Magnetotelluric response distortion over rugged topography

Daniele Rizzello[1], Patrizia Canepa[1] and Egidio Armadillo[1]

1. Dipartimento di Scienze della Terra, dell'Ambiente e della Vita, Universita' di Genova, V.le Benedetto XV, 5, Genova, Italy.

Abstract

Topographic effects on magnetotelluric responses may be severe on rugged terrains.
Finite elements simulation is a valuable tool to quantify this effect, due to its capability to match real morphologies. To do the estimate of the distortion, the AC/DC Module of COMSOL has been employed, using a model of homogeneous resistivity on which a DEM (Digital Elevation Model) of the Deep Freeze Range (Victoria Land, Antarctica) has been superimposed. Then, the MT responses at several surface sites has been computed.

Introduction

The aim of this work has been to evaluate the capability of Comsol to properly model the Magnetotelluric (MT) topographic response distortion in a peculiar context, given by a rugged earth topography underlying an ice cap.

MT method is a passive geophysical technique that exploits the natural EM field to investigate the subsurface resistivity distribution.

The spatial components of the electric field (E) associated with induced currents (fig.1), are measured at surface by means of electrical dipoles, while magnetic field (H) components are detected by magnetometers (fig.2). The so measured fields are transformed into frequency domain and their ratios (*transfer functions*) computed.

The ratio E/H at a particular frequency is proportional to the resistivity of the subsurface materials up to a depth proportional to the EM *skin depth* at that frequency. This quantity is

computed as $\delta=\sqrt{\frac{2}{\omega\mu\sigma}}$, where ω is the angular frequency, and μ and σ are the magnetic permeability and the electrical conductivity of the investigated material, respectively. MT data acquisition is usually designed to span a frequency range from 10e-4 to 10e3 Hz.

Thus, considering different fields components and frequencies, an *impedance tensor*, in which the transfer functions are stored, is built:

$$Z(f)=\begin{pmatrix} Z_{xx}(f) & Z_{xy}(f) \\ Z_{yx}(f) & Z_{yy}(f) \end{pmatrix}$$

In an hypothetical 2D context, the diagonal components of the Z tensor are null, so only the $Z_{xy}(f)$ and $Z_{yx}(f)$ transfer functions are considered. Normally, for data interpretation a real-valued *apparent resistivity* is computed, instead of complex tensor components; it's given by

$$\rho=\frac{1}{\mu\omega}|Z|^2$$

The ρ_a curves are then inverted by means of numerical algorithms to retrieve 1D, 2D or 3D resistivity models.

As known, telluric currents patterns are distorted by topography (see e.g. Vallianatos, 2002, Wannamaker, 1986). Considering, for instance current flow in the 2D plane, below a valley an increase of the current density occurs and contrarily below a topographic high (fig.3). Thus, in the first case we'll have an E-field increase, while in the latter case, a decrease. Apparent resistivity will show a similar behaviour too, according to its definition. Thus, prior to any 1D inversion a correction is necessary. Several schemes of 2D/3D topographic correction have been proposed (Gurer,1997; Chocteau,1988; Nam, 2008) and they make use of suitable 2D/3D forward codes to estimate the MT response distortion. Among these, FEM method (e.g. Wannamaker,1986) is particularly suitable for its capability to reproduce rugged topography models.

The basic approach to topographic correction is to compute a distortion matrix *D*, defined as the mathematical operator that transforms the normal impedance Z_0 into the distorted one Z_D: $Z_D=D\,Z_0$. To calculate the D matrix, we must consider the impedance of an homogeneous half space, Z_0 and the impedance obtained numerically from a topographic model, Z_t: $D=Z_t/Z_0$ (Chocteau,1988).

Note that the Z_t impedance is usually computed over an homogeneous topographic model, but other solutions are possible. For instance, Gurer et al. (1997) used a layered topographic model inferred from the supposed geological setting. In this study, we employed a two-layers topographical model with an homogeneous earth underlying an ice layer. To establish the earth resistivity value, we referred to a paper by Armadillo et al. (2004), relevant to studies accomplished in a near area (Rennick Graben), from which we chose a resistivity of 1000 Ohm m. For the ice layer, we assumed a value of 70 kOhm m, as inferred from consolidated petrophysics tables.

