# Models with Helical Symmetry Studied in a 2D Plane

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## Abstract

Tubes with helically corrugated walls are used in offshore technology for their strength and flexibility and in heat exchanger technology for their higher rate of heat transfer. The study of a computational problem with a helical symmetric geometry may be performed in a lower dimension as is commonly done with cylindrical and spherical symmetric geometries. However, the resulting equations can become complicated. For CFD problems the symmetry has only been fully used for study of flow at low Reynolds numbers (Wang 2006, Pozrikidis 2006). Formulations with full applicability, like turbulent flow, currently make use of a periodic symmetry group only. This is done by studying a small piece of the tube and applying periodic boundary conditions (Kim 2000, Pisarenco 2007).

The use of a helical symmetry could be used more efficiently, without generating complicated equations, by applying only a minimal amount of transformations instead of a full transformation to helical coordinates. The set of Euclidian coordinates can be retained allowing easy implementation into computational codes. Only the derivative in the direction of the tube-axis needs to be rewritten in terms of the derivatives in the transverse plane. Without the derivative in the direction of the tube-axis the resulting equations are only dependent on two coordinates. See figure 1 for a schematic representation of the transformation.

As an example the transformation method is applied to the problem of turbulent flow in a helically symmetric corrugated tube. The k-g turbulent model (Kalitzin 1997, cited in Kalitzin and Iacarrino 2002) is used to describe a stationary state of the fluid motion.

The computations are performed using the Weak Form PDE interface of COMSOL Multiphysics®. Pseudo-time stepping, adaptive mesh refinement and a sweep analysis of the pressure drop parameter were used to improve stability of convergence, which was difficult at high Reynolds numbers and very fine mesh size.

The friction factor from the model is plotted in a moody chart. At low Reynolds numbers ($Re < 100$) the friction factor is related to 1/Re but unlike non corrugated smooth tubes the friction is higher. At high Reynolds numbers ($Re > 1000$) the friction factor can be related to the, experimental, Colebrook Equation. However, the exact relation does not depend only on roughness but also on the pitch angle and number of corrugations. At intermediate Reynolds numbers ($100 < Re < 1000$) the computational analysis shows a typical transition that is in agreement with experimental data from literature (Garcia et al. 2012) and initial experiments in our

own lab.

This model is very efficient and proofs to be promising for further analysis of a full range of dimensionless parameters describing the corrugation geometry as well as studying differences of corrugations shapes. It is also desirable to include heat transfer into the two dimensional analysis but this may be more difficult since the temperature distribution along a tube is not constant and hence the solution has no helical symmetry.

## Reference

C. Y. Wang. Effect of helical corrugations on the low reynolds number flow in a tube. AiChe J, 52(4):2008–2012, 2006.

C. Pozrikidis. Stokes flow through a twisted tube. J fluid Mech, 567:261–280, 2006.

J.-H. Kim, Fin shape effects in turbulent heat transfer in tubes with helical fins, Phd. Thesis Rensselaer Polytechnic Institute, 2000

M. Pisarenco, Friction Factor Estimation for Turbulent Flows in Corrugated Pipes with Rough Walls, Msc. Thesis Eindhoven University, 2007

G. Kalitzin and D. Iaccarino, Turbulence modeling in an immersed-boundary RANS method. Center for Turbulence Research Annual Research Briefs, 415-426 2002

A. Garcia, et al. The influence of artificial roughness shape on heat transfer enhancement: Corrugated tubes, dimpled tubes and wire coils. Appl Therm Eng, 35:196-201 2012

## Figures used in the abstract

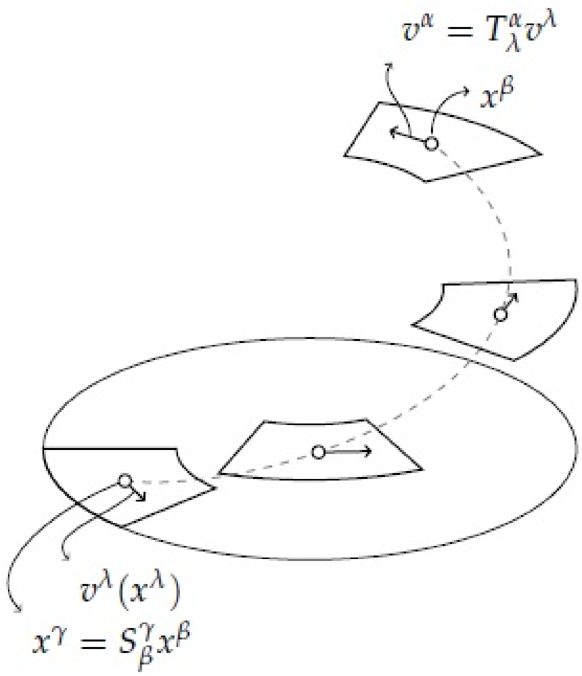


**Figure 1**: The transformation rule is found using the symmetric property v^α(x^β) = T^α_λv^λ(S^γ_β x^β) where T^α_λ and S^γ_β are transformation matrixes that rotate the coordinates and vectors. The solution of the vector v^α in point x^β is related to the solution of the vector v^λ in point x^γ. By transforming the point x^β to x^γ we find v^λ from which we calculate v^α. Applying the chain rule for derivatives we find the transformation rule that transforms the derivatives in the direction of the tube axis.