

# Geometric Optimization of Piezoelectric Energy Harvesting System

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## Abstract

The aim of this paper is to study the effect of geometrical optimization of an array configuration for a unimorph piezoelectric cantilever element. The process involves connecting a previously optimized unimorph piezoelectric cantilever in a two-element array, then in a three-element array. Geometrical configuration for the model, simulation results, and data analysis are provided. When the array is designed with an optimum spacing between each element and excited at its fundamental resonance frequency, the total energy stored was greatly increased. In comparison to a single piezoelectric element, the total stored energy has increased for the two element array by more than 30 times, and by more than 1800 times for the three-element array.

*Keywords: Piezoelectric energy harvester; Energy harvesting; Array structure; Geometric optimization*

## 1. Introduction

Piezoelectric energy harvesting systems are able to provide a clean source of energy by transforming mechanical vibrations into electrical charges that can be used to operate ultra-low power devices. However; innovative technical approaches need to be developed in order to increase the efficiency of piezoelectric converters, as the percentage of output power in comparison to the input mechanical vibrations is considerably low [1, 2]. The most popular form of piezoelectric energy harvesters is the cantilever beam with one or two piezoceramic layers; these are also called unimorph and bimorph respectively. When a dynamic strain is induced in piezoelectric energy harvester; this results in an alternating voltage output on the device's terminals, and electrical charges are accumulated on the piezoceramic layer/layers [1]. Having low coupling coefficients is the main obstacle that is hindering piezoelectric energy harvesters from being high

output energy scavengers. It has been found that through proper geometric design, the energy harvested from the piezoelectric device could be greatly optimized, giving more current output and hence more power.

This study will analyse MEMS cantilevers, using the piezoelectric effect to harvest enough energy to power up ultra-low power (ULP) devices. This is achieved by connecting a previously optimized unimorph cantilever element that has been proposed in [2], in two different array structures. The array structure is then optimized in terms of the adjacent element spacing. Results for the analysis include measurements for the output charge, open circuit voltage, maximum tip displacement and total stored energy.

## 2. Array Configuration for a Piezoelectric Energy Harvester

Given that the single element is a unimorph piezoelectric cantilever, two array configurations are proposed: the first is a connection of two identical elements and the second is a connection of three identical elements. The setups for both configurations are shown in figure 1; a) for the two elements and b) for the three elements.

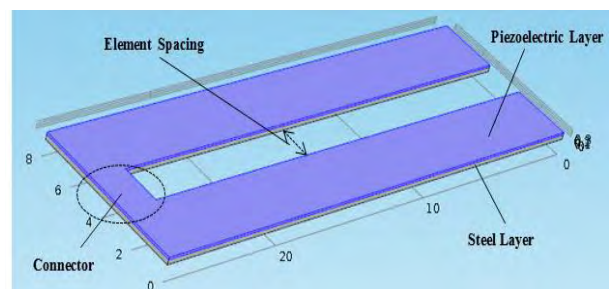


Figure 1: a) Geometrical structure for the two-element array

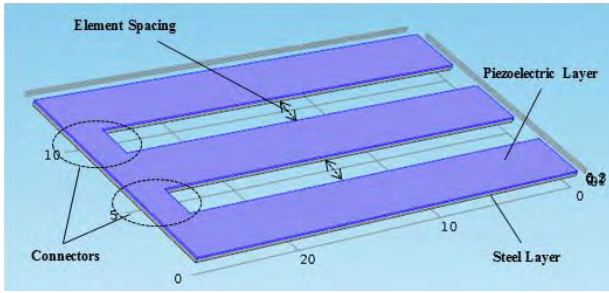


Figure 1: b) Geometrical structure for the three-element array

In the array configuration, shown in figure 1, the results for charge output and the total stored energy are expected to increase since each element will contribute to increase the stored accumulated charge.

### 2.1. Modelling Equations

In order to calculate the accumulated output charge, a customizable variable “D” had to be defined on the top boundary of the piezoelectric layer. This is to measure the electrical displacement of the cantilever.

$$D = q/A \text{ C/m}^2 \quad (2a)$$

Where, D is the electrical displacement (C/m<sup>2</sup>), q is the amount of electric charge (C), and A is the surface area in m<sup>2</sup>. The charge (q) can then be easily calculated from the previous equation using the following surface integration,  $q = \iint_S D \, dA$ .