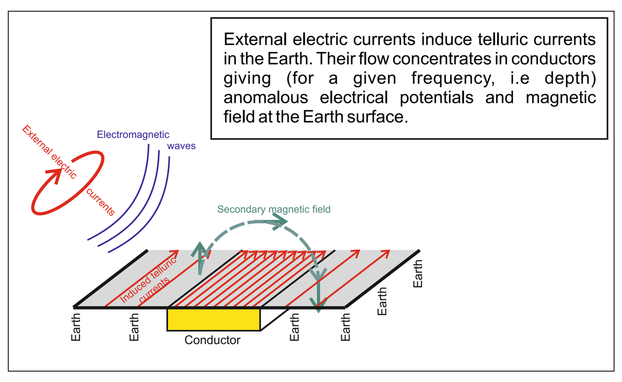


Figure 1. Basic physical principle of magnetotellurics. External electric currents induce telluric currents in the earth. Their flow concentrates in conductors giving at the Earth surface anomalous electrical potentials and magnetic field,for a given frequency, i.e depth (from Armadillo, 2007).

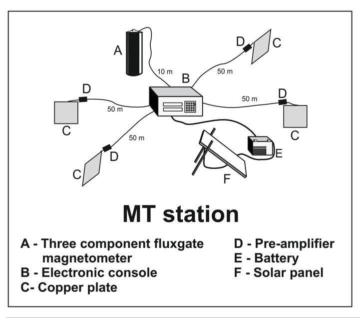


Figure 2. Scheme of MT data acquisition field setup (from Armadillo, 2007).

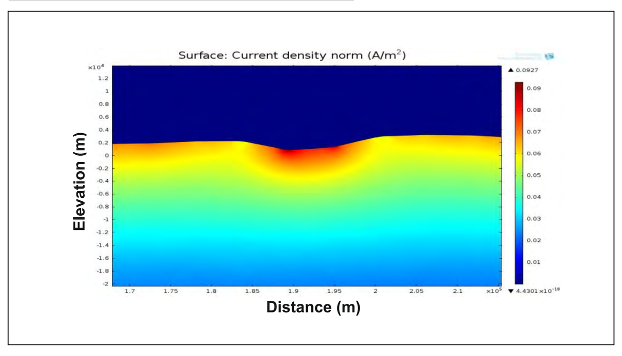


Figure 3. EM induced current density distribution in a 2D topography. Under a valley a current density increase occurs, and contrarily under a hill.

Physical settings

Magnetotellurics is a low-frequency case of EM induction, so one may use the magnetic field interface of the AC/DC module of Comsol Multiphysics®, suited to the quasi-static field conditions. In this interface, the PDE used is the Ampère law expressed in terms of the magnetic vector potential, **A**:

$$(j\omega\sigma-\omega^2\epsilon_0\epsilon_r)\boldsymbol{A}+\nabla\times(\mu_r^{-1}\mu_0^{-1}\boldsymbol{B})-\sigma\boldsymbol{v}\,\boldsymbol{B}=\boldsymbol{J}_e$$

where $\boldsymbol{J}_e$ (external current density) and $\boldsymbol{v}$ (conductor velocity) are set equal to 0. For earth material, μ_r and ε_r set equal to 1.

In a MT 2D problem, two cases occur, depending on the direction of the primary (inducing) magnetic field, **H**. The first occurs when the H field is ideally oriented perpendicular to the 2D plane and the induced currents and the associated E field will lie over it *(TM mode)*. The other case occurs when the H field is parallel to the 2D plane and the electric field is oriented perpendicular to it *(TE mode)*. The relevant boundary conditions will change, as summarized in table 1.

Table 1. Boundary conditions for the MT TE and TM modes.

MT mode	TE	TM
Top surface condition	H_x= 1A/m	H_z = 1A/m
Left and right side boundaries condition	Perfect magnetic conductor ($n \times H=0$)	Magnetic insulation ($n \times A=0$)
Bottom condition	Magnetic insulation ($n \times A=0$)	Magnetic insulation ($n \times A=0$)

This conditions are also illustrated in fig.4. Internal b.c. are set to fields continuity condition.

