

$$(\partial_t - ix)\mathbf{b} = -\frac{3}{2\bar{\omega}_A}b_x\hat{y} + i\mathbf{u} \quad (1)$$

$$(\partial_t - ix)\mathbf{u} = \frac{2}{\bar{\omega}_A}u_y\hat{x} - \frac{1}{2\bar{\omega}_A}u_x\hat{y} - \frac{3}{2\bar{\omega}_A}\partial_x Q\hat{x} - iQ\hat{y} - i\frac{1}{\kappa}Q\hat{z} + i\mathbf{b} \quad (2)$$

$$\frac{3}{2}\partial_x u_x + i\bar{\omega}_A u_y + \frac{i\bar{\omega}_A}{\kappa}u_z = 0 \quad (3)$$

This last equation may look a little odd but it is really just $\nabla \cdot \mathbf{b} = 0$ after making the original equations dimensionless.

After putting them in component form:

$$\begin{aligned} \frac{\partial}{\partial t}b_x &= ix b_x + iu_x \\ \frac{\partial}{\partial t}b_y &= ixu_y - \frac{3}{2\bar{\omega}_A}b_x + iu_y \\ \frac{\partial}{\partial t}b_z &= ix b_z + iu_z \\ \frac{\partial}{\partial t}u_x &= ixu_x + \frac{2}{\bar{\omega}_A}u_y - \frac{3}{2\bar{\omega}_A}\frac{\partial Q}{\partial x} + ib_x \\ \frac{\partial}{\partial t}u_y &= ixu_y - \frac{1}{2\bar{\omega}_A}u_x - iQ + ib_y \\ \frac{\partial}{\partial t}u_z &= ixu_z - \frac{i}{\kappa}Q + ib_z \end{aligned}$$

Subscripts x,y and z indicate the x,y and z components of \mathbf{b} and \mathbf{u} .

Q is found by taking the divergence of (2) to get

$$\frac{\partial^2 Q}{\partial x^2} - K^2 Q - F = 0$$

where $K^2 = \frac{4}{3}i\bar{\omega}_A\sqrt{1 + \frac{1}{\kappa^2}}$ and $F = \frac{4}{3}\frac{\partial u_y}{\partial x} + \frac{4i\bar{\omega}_A u_x}{9}$

The initial conditions are

$$\begin{aligned} b_x(x, 0) &= e^{-x^2} \\ b_y(x, 0) &= \frac{3i}{2\bar{\omega}_A}\frac{\partial b_x(x, 0)}{\partial x} \\ b_z(x, 0) &= u_x(x, 0) = u_y(x, 0) = u_z(x, 0) = 0 \end{aligned}$$

With everything going to zero at infinity. As for Q, it should go to zero at + and - infinity as well.