The electrical displacement “D” can be calculated from the linear piezoelectric equations in the Strain-charge, given by:

$$D = dT + \epsilon_T E \quad (2b)$$

Where,  $S_E$  is the elastic compliance tensor (Pa<sup>-1</sup>),  $T$  is the mechanical stress vector (N/m<sup>2</sup>),  $\epsilon_T$  is the dielectric permittivity tensor (F/m),  $E$  is the electric field vector (V/m),  $d$  is the transverse piezoelectric coefficient tensor (C/N). The open circuit voltage from a piezoelectric material can be calculated from the following equation:

$$V = q/C \quad (2c)$$

Where, V is the piezoelectric generated voltage (Volts), q is the accumulated charge on piezoelectric

terminal (C) and C is the Capacitance of piezoelectric device (μF).

### 2.2. Geometrical Setup

The previously optimized unimorph cantilever [2] was used as the basic element from which the array will be composed. The length, width and thickness of the single elements are going to remain constant throughout the simulation. However, for optimum energy output, it is necessary to optimize the element spacing between the cantilevers so that the generated electric fields would be added up together from both elements. Therefore, the element spacing between the connected cantilevers has been varied in the range of (0.5-2) mm. The optimum element spacing is then applied to the model and the process of simulation and analysis will then be carried out.

### 2.3. Boundary Conditions and Mesh

The piezoelectric energy harvesting device was built and simulated in 3D configuration using COMSOL Multiphysics software tool. The model was simulated in such way that involves the cantilever to be clamped from one end leaving the rest of the cantilever to vibrate freely. This was achieved by applying a fixed constraint on the vertical side of the device along the width. In order to perform the necessary measurements, two electrodes were defined at the top and bottom of the piezoelectric layer. This was achieved by defining a floating potential on the top of the piezoelectric layer and a ground terminal on the bottom of the piezoelectric layer.

The simulation utilizes the sinusoidal z-axis vibrations, where the total vertical force remained constant for both array configurations. The force  $f_r$  for the two-element array and the three-element array was 150 N/m<sup>3</sup> and 100 N/m<sup>3</sup> respectively.

As for the mesh, a slight modification in the meshing structure was essential in order to increase the number of mesh elements. For the two-element array, mesh elements along the width were doubled. While for the three-element array, mesh elements along the width were tripled with respect to that of the single element structure.

### 3. Parametric Study Results

As previously mentioned, the parameter sweep function has been used to find the optimum value for the element spacing between the single cantilevers. The value for element spacing is varied in the range of (0.5-2) mm.

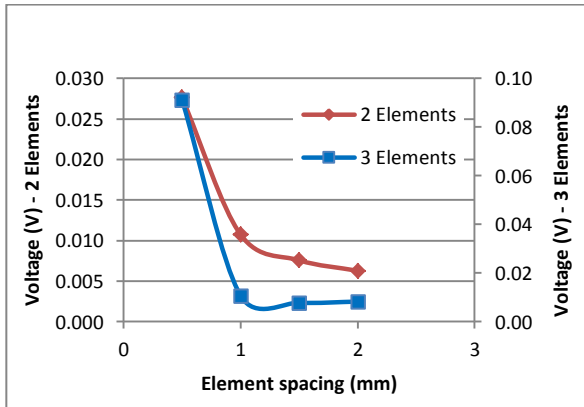


Figure 2: Output voltage for the two-element array and the three-element array at different element spacing

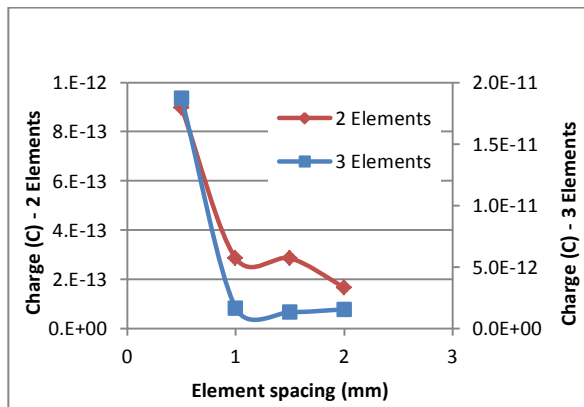


Figure 3: Output charge for the two-element array and the three-element array at different element spacing

From the obtained simulation results shown in both figures 2 and 3, it is clear that the optimum element spacing for both array configurations is at 0.5 mm. This result is obtained when exciting at 300 Hz.