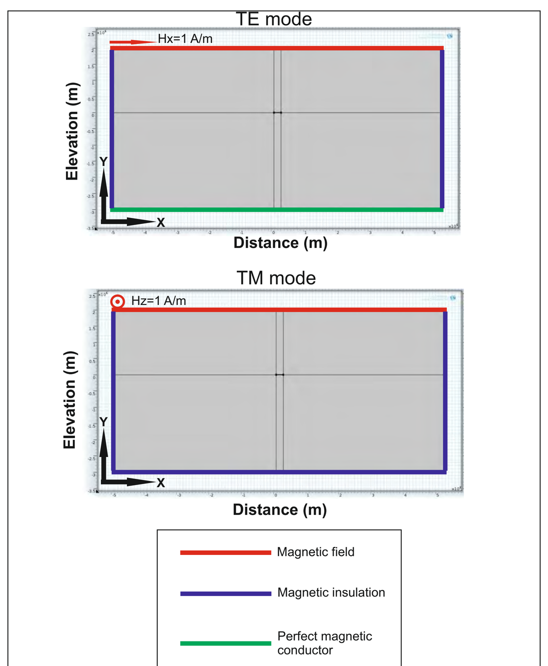


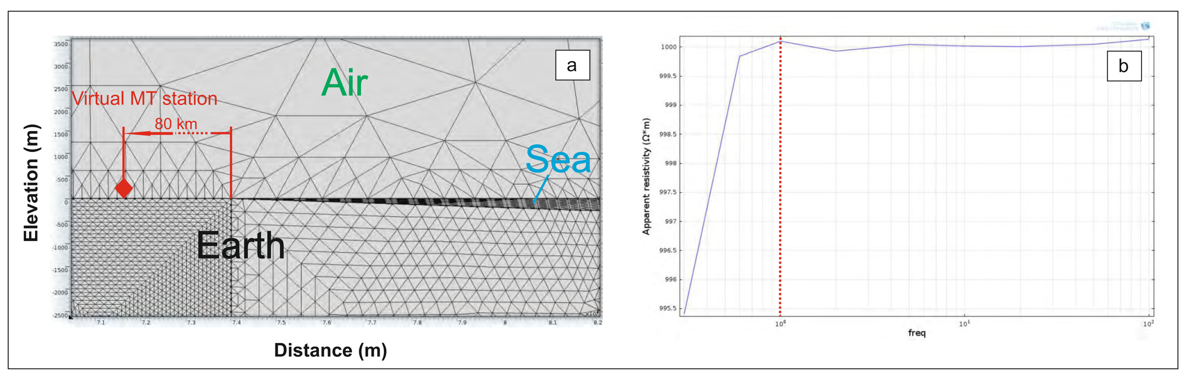
Figure 4. Boundary conditions used to set magnetotellurics numerical simulation by Comsol Multiphysics®.

Model and mesh sizing

The choice of mesh and model sizes has been done taking into account different and somewhat contrasting claims: to retrieve an adequate topographic response, exclude the effect of the sea on the MT response and perform a reliable numerical simulation. Two-dimensional topography has been recovered from a 5km-gridded DEM file of the area.

- To estimate the frequencies suited to investigate the topographic effect, we first evaluated an "average" topographic feature, in terms of height and slope of an "average" hill (respectively about 1300m and 13°). Then, referring to Gurer et al. (1997), we computed the minimum and maximum *h/sd* ratio to properly describe the effect (where *h* is the height of the hill and *sd* is the skin depth, as defined above). Considering a resistivity of 1000 Ohm m, we found that the topographic effect would be quite well detectable from 0.1 to 350 Hz. To limit the computational weight of the simulation, we initially restricted this frequencies from 0.1 to 100 Hz.
- The presence of the sea is a well known problem when MT surveys are carried out in its neighbourhood (e.g. Yang et al., 2008), due to its high conductivity, involving a bias on the apparent resistivity curves. In our simulation, to exclude its effect, we considered a flat model over its land part and the real bathymetry in its underwater part (fig.5a). The profile on which the model was built (fig. 6c) has been chosen perpendicular to the coastline and including the MT station nearest to the sea (about 80 km). Setting the sea resistivity to 0.2 Ohm m, we found that its effect was theoretically detectable up to a frequency of about 1 Hz (fig.5b).
- For a proper numerical simulation of the EM induction, we cared to size the triangular mesh in order to have at least four elements for the minimum skin depth (equal to about 1600 m, corresponding to the max frequency of 100 Hz). To do so, a flat model was set and the response computed at diminishing mesh sizes until a constant response (equal to the resistivity of the model) over the considered frequency range was obtained. We found that the maximum acceptable element size was about 400 m.
The total model size (that includes also lateral blocks necessary to properly simulate MT phenomenon) was chosen as at least twice the maximum *sd*. In practice, this dimension estimation revealed not sufficient during the real model simulation, so it was extended by trial and error until convergence to the solution was obtained. The chosen dimensions of the model are about 10000 km of width and 5000 km of height

