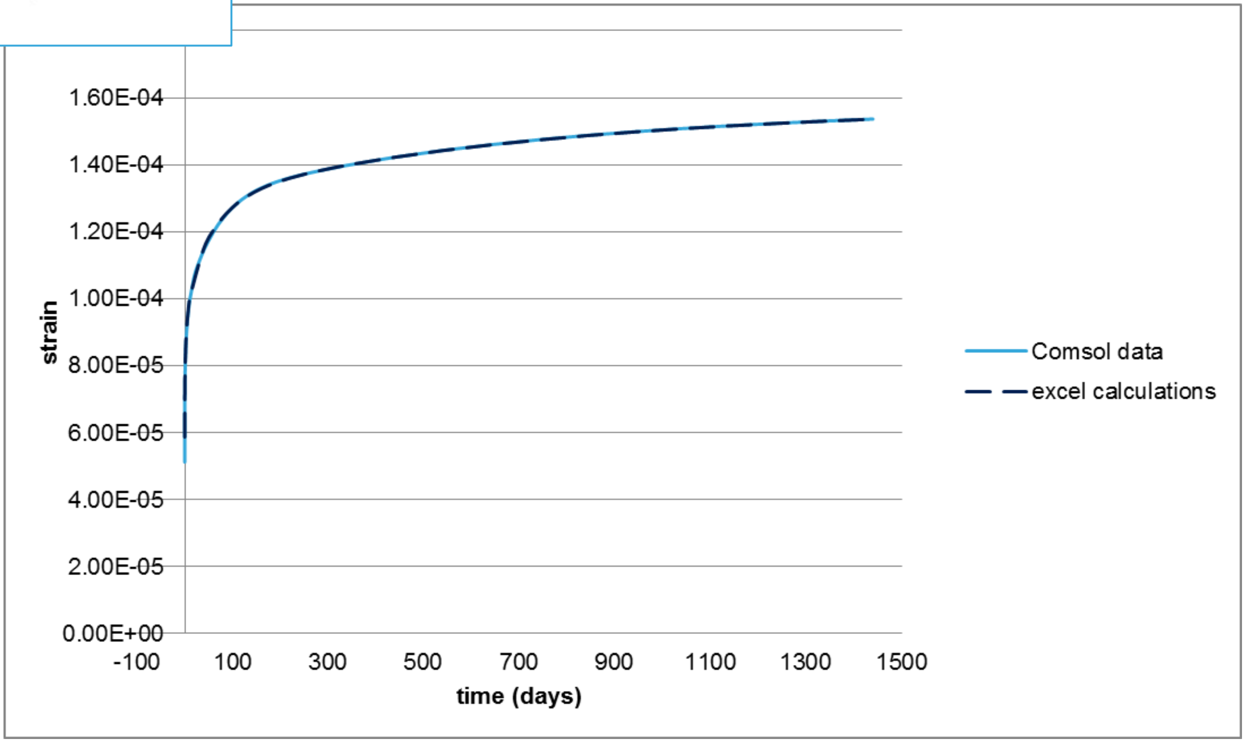
COMSOL results *vs* excel calculations of the Kelvin model (1/2)

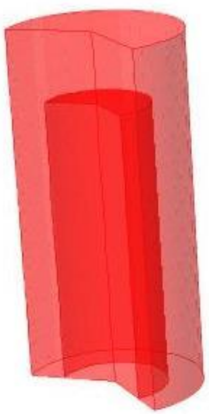
The model has been applied to a cylinder (with given concrete material parameters), L=20cm, R=5cm, with an applied load of 1MPa.