### 4. Eigenfrequency Analysis

An eigenfrequency analysis was conducted in order to identify the resonant frequencies of the piezoelectric device. Knowledge of the resonance frequencies is essential, since operating the piezoelectric harvester at those frequencies will give optimum results with the highest possible energy conversion efficiency. Results for the first four resonance frequencies are shown in figures 4 and 5, for both array configurations, respectively.

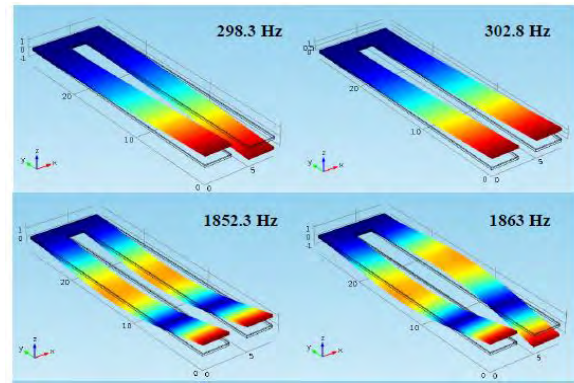


Figure 4: Total displacement for the first four eigenfrequency values for the two-element array

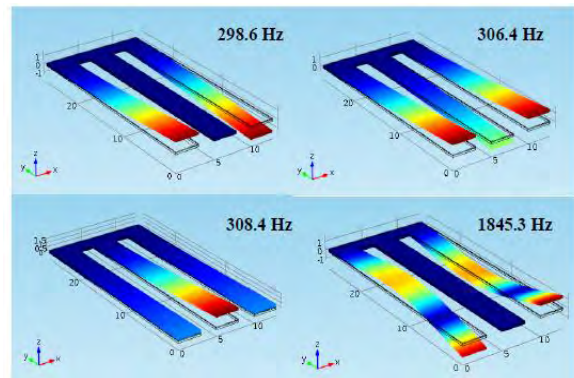


Figure 5: Total displacement for the first four eigenfrequency values for the three-element array

In the previous simulations, the piezoelectric device was excited at 300 Hz as it's the frequency at which all three elements are excited together.

### 5. Simulation Results

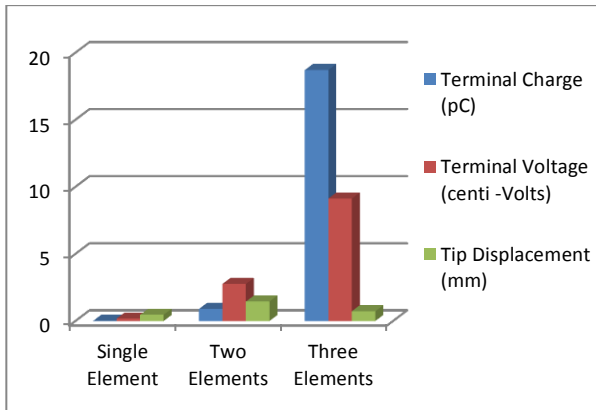
In comparison to a single piezoelectric element, the total stored energy has increased for the two element array by more than 30 times. However, for the three-element array the total stored energy has increased by more than 1800 times. The total stored energy was calculated using the following equation:

$$E = \frac{1}{2} QV$$

Where Q is the accumulated charge (C), V is the open circuit voltage (Volts), and E is total stored energy (J).

**Table 1: Results for the total stored energy**

	Total stored energy
Single element	0.04716 fJ
Two element array	1.54 fJ
Three element array	854.5 fJ



**Figure 6: Comparing charge, voltage and maximum tip displacement for different geometrical structures**

The three element array gave the highest results in terms of output charge, voltage and total amount of stored energy. Figure 6 shows that the stored energy is too high to be comparable to that obtained from the other two structures.

## 6. Conclusion

The proposed array structures in this study provided increased energy output using two different array configurations. When the array structures were excited at their fundamental resonance frequencies, and designed with an optimum spacing between each element, the total energy stored was increased to more than 30 times for the two-element array and to more than 1800 times for the three element array (in comparison to a single element). Further investigations can include the same structure but with different sized cantilevers, which is expected to provide a wider bandwidth of operation. This will permit the use of such array configurations for low as well as for high vibration frequencies and corresponding devices.

## References

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