Figure 5. Model used for sea effect estimation. The considered profile is shown in fig.6c (profile T). a): detail of the model and mesh used; b): MT response at the nearest station to the coastline. The sea effect, visible as an apparent resistivity strong decrease, is present up to a frequency of about 1Hz.



Conclusions

- FEM analysis by Comsol Multiphysics has revealed a useful tool to evaluate MT response distortion in the presence of a rugged topography. The obtained responses are consistent with those available from geophysical literature.

- Once the simulation has been carried out, a great amount of physical quantities can be easily retrieved for better understanding of the phenomenon and for further processing.

- The combined effect of ice and earth topography on MT responses has been estimated. However, it must be noticed that at the used frequencies the skin depth in the ice is much greater than the ice-cap topography wavelength, so we can expect that the response distortion is mainly due to earth topography. Thus, the differences between the estimated response at the surface (real stations) and at the bedrock, are supposed mainly yielded by the presence of the ice under the stations and not by the distortion effect due to the ice surface roughness.

- These results might be subsequently incorporated into a two-layer topographic data correction procedure.

- Finally, we assumed a "regional" 2D setting, but it should be advisable take into account local 3D effects detectable at the higher frequency used, and subsequently a 3D modeling should be employed.

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Case study

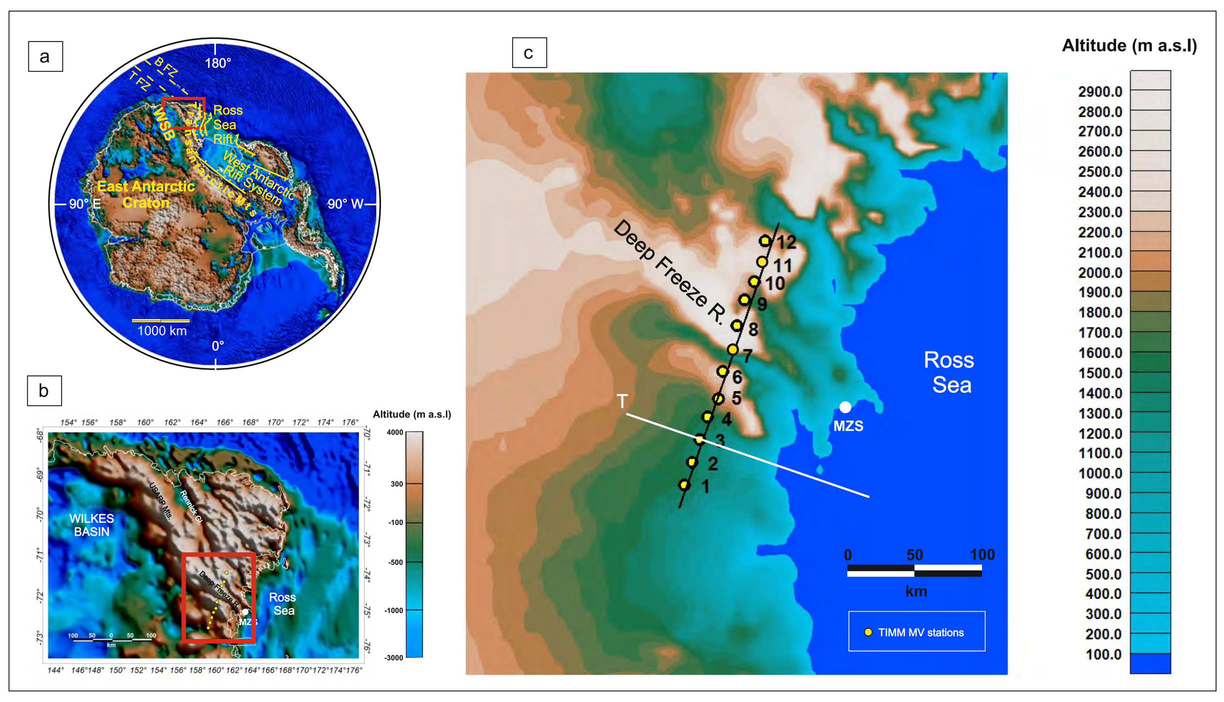
The chosen 2D profile for this study (fig.6c) has the same location of the TIMM (Tectonics and Interior of Mt. Melbourne area) magnetovariational (MV) transept, carried out during the 2002-2003 Italian Antarctic campaign (Armadillo et al, 2006); the strike of the transept is near perpendicular to the main 2D topographical features, so the 2D modeling is deemed quite suitable.