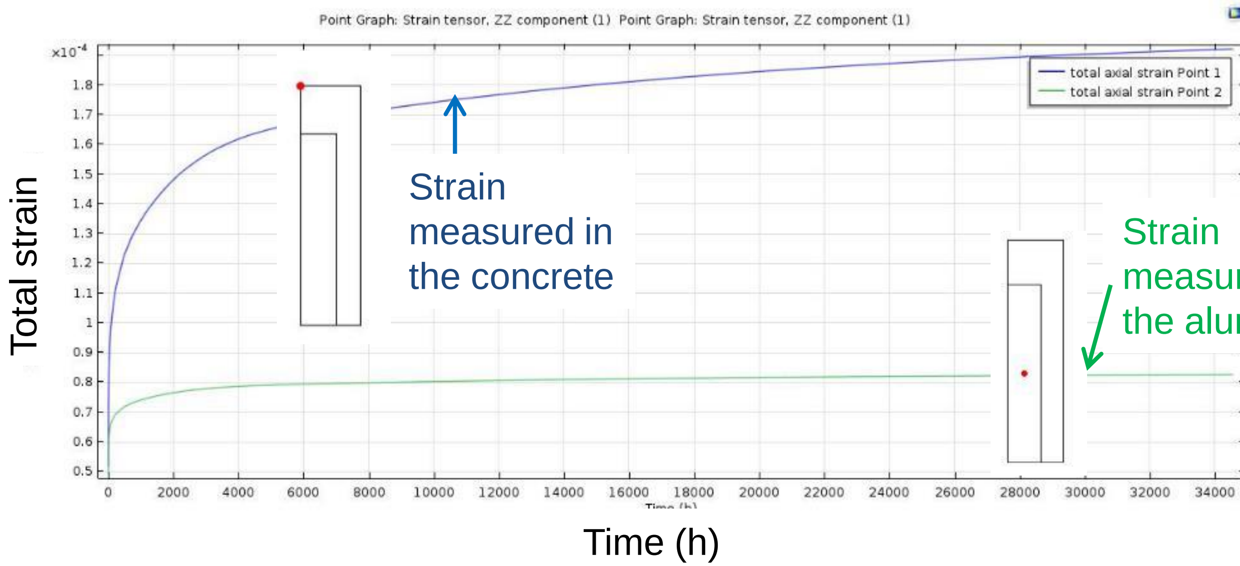


Additional notes

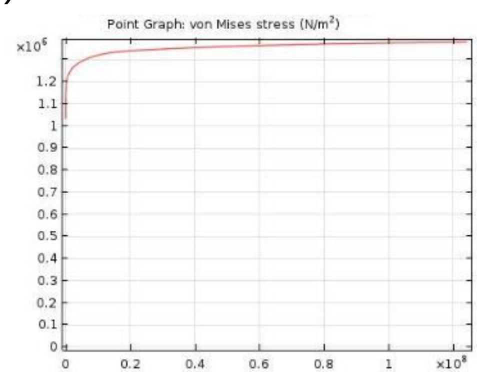
- The viscoelastic material can be only a portion of the modelled structure. Other domains, having no viscoelastic behaviour, can be built in the same COMSOL file.

EXAMPLE: model consisting of the same concrete cylinder, with an alumina (not viscoelastic) concentric cylinder inside. A constant 1MPa was applied on top of the concrete:





The time dependence of the strain in alumina is consistent with the time-variable stress on the top of alumina:



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Concrete shrinkage theory in ModelCode

Total shrinkage in concrete structures:

$$\varepsilon_{cs}(t,t_s) = \varepsilon_{cas}(t) + \varepsilon_{cds}(t,t_s)$$

where shrinkage is subdivided into the autogenous shrinkage $\varepsilon_{cas}(t)$:

$$\varepsilon_{cas}(t) = \varepsilon_{cas0}(f_{cm}) \cdot \beta_{as}(t)$$

and the drying shrinkage $\varepsilon_{cds}(t,t_s)$:

$$\varepsilon_{cds}(t,t_s) = \varepsilon_{cds0}(f_{cm}) \cdot \beta_{RH}(RH) \cdot \beta_{ds}(t-t_s)$$

MAIN PARAMETERS INFLUENCING THE SHRINKAGE:

- t is the concrete age (in days)
- t_s is the concrete age at the beginning of drying (in days)
- $(t-t_s)$ is the duration of drying (in days)
- the coefficient $\beta_{RH}(RH)$ takes into account the effect of the ambient relative humidity RH
- the function $\beta_{ds}(t-t_s)$ describing the time-development, is a function of the notional size h of the sample

In COMSOL, shrinkage has been modelled as a thermal contraction, introducing an effective thermal variation ΔT.