The area is situated in North Victoria Land (East Antartica) and is characterized by the presence of the Deep Freeze Range mountain chain, which reaches, along the profile, an elevation of about 3300 m a.s.l.

At the time of the MV measurements, along the profile a continuous ice cap was present, so we reproduced the same physical conditions in our numerical simulation. We chose this area in order to evaluate the topographic distortion for future MT data acquisition and processing.

The transept has a length of about 180 km, and the hypothetical MT stations, having the same location of the MV stations, are 16 -20 km apart; the total length of the 2D model we used to compute MT response is 220 km.

Figure 6. Locations of TIMM project magnetovariational (MV) stations. In fig. 6c the MV stations, the considered profile for 2D model in this study (black line) and the profile considered for the sea effect test (white line, profile "T") are shown; white circle indicates the Mario Zucchelli Italian Station (fig.6a and 6b modified from Armadillo et al., 2007).



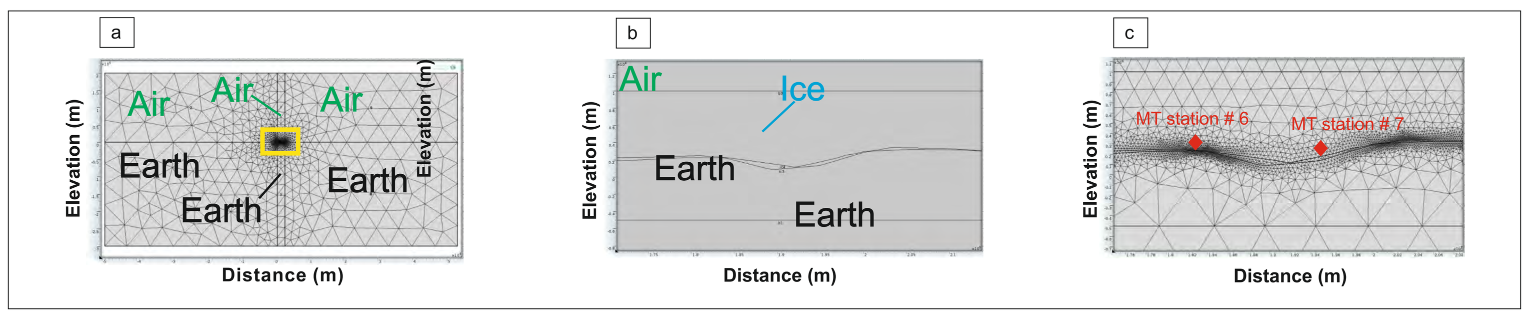


Figure 7. Two dimensional model used for the simulation. a) Entire model mesh and domains. b) Detail of the model geometry and domains. c) Detail of the mesh, with location of 2 MT station.

Results

TE and TM apparent resistivity profiles for the considered frequencies (1,5,10,20,100 Hz) have been yielded (fig. 8a and 8b), to correlate MT response distortion with topography. Note that in the case of a flat topography, the curves would have a constant value equal to 1000 Ohm for both MT modes, that's the bedrock resistivity.

To allow a direct comparison between TE and TM modes behaviours over the topography, four frequencies have been selected and apparent resistivity profiles shown together (fig.9). At the same time, as a reference, the hypothetical bedrock responses have been plotted. From the figure a nearly flat response at bedrock is visible up to a distance of 50 km, with a constant value equal to that of bedrock (1000 Ohm m). As expected, it might also be noted that TE and TM responses often have opposite trends, as can be seen clearly over the main valley, situated about at the middle of the profile (at a distance of 110 km).

A comparison between the surface response (over the ice cap) and the hypothetical bedrock response is shown in fig.10, for the lower and higher frequencies. As can be seen, the apparent resistivity enhancement from bedrock to surface is stronger for the high frequency, since the EM skin depth is smaller, and should be noted that this enhancement is stronger for the TM polarization than TE. Further, while the TM mode maintains a quite clear signature of main valley at 110 km, the TE surface response at 100 Hz doesn't give a clear indication of the valley presence.

For each station and MT mode, apparent resistivity curves are visible in fig.11. All the curves show an increasing trend toward high frequencies, due mainly to the presence of the resistive ice cap; in fact, for higher frequencies, EM skin depth decreases and the MT response is dominated by the shallower materials.

In general, from a qualitative point of view, TE-TM (surface) responses match is better when the s.d. is small respect to the topographic wavelength, and MT local response is nearly 1D; for instance, in the flatter part of the profile (about 0-40 km), the match is good for all considered frequencies, in the central portion, where the topography is rougher, it's never good and in its third part (from 130 km to end) becomes to be quite good from 20 Hz, because s.d. is definitely smaller than the topographic wavelength. Ultimately, when the match gets better, we can hypothesize that the apparent resistivity variations were due to changes of ice thickness.

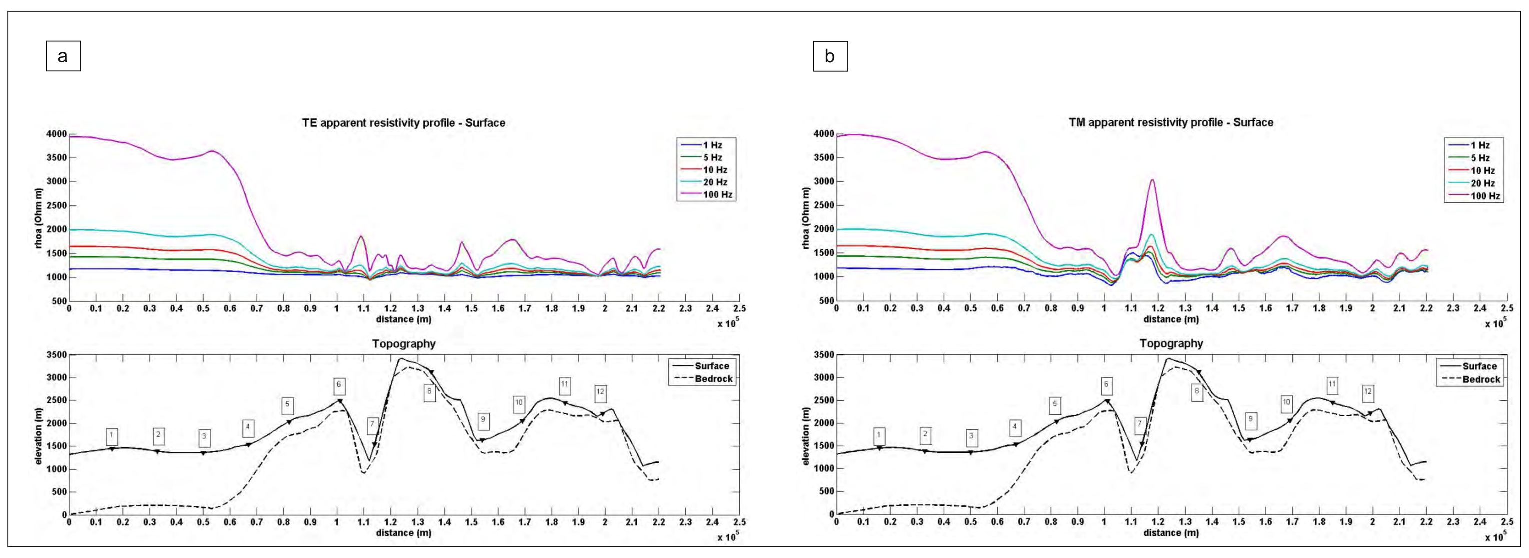
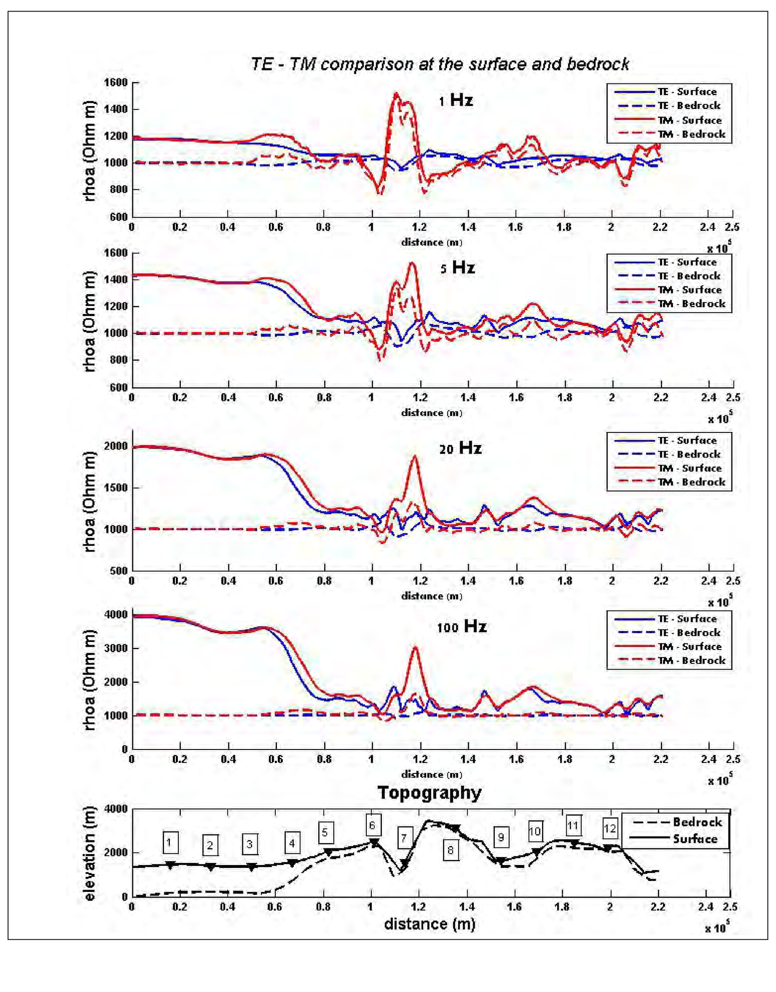