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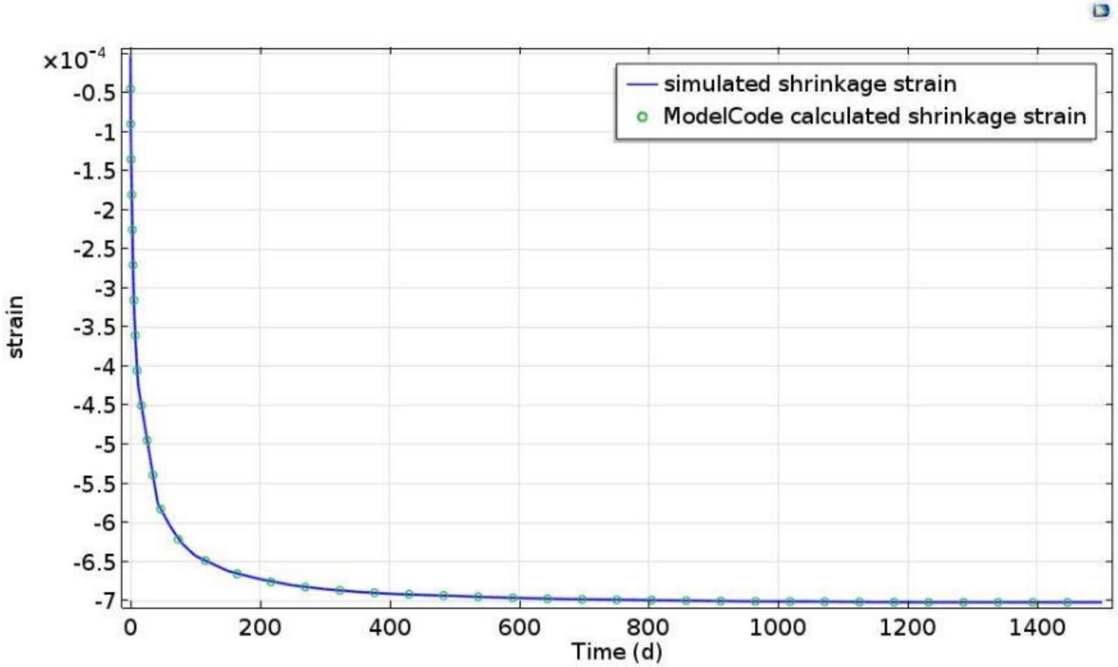
Shrinkage strain in concrete

The model has been applied to a cylinder (with concrete material parameters), L=20cm, R=5cm.



No load applied.

ONLY SHRINKAGE STRAIN

maximum strain ~-7e-4

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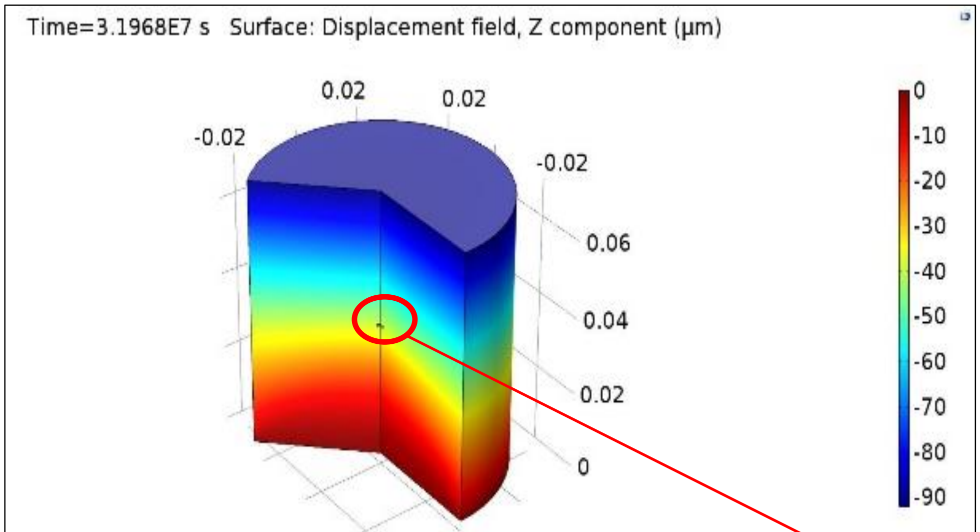
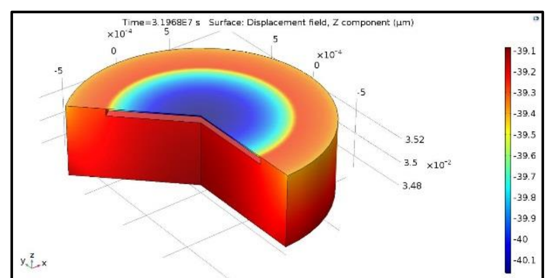
Analysis of a sensor in a concrete environment

Pressure Sensor structure (a Silicon membrane) in a concrete sample:



10 MPa load applied on top of the concrete sample



Radius: 2mm
Height: 600µm

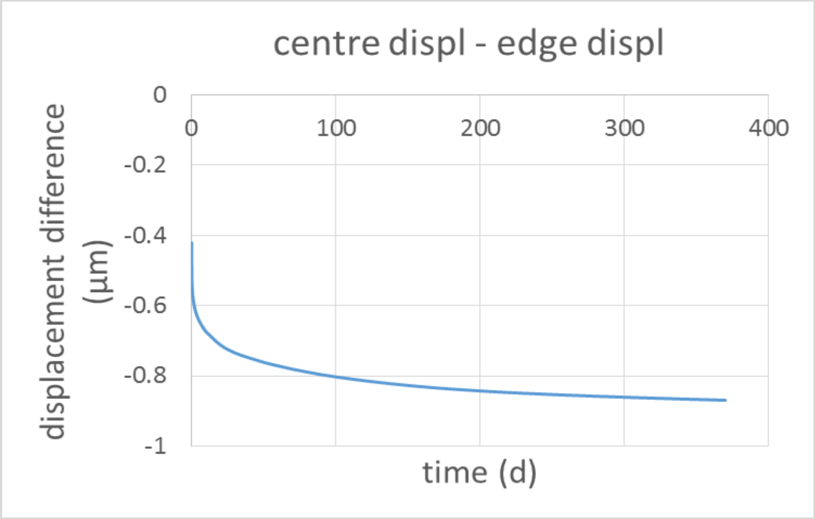
Membrane:

Thickness: 10µm
Radius: 700µm
Cavity depth: 50µm

Modelling results: creep effects (1/2)

370 days time span for the time-dependent simulation
ONLY CREEP considered (no shrinkage strain)
10MPa CONSTANT EXTERNAL LOAD APPLIED

First observation: Modification in the membrane displacement at its centre over time (assuming the membrane edge as a reference)



Initial value: ~ -0.42 µm
Final value: ~ -0.87 µm

Membrane effective deformation has more than doubled its value

Correlation with a change in the stress state of the membrane ?

Modelling results: creep effects (2/2)

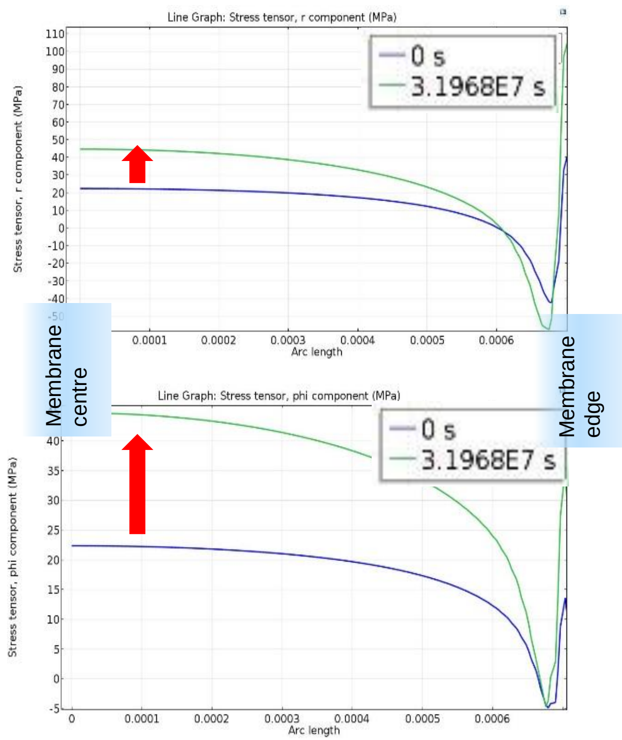
Modifications in the membrane radial and angular stress distributions:

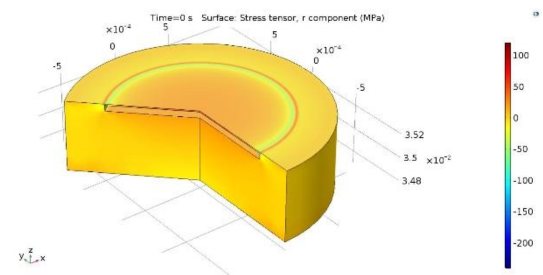
Time variations of the radial and angular stress components (taken along a radius)

3D plots

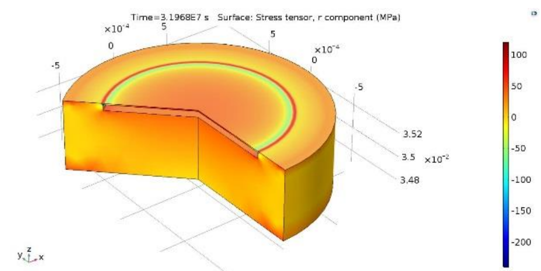
RADIAL STRESS:

ANGULAR STRESS:





Radial stress t=0



Radial stress t=370days

<u>If piezoresistors are fabricated on the membrane:</u>

Creep-induced time dependence of the stress will be observed in piezoresistors.

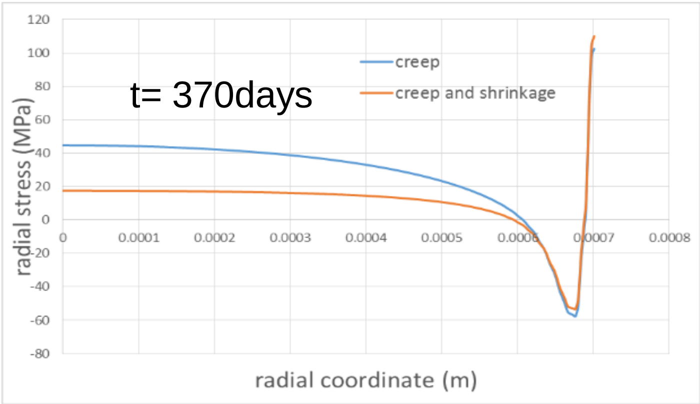
(relative weight of the two components depending on the actual position of piezoresistors)

Modelling results: adding the shrinkage effect

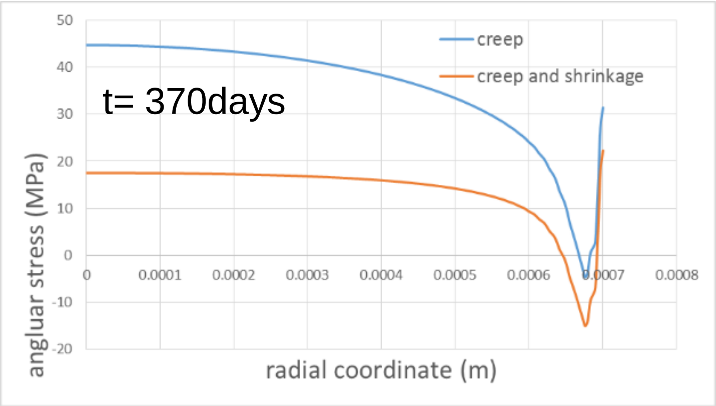
Only a very small additional modification in the membrane displacement at its centre over time (from –0.42 µm to -0.9 µm)

BUT: Relevant changes are also in this case observed in the stress distributions in the membrane:

RADIAL STRESS:



ANGULAR STRESS:



<u>If piezoresistors are fabricated on the membrane:</u>

Both creep-induced and shrinkage-induced time dependence of the stress will be observed in piezoresistors.

Time dependent output voltage of the sensor

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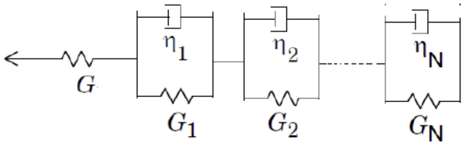
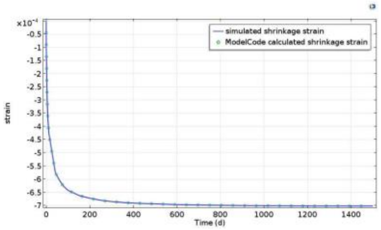
Conclusions

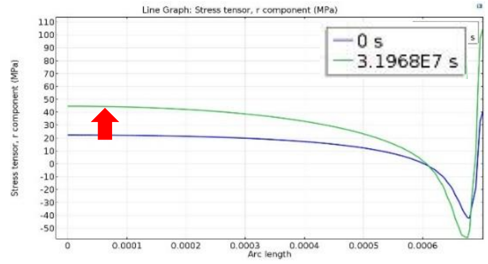
- Concrete creep was modelled by means of a Kelvin-chain model approach exploiting the *Equation Based Modelling* of COMSOL



- Shrinkage strain of concrete was modelled using an equivalent thermal contraction



- Both viscoelastic creep and shrinkage strain are *critical phenomena* to take into account for the design of reliable sensors for concrete.



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THANK YOU!!!

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