Figure 8. Apparent resistivity horizontal profiles, computed for five different frequencies at the surface of the model. a) TE response. b) TM response. Topography of the ice cap (surface) and of the underlying earth (bedrock) are also shown. MT stations positions along the profile are indicated by framed numbers.



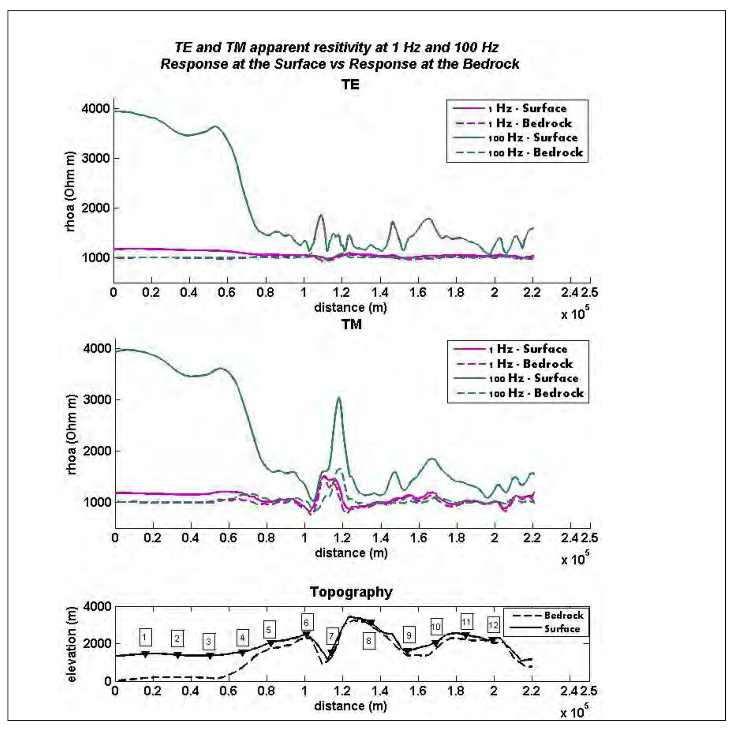


Figure 9 (above). Comparison between TE and TM responses, at the surface of the model for all the considered frequencies (continuous lines). Hypothetical responses at bedrock is also shown (dashed lines).

Figure 10 (left). Comparison between MT responses at the surface (continuous lines) and at bedrock of the model (dashed lines), for the lower and higher considered frequencies (top, TE mode, bottom, TM mode).

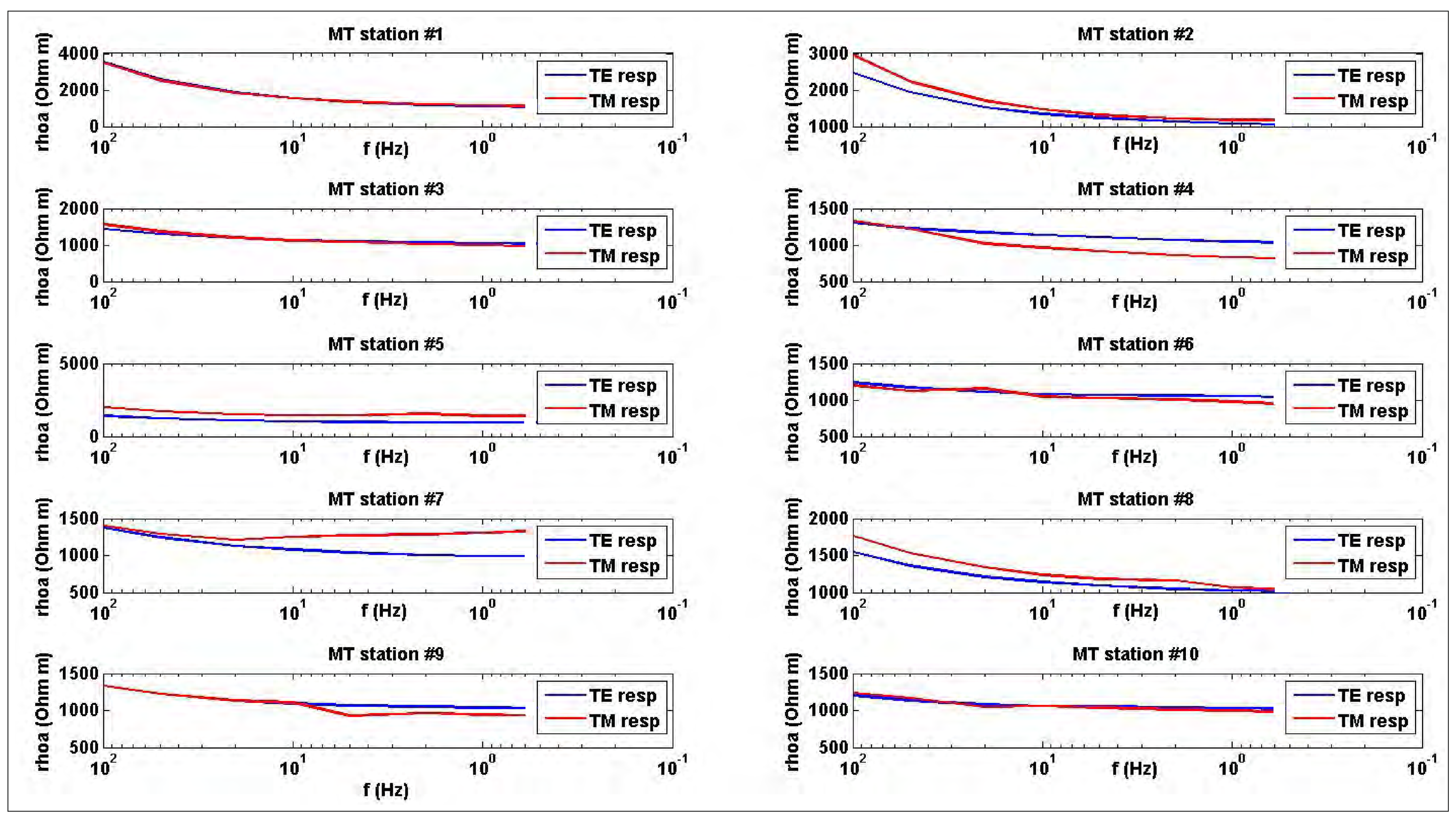


Figure 11. Apparent resistivity curves at each MT station.